

Monday 10 June 2013 – Morning

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

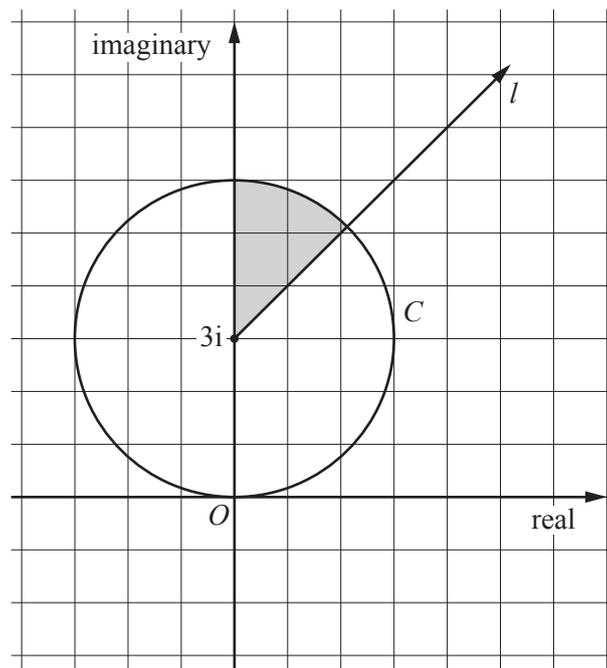
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- 1 The complex number $3 + ai$, where a is real, is denoted by z . Given that $\arg z = \frac{1}{6}\pi$, find the value of a and hence find $|z|$ and $z^* - 3$. [6]
- 2 The matrices \mathbf{A} , \mathbf{B} and \mathbf{C} are given by $\mathbf{A} = \begin{pmatrix} 5 & 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & -5 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.
- (i) Find $3\mathbf{A} - 4\mathbf{B}$. [2]
- (ii) Find \mathbf{CB} . Determine whether \mathbf{CB} is singular or non-singular, giving a reason for your answer. [5]
- 3 Use an algebraic method to find the square roots of $11 + (12\sqrt{5})i$. Give your answers in the form $x + iy$, where x and y are exact real numbers. [6]
- 4 The matrix \mathbf{M} is given by $\mathbf{M} = \begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}$. Prove by induction that, for $n \geq 1$,

$$\mathbf{M}^n = \begin{pmatrix} 2^n & 2^{n+1} - 2 \\ 0 & 1 \end{pmatrix}. \quad [6]$$

- 5 Find $\sum_{r=1}^n (4r^3 - 3r^2 + r)$, giving your answer in a fully factorised form. [6]

6



The Argand diagram above shows a half-line l and a circle C . The circle has centre $3i$ and passes through the origin.

- (i) Write down, in complex number form, the equations of l and C . [4]
- (ii) Write down inequalities that define the region shaded in the diagram. [The shaded region includes the boundaries.] [3]

- 7 (i) Find the matrix that represents a rotation through 90° clockwise about the origin. [2]
- (ii) Find the matrix that represents a reflection in the x -axis. [2]
- (iii) Hence find the matrix that represents a rotation through 90° clockwise about the origin, followed by a reflection in the x -axis. [2]
- (iv) Describe a **single** transformation that is represented by your answer to part (iii). [2]

- 8 The cubic equation $kx^3 + 6x^2 + x - 3 = 0$, where k is a non-zero constant, has roots α , β and γ .
Find the value of $(\alpha + 1)(\beta + 1) + (\beta + 1)(\gamma + 1) + (\gamma + 1)(\alpha + 1)$ in terms of k . [6]

- 9 (i) Show that $\frac{1}{3r-1} - \frac{1}{3r+2} \equiv \frac{3}{(3r-1)(3r+2)}$. [2]

- (ii) Hence show that $\sum_{r=1}^{2n} \frac{1}{(3r-1)(3r+2)} = \frac{n}{2(3n+1)}$. [6]

- 10 The matrix \mathbf{A} is given by $\mathbf{A} = \begin{pmatrix} a & 2 & 1 \\ 1 & 3 & 2 \\ 4 & 1 & 1 \end{pmatrix}$.

- (i) Find the value of a for which \mathbf{A} is singular. [5]
- (ii) Given that \mathbf{A} is non-singular, find \mathbf{A}^{-1} and hence solve the equations

$$\begin{aligned} ax + 2y + z &= 1, \\ x + 3y + 2z &= 2, \\ 4x + y + z &= 3. \end{aligned}$$

[7]

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE



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