OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS   **4725**
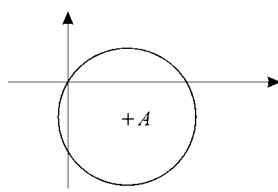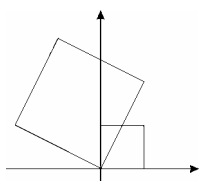
Further Pure Mathematics 1

MARK SCHEME

Specimen Paper

| MAXIMUM MARK | 72 |

This mark scheme consists of 4 printed pages.

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1	$\displaystyle\sum_{r=1}^{n} r(r+1) = \sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r = \tfrac{1}{6}n(n+1)(2n+1) + \tfrac{1}{2}n(n+1)$	M1	For considering the two separate sums
		A1	For either correct sum formula stated
		A1	For completely correct expression
	$= \tfrac{1}{6}n(n+1)(2n+1+3) = \tfrac{1}{3}n(n+1)(n+2)$	M1	For factorising attempt
		A1 **5**	For showing given answer correctly
		5	

2	**(i)**	$(p-q)+p+(p+q) = 6 \Rightarrow p = 2$	M1	For use of $\Sigma\alpha = -b/a$
			A1 **2**	For correct answer
	(ii)	$2(2-q)(2+q) = -10$	B1√	For use of $\alpha\beta\gamma = -d/a$
		Hence $4 - q^2 = -5 \Rightarrow q = 3$	M1	For expanding and solving for q^2
			A1 **3**	For correct answer
	(iii)	*EITHER*: Roots are $-1, 2, 5$	B1√	For stating or using three numerical roots
		$-1\times2 + 2\times5 + -1\times5 = k$	M1	For use of $\Sigma\alpha\beta = c/a$
		i.e. $k = 3$	A1√	For correct answer from their roots
		OR: Roots are $-1, 2, 5$	B1√	For stating or using three numerical roots
		Equation is $(x+1)(x-2)(x-5) = 0$	M1	For stating and expanding factorised form
		Hence $k = 3$	A1√ **3**	For correct answer from their roots
			8	

3	**(i)**	$z^2 = (2+\mathrm{i})^2 = 4 + 4\mathrm{i} + \mathrm{i}^2 = 3 + 4\mathrm{i}$	M1	For showing 3-term or 4-term expansion
			A1 **2**	For correct answer
	(ii)	$4z - z^2 = 8 + 4\mathrm{i} - 3 - 4\mathrm{i} = 5$	B1	For correct value 5
		$zz^* = (2+\mathrm{i})(2-\mathrm{i}) = 5$	B1	For stating or using $z^* = 2 - \mathrm{i}$
			B1 **3**	For correct verification of given restult
	(iii)	$\dfrac{z+1}{z-1} = \dfrac{3+\mathrm{i}}{1+\mathrm{i}} = \dfrac{(3+\mathrm{i})(1-\mathrm{i})}{(1+\mathrm{i})(1-\mathrm{i})} = \dfrac{4-2\mathrm{i}}{2} = 2 - \mathrm{i}$	B1	For correct initial form $\dfrac{3+\mathrm{i}}{1+\mathrm{i}}$
			M1	For multiplying top and bottom by $1-\mathrm{i}$
			A1 **3**	For correct answer $2 - \mathrm{i}$
			8	

4	**(i)**	$u_1 = 8$	B1 **1**	For correct value stated
	(ii)	$3^{2(n+1)} - 1 - (3^{2n} - 1) = 9\times3^{2n} - 3^{2n} = 8\times3^{2n}$	B1	For stating or using $u_{n+1} = 3^{2(n+1)} - 1$
			M1	For relevant manipulation of indices in u_{n+1}
			A1 **3**	For showing given answer correctly
	(iii)	u_1 is divisible by 8, from **(i)**	B1	For explicit check for u_1
		Suppose u_k is divisible by 8, i.e. $u_k = 8a$	M1	For induction hypothesis u_k is mult. of 8
		Then $u_{k+1} = u_k + 8\times3^{2k} = 8(a + 3^{2k}) = 8b$	M1	For obtaining and simplifying expr. for u_{k+1}
		i.e. u_{k+1} is also divisible by 8, and result follows		
		by the induction principle	A1 **4**	For correct conclusion, stated and justified
			8	

5	(i)	$\text{LHS} = \dfrac{2r+1-(2r-1)}{(2r-1)(2r+1)} = \dfrac{2}{4r^2-1} = \text{RHS}$	M1		For correct process for adding fractions
			A1	**2**	For showing given result correctly
	(ii)	$\text{Sum is } \left(\dfrac{1}{1}-\dfrac{1}{3}\right)+\left(\dfrac{1}{3}-\dfrac{1}{5}\right)+\left(\dfrac{1}{5}-\dfrac{1}{7}\right)+\ldots+\left(\dfrac{1}{2n-1}-\dfrac{1}{2n+1}\right)$	M1		For expressing terms as differences using **(i)**
			A1		For at least first two and last terms correct
		$\text{This is } 1-\dfrac{1}{2n+1}$	M1		For cancelling pairs of terms
			A1	**4**	For any correct form
	(iii)	**(a)** Sum to infinity is 1	B1✓	**1**	For correct value; follow their **(ii)** if cnvgt
		(b) Required sum is $\dfrac{1}{2n+1}$	B1✓	**1**	For correct difference of their **(iii)(a)** and **(ii)**
				8	
6	(i)	(See diagram in part **(ii)** below)	B1		For point A correctly located
		$|a| = \sqrt{(3^2+4^2)} = 5$	B1		For correct value for the modulus
		$\arg a = -\tan^{-1}\left(\frac{3}{4}\right) = -0.644$	M1		For any correct relevant trig statement
			A1	**4**	For correct answer (radians or degrees)
	(ii)		B1		For any indication that locus is a circle
			B1		For any indication that the centre is at A
			B1	**3**	For a completely correct diagram
	(iii)	**(a)** $z = -6i$	B1	**1**	For correct answer
		(b) $z = 8 - 6i$	M1		For identification of end of diameter thru A
			A1	**2**	For correct answer
				10	
7	(i)	$\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & -2 & -1 \\ 0 & 2 & 1 & 3 \end{pmatrix}$	M1		For at least one correct image
			A1		For all vertices correct
			A1	**3**	For correct diagram
	(ii)	The area scale-factor is 5	B1		For identifying det as area scale factor
		The transformed square has side of length $\sqrt{5}$	M1		For calculation method relating to large sq.
		So its area is 5 times that of the unit square	A1	**3**	For a complete explanataion
	(iii)	Angle is $\tan^{-1}(2) = 63.4°$	B1		For $\tan^{-1}(2)$, or equivalent
		Enlargement with scale factor $\sqrt{5}$	B1		For stating 'enlargement'
			B1	**3**	For correct (exact) scale factor
	(iv)	$\begin{pmatrix} \dfrac{1}{\sqrt{5}} & -\dfrac{2}{\sqrt{5}} \\ \dfrac{2}{\sqrt{5}} & \dfrac{1}{\sqrt{5}} \end{pmatrix}$	M1		For correct $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ pattern
			A1	**2**	For correct matrix in exact form
				11	

[Turn over

8	(i)	$\det \mathbf{M} = a(3-1) - 2(2-(-2)) - 1(-2-6)$	M1		For correct expansion process
		$= 2a$	A1	**2**	For showing given answer correctly
	(ii)	$\mathbf{M}^{-1} = \dfrac{1}{2a}\begin{pmatrix} 2 & -1 & 1 \\ -4 & a+2 & a-2 \\ -8 & a+4 & 3a-4 \end{pmatrix}$	M1		For correct process for adjoint entries
			A1		For at least 4 correct entries in adjoint
			B1		For dividing by the determinant
			A1	**4**	For completely correct inverse
	(iii)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$, with $a = 1$	B1		For correct statement involving inverse
		So $x = 0$, $y = 1$, $z = 1$	M1		For carrying out the correct multiplication
			A1	**3**	For all three correct values
	(iv)	Eliminating x gives $4y - 2z = 2$	M1		For eliminating x from 2nd and 3rd equns
		So for consistency with 1st equn, $k = 1$	M1		For comparing two y-z equations
			A1	**3**	For correct value for k
	(v)	Solving $x + 3y = 2$, $3x - y = 0$ gives $x = \frac{1}{5}$, $y = \frac{3}{5}$	M1		For using $x = z$ to solve a pair of equns
		These values check in $2y - x = 1$, so soln exists	A1	**2**	For a completely correct demonstration
				14	