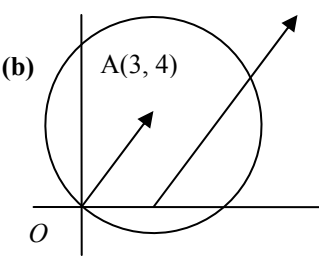


4725 Further Pure Mathematics 1

<p>1 (i) $\begin{pmatrix} 1 & 1 \\ 5 & -1 \end{pmatrix}$</p>	<p>B1 Two elements correct B1 All four elements correct 2</p>
<p>(ii) EITHER</p> $\frac{1}{3} \begin{pmatrix} 2 & -1 \\ -5 & 4 \end{pmatrix}$ <p>OR</p>	<p>B1 Both diagonals correct B1 Divide by determinant 2 B1 Solve sim. eqns. 1st column correct B1 2nd column correct</p>
<p>2 (i) 5 0.927 or 53.1°</p>	<p>B1 Correct modulus B1 Correct argument, any equivalent form 2</p>
<p>(ii)(a)</p> <p>(b) </p>	<p>B1 Circle centre A (3, 4) B1 Through O, allow if centre is (4, 3) 2 B1 Half line with +ve slope B1 Starting at (3, 0) B1 Parallel to OA, (implied by correct arg shown) 3</p>
<p>3 (i) $\frac{r}{(r+1)!}$</p>	<p>M1 Common denominator of $(r+1)!$ or $r!(r+1)!$ A1 Obtain given answer correctly 2</p>
<p>(ii) $1 - \frac{1}{(n+1)!}$</p>	<p>M1 Express terms as differences using (i) A1 At least 1st two and last term correct M1 Show pairs cancelling A1 Correct answer a.e.f. 4</p>
<p>4</p>	<p>B1 Establish result is true, for $n = 1$ (or 2 or 3) M1 Attempt to multiply \mathbf{A} and \mathbf{A}^n, or vice versa M1 Correct process for matrix multiplication A1 Obtain 3^{n+1}, 0 and 1 A1 Obtain $\frac{1}{2}(3^{n+1} - 1)$ A1 Statement of Induction conclusion, only if 5 marks earned, but may be in body of working 6</p>

5		M1 Express as difference of two series M1 Use standard results
	$\frac{1}{4}n^2(n+1)^2 - \frac{1}{6}n(n+1)(2n+1)$	A1 Correct unsimplified answer
	$\frac{1}{12}n(n+1)(3n+2)(n-1)$	M1 Attempt to factorise A1 At least factor of $n(n+1)$ A1 Obtain correct answer
		6
6 (i)	$3 - i$	B1 Conjugate stated 1
(ii)	<i>EITHER</i>	M1 Use sum of roots A1 Obtain correct answer M1 Use sum of pairs of roots A1 Obtain correct answer M1 Use product of roots A1 Obtain correct answers 6
	$a = -8, b = 22, c = -20$	M1 Attempt to find a quadratic factor A1 Obtain correct factor M1 Expand linear and quadratic factors A1A1A1 Obtain correct answers
	<i>OR</i>	M1 Substitute 1 imaginary & the real root into eqn M1 Equate real and imaginary parts M1 Attempt to solve 3 eqns. A1A1A1 Obtain correct answers
	$a = -8, b = 22, c = -20$	
	<i>OR</i>	
	$a = -8, b = 22, c = -20$	
7 (i)		B1 Enlargement (centre O) scale factor 6 1
(ii)		B1 Reflection B1 Mirror line is $y = x$ 2
(iii)		B1 Stretch in y direction B1 Scale factor 6, must be a stretch 2
(iv)		B1 Rotation B1 36.9° clockwise or equivalent 2

8	$\alpha + \beta = -k$ $\alpha\beta = 2k$	B1 State or use correct value B1 State or use correct value M1 Attempt to express sum of new roots in terms of $\alpha + \beta$, $\alpha\beta$ A1 Obtain correct expression A1 Obtain correct answer a.e.f. B1 Correct product of new roots seen B1ft Obtain correct answer, must be an eqn.
	$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$ $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{1}{2}(k - 4)$ $\alpha'\beta' = 1$ $x^2 - \frac{1}{2}(k - 4)x + 1 = 0$	<div style="border: 1px solid black; width: 20px; height: 20px; display: flex; align-items: center; justify-content: center; margin: 5px 0;">7</div> <p style="margin-left: 40px;">Alternative for last 5 marks</p> M1 Obtain expression for $u = \frac{\alpha}{\beta}$ in terms of k and α or k and β A1 Obtain a correct expression A1 rearrange to get α in terms of u M1 Substitute into given equation A1 Obtain correct answer
9 (i)	$x^2 - y^2 = 5$ and $xy = 6$ $\pm(3 + 2i)$	M1 Attempt to equate real and imaginary parts of $(x + iy)^2$ and $5 + 12i$ A1 Obtain both results M1 Eliminate to obtain a quadratic in x^2 or y^2 M1 Solve a 3 term quadratic & obtain x or y A1 Obtain correct answers as complex nos.
(ii)	$5 - 12i$	B1B1 Correct real and imaginary parts <div style="border: 1px solid black; width: 20px; height: 20px; display: flex; align-items: center; justify-content: center; margin: 5px 0;">2</div>
(iii)	$x^2 = 5 \pm 12i$ $x = \pm(3 \pm 2i)$	M1 Attempt to solve a quadratic equation A1 Obtain correct answers A1A1 Each pair of correct answers a.e.f.
		<div style="border: 1px solid black; width: 20px; height: 20px; display: flex; align-items: center; justify-content: center; margin: 5px 0;">4</div>

10 (i)

M1 Find value of det **AB****A1** Correct value 2 seen**2**

(ii)

M1 Show correct process for adjoint entries**A1** Obtain at least 4 correct entries in adjoint**B1** Divide by their determinant

$$(\mathbf{AB})^{-1} = \frac{1}{2} \begin{pmatrix} 0 & 3 & -1 \\ 0 & -1 & 1 \\ 2 & 6-3a & a-6 \end{pmatrix}$$

A1 Obtain completely correct answer**4**

(iii) EITHER

M1 State or imply $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ **A1** Obtain $\mathbf{B}^{-1} = (\mathbf{AB})^{-1} \times \mathbf{A}$ **M1** Correct multiplication process seen**A1** Obtain three correct elements

$$\mathbf{B}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ -6 & 2 & -2 \end{pmatrix}$$

A1 All elements correct**5**

OR

M1 Attempt to find elements of **B****A1** All correct**M1** Correct process for \mathbf{B}^{-1} **A1** 3 elements correct**A1** All elements correct