RECOGNISING ACHIEVEMENT

**ADVANCED SUBSIDIARY GCE UNIT** **4725/01**

## MATHEMATICS

Further Pure Mathematics 1

**MONDAY 11 JUNE 2007** Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

**ADVICE TO CANDIDATES**

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

**1** The complex number $a + \mathrm{i}b$ is denoted by $z$. Given that $|z| = 4$ and $\arg z = \frac{1}{3}\pi$, find $a$ and $b$. [4]

**2** Prove by induction that, for $n \geqslant 1$, $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$. [5]

**3** Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers $n$,

$$\sum_{r=1}^{n}(3r^2 - 3r + 1) = n^3.$$ [6]

**4** The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 3 & 5 \end{pmatrix}$.

**(i)** Find $\mathbf{A}^{-1}$. [2]

The matrix $\mathbf{B}^{-1}$ is given by $\mathbf{B}^{-1} = \begin{pmatrix} 1 & 1 \\ 4 & -1 \end{pmatrix}$.

**(ii)** Find $(\mathbf{AB})^{-1}$. [4]

**5** **(i)** Show that

$$\frac{1}{r} - \frac{1}{r+1} = \frac{1}{r(r+1)}.$$ [1]

**(ii)** Hence find an expression, in terms of $n$, for

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots + \frac{1}{n(n+1)}.$$ [3]

**(iii)** Hence find the value of $\displaystyle\sum_{r=n+1}^{\infty} \frac{1}{r(r+1)}$. [3]

**6** The cubic equation $3x^3 - 9x^2 + 6x + 2 = 0$ has roots $\alpha$, $\beta$ and $\gamma$.

**(i)** **(a)** Write down the values of $\alpha + \beta + \gamma$ and $\alpha\beta + \beta\gamma + \gamma\alpha$. [2]

**(b)** Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [2]

**(ii)** **(a)** Use the substitution $x = \dfrac{1}{u}$ to find a cubic equation in $u$ with integer coefficients. [2]

**(b)** Use your answer to part **(ii)** **(a)** to find the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$. [2]

**7** The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} a & 4 & 0 \\ 0 & a & 4 \\ 2 & 3 & 1 \end{pmatrix}$.

**(i)** Find, in terms of $a$, the determinant of $\mathbf{M}$. [3]

**(ii)** In the case when $a = 2$, state whether $\mathbf{M}$ is singular or non-singular, justifying your answer. [2]

**(iii)** In the case when $a = 4$, determine whether the simultaneous equations

$$\begin{aligned} ax + 4y \quad\quad &= 6, \\ ay + 4z &= 8, \\ 2x + 3y + \ z &= 1, \end{aligned}$$

have any solutions. [3]

**8** The loci $C_1$ and $C_2$ are given by $|z - 3| = 3$ and $\arg(z - 1) = \frac{1}{4}\pi$ respectively.

**(i)** Sketch, on a single Argand diagram, the loci $C_1$ and $C_2$. [6]

**(ii)** Indicate, by shading, the region of the Argand diagram for which

$$|z - 3| \leqslant 3 \quad \text{and} \quad 0 \leqslant \arg(z - 1) \leqslant \tfrac{1}{4}\pi.$$ [2]

**9** **(i)** Write down the matrix, $\mathbf{A}$, that represents an enlargement, centre $(0, 0)$, with scale factor $\sqrt{2}$. [1]

**(ii)** The matrix $\mathbf{B}$ is given by $\mathbf{B} = \begin{pmatrix} \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix}$. Describe fully the geometrical transformation represented by $\mathbf{B}$. [3]

**(iii)** Given that $\mathbf{C} = \mathbf{AB}$, show that $\mathbf{C} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$. [1]

**(iv)** Draw a diagram showing the unit square and its image under the transformation represented by $\mathbf{C}$. [2]

**(v)** Write down the determinant of $\mathbf{C}$ and explain briefly how this value relates to the transformation represented by $\mathbf{C}$. [2]

**10** **(i)** Use an algebraic method to find the square roots of the complex number $16 + 30\mathrm{i}$. [6]

**(ii)** Use your answers to part **(i)** to solve the equation $z^2 - 2z - (15 + 30\mathrm{i}) = 0$, giving your answers in the form $x + \mathrm{i}y$. [5]