

**ADVANCED SUBSIDIARY GCE
MATHEMATICS**

4725/01

Further Pure Mathematics 1

MONDAY 2 JUNE 2008

Morning
Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of 4 printed pages.

1 The matrix \mathbf{A} is given by $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 5 & 2 \end{pmatrix}$ and \mathbf{I} is the 2×2 identity matrix. Find

(i) $\mathbf{A} - 3\mathbf{I}$, [2]

(ii) \mathbf{A}^{-1} . [2]

2 The complex number $3 + 4i$ is denoted by a .

(i) Find $|a|$ and $\arg a$. [2]

(ii) Sketch on a single Argand diagram the loci given by

(a) $|z - a| = |a|$, [2]

(b) $\arg(z - 3) = \arg a$. [3]

3 (i) Show that $\frac{1}{r!} - \frac{1}{(r+1)!} = \frac{r}{(r+1)!}$. [2]

(ii) Hence find an expression, in terms of n , for

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{n}{(n+1)!}. \quad [4]$$

4 The matrix \mathbf{A} is given by $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$. Prove by induction that, for $n \geq 1$,

$$\mathbf{A}^n = \begin{pmatrix} 3^n & \frac{1}{2}(3^n - 1) \\ 0 & 1 \end{pmatrix}. \quad [6]$$

5 Find $\sum_{r=1}^n r^2(r-1)$, expressing your answer in a fully factorised form. [6]

6 The cubic equation $x^3 + ax^2 + bx + c = 0$, where a , b and c are real, has roots $(3 + i)$ and 2 .

(i) Write down the other root of the equation. [1]

(ii) Find the values of a , b and c . [6]

7 Describe fully the geometrical transformation represented by each of the following matrices:

(i) $\begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$, [1]

(ii) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, [2]

(iii) $\begin{pmatrix} 1 & 0 \\ 0 & 6 \end{pmatrix}$, [2]

(iv) $\begin{pmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix}$. [2]

8 The quadratic equation $x^2 + kx + 2k = 0$, where k is a non-zero constant, has roots α and β . Find a quadratic equation with roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [7]

9 (i) Use an algebraic method to find the square roots of the complex number $5 + 12i$. [5]

(ii) Find $(3 - 2i)^2$. [2]

(iii) Hence solve the quartic equation $x^4 - 10x^2 + 169 = 0$. [4]

10 The matrix \mathbf{A} is given by $\mathbf{A} = \begin{pmatrix} a & 8 & 10 \\ 2 & 1 & 2 \\ 4 & 3 & 6 \end{pmatrix}$. The matrix \mathbf{B} is such that $\mathbf{AB} = \begin{pmatrix} a & 6 & 1 \\ 1 & 1 & 0 \\ 1 & 3 & 0 \end{pmatrix}$.

(i) Show that \mathbf{AB} is non-singular. [2]

(ii) Find $(\mathbf{AB})^{-1}$. [4]

(iii) Find \mathbf{B}^{-1} . [5]

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