

**Friday 16 May 2014 – Afternoon**

**AS GCE MATHEMATICS**

**4725/01** Further Pure Mathematics 1

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4725/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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- 1 Find the determinant of the matrix  $\begin{pmatrix} a & 4 & -1 \\ 3 & a & 2 \\ a & 1 & 1 \end{pmatrix}$ . [3]
- 2 The complex number  $7 + 3i$  is denoted by  $z$ . Find
- (i)  $|z|$  and  $\arg z$ , [2]
- (ii)  $\frac{z}{4-i}$ , showing clearly how you obtain your answer. [3]
- 3 The matrices  $\mathbf{A}$  and  $\mathbf{B}$  are given by  $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -4 & 5 \end{pmatrix}$ ,  $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ 2 & 3 \end{pmatrix}$  and  $\mathbf{I}$  is the  $2 \times 2$  identity matrix. Find
- (i)  $4\mathbf{A} - \mathbf{B} + 2\mathbf{I}$ , [2]
- (ii)  $\mathbf{A}^{-1}$ , [2]
- (iii)  $(\mathbf{AB}^{-1})^{-1}$ . [3]
- 4 (a) Find the matrix that represents a shear with the  $y$ -axis invariant, the image of the point  $(1, 0)$  being the point  $(1, 4)$ . [2]
- (b) The matrix  $\mathbf{X}$  is given by  $\mathbf{X} = \begin{pmatrix} \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix}$ .
- (i) Describe fully the geometrical transformation represented by  $\mathbf{X}$ . [2]
- (ii) Find the value of the determinant of  $\mathbf{X}$  and describe briefly how this value relates to the transformation represented by  $\mathbf{X}$ . [2]
- 5 The cubic equation  $2x^3 + 3x + 3 = 0$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ .
- (i) Use the substitution  $x = u + 2$  to find a cubic equation in  $u$ . [3]
- (ii) Hence find the value of  $\frac{1}{\alpha-2} + \frac{1}{\beta-2} + \frac{1}{\gamma-2}$ . [4]
- 6 (i) Show that  $\frac{1}{r^2} - \frac{1}{(r+2)^2} \equiv \frac{4(r+1)}{r^2(r+2)^2}$ . [2]
- (ii) Hence find an expression, in terms of  $n$ , for  $\sum_{r=1}^n \frac{4(r+1)}{r^2(r+2)^2}$ . [6]
- (iii) Find  $\sum_{r=5}^{\infty} \frac{4(r+1)}{r^2(r+2)^2}$ , giving your answer in the form  $\frac{p}{q}$  where  $p$  and  $q$  are integers. [2]

- 7 The loci  $C_1$  and  $C_2$  are given by  $\arg(z-2-2i) = \frac{1}{4}\pi$  and  $|z| = |z-10|$  respectively.
- (i) Sketch on a single Argand diagram the loci  $C_1$  and  $C_2$ . [4]
- (ii) Indicate, by shading, the region of the Argand diagram for which
- $$0 \leq \arg(z-2-2i) \leq \frac{1}{4}\pi \text{ and } |z| \geq |z-10|. \quad [3]$$
- 8 (i) Show that  $\sum_{r=n}^{2n} r^3 = \frac{3}{4}n^2(n+1)(5n+1)$ . [4]
- (ii) Hence find  $\sum_{r=n}^{2n} r(r^2-2)$ , giving your answer in a fully factorised form. [5]
- 9 The roots of the equation  $x^3 - kx^2 - 2 = 0$  are  $\alpha$ ,  $\beta$  and  $\gamma$ , where  $\alpha$  is real and  $\beta$  and  $\gamma$  are complex.
- (i) Show that  $k = \alpha - \frac{2}{\alpha^2}$ . [2]
- (ii) Given that  $\beta = u + iv$ , where  $u$  and  $v$  are real, find  $u$  in terms of  $\alpha$ . [4]
- (iii) Find  $v^2$  in terms of  $\alpha$ . [4]
- 10 The sequence  $u_1, u_2, u_3, \dots$  is defined by  $u_n = 5^n + 2^{n-1}$ .
- (i) Find  $u_1, u_2$  and  $u_3$ . [2]
- (ii) Hence suggest a positive integer, other than 1, which divides exactly into every term of the sequence. [1]
- (iii) By considering  $u_{n+1} + u_n$ , prove by induction that your suggestion in part (ii) is correct. [5]

**END OF QUESTION PAPER**

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