

**Thursday 14 May 2015 – Morning**

**AS GCE MATHEMATICS**

**4725/01** Further Pure Mathematics 1

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4725/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



## INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

## INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

## INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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- 1 The complex number  $x + iy$  is denoted by  $z$ . Express  $3zz^* - |z|^2$  in terms of  $x$  and  $y$ . [3]
- 2 Find  $\sum_{r=1}^n (3r^2 - 5)$ , expressing your answer in a fully factorised form. [4]
- 3 The matrix  $\mathbf{A}$  is given by  $\mathbf{A} = \begin{pmatrix} 2 & a \\ 0 & 1 \end{pmatrix}$ , where  $a$  is a constant.
- (i) Find  $\mathbf{A}^{-1}$ . [2]
- The matrix  $\mathbf{B}$  is given by  $\mathbf{B} = \begin{pmatrix} 2 & a \\ 4 & 1 \end{pmatrix}$ .
- (ii) Given that  $\mathbf{PA} = \mathbf{B}$ , find the matrix  $\mathbf{P}$ . [3]
- 4 Prove by induction that, for  $n \geq 1$ ,  $\sum_{r=1}^n r(3r+1) = n(n+1)^2$ . [5]
- 5 The loci  $C_1$  and  $C_2$  are given by  $|z+2| = 2$  and  $\arg(z+2) = \frac{5}{6}\pi$  respectively.
- (i) Sketch, on a single Argand diagram, the loci  $C_1$  and  $C_2$ . [4]
- (ii) Find the complex number represented by the intersection of  $C_1$  and  $C_2$ . [2]
- (iii) Indicate, by shading, the region of the Argand diagram for which
- $$|z+2| \leq 2 \text{ and } \frac{5}{6}\pi \leq \arg(z+2) \leq \pi. \quad [2]$$
- 6 The matrix  $\mathbf{M}$  is given by  $\mathbf{M} = \begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix}$ .
- (i) The diagram in the Printed Answer Book shows the unit square  $OABC$ . The image of the unit square under the transformation represented by  $\mathbf{M}$  is  $OA'B'C'$ . Draw and label  $OA'B'C'$ , indicating clearly the coordinates of  $A'$ ,  $B'$  and  $C'$ . [3]
- (ii) The transformation represented by  $\mathbf{M}$  is equivalent to a transformation  $P$  followed by a transformation  $Q$ . Give geometrical descriptions of a possible pair of transformations  $P$  and  $Q$  and state the matrices that represent them. [4]
- 7 (i) Use an algebraic method to find the square roots of the complex number  $5 + 12i$ . You must show sufficient working to justify your answers. [5]
- (ii) Hence solve the quadratic equation  $x^2 - 4x - 1 - 12i = 0$ . [5]
- 8 (i) Show that  $\frac{3}{r-1} - \frac{2}{r} - \frac{1}{r+1} \equiv \frac{4r+2}{r(r^2-1)}$ . [2]
- (ii) Hence find an expression, in terms of  $n$ , for  $\sum_{r=2}^n \frac{4r+2}{r(r^2-1)}$ . [6]
- (iii) Hence find the value of  $\sum_{r=4}^{\infty} \frac{4r+2}{r(r^2-1)}$ . [2]

9 The matrix  $\mathbf{D}$  is given by  $\mathbf{D} = \begin{pmatrix} 1 & 3 & 4 \\ 2 & a & 3 \\ 0 & 1 & a \end{pmatrix}$ .

(i) Find the values of  $a$  for which  $\mathbf{D}$  is singular.

[6]

(ii) Three simultaneous equations are shown below.

$$\begin{aligned}x + 3y + 4z &= 3 \\2x + ay + 3z &= 2 \\y + az &= 0\end{aligned}$$

For each of the following values of  $a$ , determine whether or not there is a unique solution. If a unique solution does not exist, determine whether the equations are consistent or inconsistent.

(a)  $a = 3$

(b)  $a = 1$

[4]

10 The cubic equation  $x^3 + 4x + 3 = 0$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ .

(i) Use the substitution  $x = \sqrt{u}$  to obtain a cubic equation in  $u$ .

[3]

(ii) Find the value of  $\alpha^4 + \beta^4 + \gamma^4 + \alpha\beta\gamma$ .

[7]

**END OF QUESTION PAPER**

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