OCR
Oxford Cambridge and RSA

# Friday 20 May 2016 – Morning

## AS GCE MATHEMATICS

**4725/01** Further Pure Mathematics 1

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**
- Printed Answer Book 4725/01
- List of Formulae (MF1)

**Other materials required:**
- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

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**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER / INVIGILATOR**
- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

**1** Find $\sum_{r=1}^{n}(3r+1)(r-1)$, giving your answer in a fully factorised form. **[5]**

**2** The complex number $z$ has modulus $2\sqrt{3}$ and argument $-\frac{1}{3}\pi$. Giving your answers in the form $x+\mathrm{i}y$, where $x$ and $y$ are exact real numbers, and showing clearly how you obtain them, find

**(i)** $z$, **[2]**

**(ii)** $\dfrac{1}{(z^*-5\mathrm{i})^2}$. **[5]**

**3** The quadratic equation $kx^2+x+k=0$ has roots $\alpha$ and $\beta$.

**(i)** Write down the values of $\alpha+\beta$ and $\alpha\beta$. **[1]**

**(ii)** Find the value of $\left(\alpha+\frac{1}{\alpha}\right)\left(\beta+\frac{1}{\beta}\right)$ in terms of $k$. **[5]**

**4** The matrices **A**, **B** and **C** are given by $\mathbf{A}=(a\ \ 2\ \ 3)$, $\mathbf{B}=(b\ \ 0\ \ 5)$ and $\mathbf{C}=\begin{pmatrix}6\\4\\-1\end{pmatrix}$. Find

**(i)** $5\mathbf{A}-3\mathbf{B}$, **[2]**

**(ii)** $\mathbf{BC}$, **[2]**

**(iii)** $\mathbf{CA}$. **[2]**

**5** The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1=5 \text{ and } u_{n+1}=3u_n+2 \text{ for } n \geqslant 1.$$

Prove by induction that $u_n=2\times 3^n-1$. **[4]**

**6** In an Argand diagram the points $A$ and $B$ represent the complex numbers $5+4\mathrm{i}$ and $1+2\mathrm{i}$ respectively.

**(i)** Given that $A$ and $B$ are the ends of a diameter of a circle $C$, find the equation of $C$ in complex number form. **[4]**

The perpendicular bisector of $AB$ is denoted by $l$.

**(ii)** Sketch $C$ and $l$ on a single Argand diagram. **[2]**

**(iii)** Find the complex numbers represented by the points of intersection of $C$ and $l$. **[3]**

**7** The matrix $\begin{pmatrix}1&3\\0&1\end{pmatrix}$ represents a transformation P.

**(i)** Describe fully the transformation P. **[2]**

The matrix **M** is given by $\mathbf{M}=\begin{pmatrix}-3&-1\\-1&0\end{pmatrix}$.

**(ii)** Given that **M** represents transformation Q followed by transformation P, find the matrix that represents the transformation Q and describe fully the transformation Q. **[6]**

**8** **(i)** Show that $\frac{1}{2r+1}-\frac{1}{2r+3}\equiv\frac{2}{(2r+1)(2r+3)}$. **[1]**

**(ii)** Hence find $\sum_{r=1}^{n}\frac{1}{(2r+1)(2r+3)}$, giving your answer as a single fraction. **[6]**

**(iii)** Find $\sum_{r=n}^{\infty}\frac{1}{(2r+1)(2r+3)}$, giving your answer as a single fraction. **[3]**

**9** **(i)** The matrix **X** is given by $\mathbf{X}=\begin{pmatrix} a & 3 & -2 \\ 0 & a & 5 \\ 1 & 2 & 1 \end{pmatrix}$. Show that the determinant of **X** is $a^2-8a+15$. **[3]**

**(ii)** Explain briefly why the equations

$$\begin{aligned} 3x+3y-2z &= 1 \\ 3y+5z &= 5 \\ x+2y+\ z &= 2 \end{aligned}$$

do not have a unique solution and determine whether these equations are consistent or inconsistent. **[3]**

**10** **(i)** Use an algebraic method to find the square roots of the complex number $9+40\mathrm{i}$. **[6]**

**(ii)** Show that $9+40\mathrm{i}$ is a root of the quadratic equation $z^2-18z+1681=0$. **[1]**

**(iii)** By using the substitution $z=\frac{1}{u^2}$, find the roots of the equation $1681u^4-18u^2+1=0$. Give your answers in the form $x+\mathrm{i}y$, where $x$ and $y$ are real. **[4]**

**END OF QUESTION PAPER**