

Friday 1 June 2012 – Morning

**AS GCE MATHEMATICS**

**4725** Further Pure Mathematics 1

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4725
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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- 1 The complex numbers  $z$  and  $w$  are given by  $z = 6 - i$  and  $w = 5 + 4i$ . Giving your answers in the form  $x + iy$  and showing clearly how you obtain them, find
- (i)  $z + 3w$ , [2]
- (ii)  $\frac{z}{w}$ . [3]
- 2 The matrices  $\mathbf{A}$  and  $\mathbf{B}$  are given by  $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$ . Find
- (i)  $\mathbf{AB}$ , [2]
- (ii)  $\mathbf{B}^{-1}\mathbf{A}^{-1}$ . [3]
- 3 One root of the quadratic equation  $x^2 + ax + b = 0$ , where  $a$  and  $b$  are real, is the complex number  $4 - 3i$ . Find the values of  $a$  and  $b$ . [4]
- 4 Find  $\sum_{r=1}^n (3r^2 - 3r + 2)$ , expressing your answer in a fully factorised form. [7]
- 5 Prove by induction that, for  $n \geq 1$ ,  $\sum_{r=1}^n 4 \times 3^r = 6(3^n - 1)$ . [5]
- 6 The quadratic equation  $2x^2 + x + 5 = 0$  has roots  $\alpha$  and  $\beta$ .
- (i) Use the substitution  $x = \frac{1}{u+1}$  to obtain a quadratic equation in  $u$  with integer coefficients. [3]
- (ii) Hence, or otherwise, find the value of  $\left(\frac{1}{\alpha} - 1\right)\left(\frac{1}{\beta} - 1\right)$ . [3]
- 7 The loci  $C_1$  and  $C_2$  are given by  $|z - 3 - 4i| = 4$  and  $|z| = |z - 8i|$  respectively.
- (i) Sketch, on a single Argand diagram, the loci  $C_1$  and  $C_2$ . [6]
- (ii) Hence find the complex numbers represented by the points of intersection of  $C_1$  and  $C_2$ . [2]
- (iii) Indicate, by shading, the region of the Argand diagram for which
- $$|z - 3 - 4i| \leq 4 \text{ and } |z| \geq |z - 8i|. \quad [2]$$
- 8 (i) Show that  $\frac{1}{r} - \frac{1}{r+2} \equiv \frac{2}{r(r+2)}$ . [1]
- (ii) Hence find an expression, in terms of  $n$ , for  $\sum_{r=1}^n \frac{2}{r(r+2)}$ . [6]
- (iii) Given that  $\sum_{r=N+1}^{\infty} \frac{2}{r(r+2)} = \frac{11}{30}$ , find the value of  $N$ . [4]

- 9 (i) The matrix  $\mathbf{X}$  is given by  $\mathbf{X} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ . Describe fully the geometrical transformation represented by  $\mathbf{X}$ . [2]

- (ii) The matrix  $\mathbf{Z}$  is given by  $\mathbf{Z} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2}(2 + \sqrt{3}) \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2}(1 - 2\sqrt{3}) \end{pmatrix}$ . The transformation represented by  $\mathbf{Z}$  is

equivalent to the transformation represented by  $\mathbf{X}$ , followed by another transformation represented by the matrix  $\mathbf{Y}$ . Find  $\mathbf{Y}$ . [5]

- (iii) Describe fully the geometrical transformation represented by  $\mathbf{Y}$ . [2]

- 10 The matrix  $\mathbf{D}$  is given by  $\mathbf{D} = \begin{pmatrix} a & 2 & -1 \\ 2 & a & 1 \\ 1 & 1 & a \end{pmatrix}$ .

- (i) Find the determinant of  $\mathbf{D}$  in terms of  $a$ . [3]

- (ii) Three simultaneous equations are shown below.

$$ax + 2y - z = 0$$

$$2x + ay + z = a$$

$$x + y + az = a$$

For each of the following values of  $a$ , determine whether or not there is a unique solution. If the solution is not unique, determine whether the equations are consistent or inconsistent.

(a)  $a = 3$

(b)  $a = 2$

(c)  $a = 0$

[7]

**THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE**



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