OCR
Oxford Cambridge and RSA

Thursday 14 May 2015 – Morning

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The complex number $x+\mathrm{i}y$ is denoted by z. Express $3zz^*-|z|^2$ in terms of x and y. **[3]**

2 Find $\sum_{r=1}^{n}(3r^2-5)$, expressing your answer in a fully factorised form. **[4]**

3 The matrix **A** is given by $\mathbf{A}=\begin{pmatrix}2 & a\\ 0 & 1\end{pmatrix}$, where a is a constant.

(i) Find $\mathbf{A}^{-1}$. **[2]**

The matrix **B** is given by $\mathbf{B}=\begin{pmatrix}2 & a\\ 4 & 1\end{pmatrix}$.

(ii) Given that $\mathbf{PA}=\mathbf{B}$, find the matrix **P**. **[3]**

4 Prove by induction that, for $n \geqslant 1$, $\sum_{r=1}^{n} r(3r+1)=n(n+1)^2$. **[5]**

5 The loci C_1 and C_2 are given by $|z+2|=2$ and $\arg(z+2)=\frac{5}{6}\pi$ respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. **[4]**

(ii) Find the complex number represented by the intersection of C_1 and C_2. **[2]**

(iii) Indicate, by shading, the region of the Argand diagram for which

$$|z+2| \leqslant 2 \text{ and } \tfrac{5}{6}\pi \leqslant \arg(z+2) \leqslant \pi.$$ **[2]**

6 The matrix **M** is given by $\mathbf{M}=\begin{pmatrix}0 & 2\\ -1 & 0\end{pmatrix}$.

(i) The diagram in the Printed Answer Book shows the unit square $OABC$. The image of the unit square under the transformation represented by **M** is $OA'B'C'$. Draw and label $OA'B'C'$, indicating clearly the coordinates of A', B' and C'. **[3]**

(ii) The transformation represented by **M** is equivalent to a transformation P followed by a transformation Q. Give geometrical descriptions of a possible pair of transformations P and Q and state the matrices that represent them. **[4]**

7 **(i)** Use an algebraic method to find the square roots of the complex number $5+12\mathrm{i}$. You must show sufficient working to justify your answers. **[5]**

(ii) Hence solve the quadratic equation $x^2-4x-1-12\mathrm{i}=0$. **[5]**

8 **(i)** Show that $\frac{3}{r-1}-\frac{2}{r}-\frac{1}{r+1} \equiv \frac{4r+2}{r(r^2-1)}$. **[2]**

(ii) Hence find an expression, in terms of n, for $\sum_{r=2}^{n}\frac{4r+2}{r(r^2-1)}$. **[6]**

(iii) Hence find the value of $\sum_{r=4}^{\infty}\frac{4r+2}{r(r^2-1)}$. **[2]**

9 The matrix **D** is given by $\mathbf{D} = \begin{pmatrix} 1 & 3 & 4 \\ 2 & a & 3 \\ 0 & 1 & a \end{pmatrix}$.

(i) Find the values of a for which **D** is singular. **[6]**

(ii) Three simultaneous equations are shown below.

$$\begin{aligned} x + 3y + 4z &= 3 \\ 2x + ay + 3z &= 2 \\ y + az &= 0 \end{aligned}$$

For each of the following values of a, determine whether or not there is a unique solution. If a unique solution does not exist, determine whether the equations are consistent or inconsistent.

(a) $a = 3$

(b) $a = 1$

[4]

10 The cubic equation $x^3 + 4x + 3 = 0$ has roots α, β and γ.

(i) Use the substitution $x = \sqrt{u}$ to obtain a cubic equation in u. **[3]**

(ii) Find the value of $\alpha^4 + \beta^4 + \gamma^4 + \alpha\beta\gamma$. **[7]**

END OF QUESTION PAPER