Mark Scheme 4725
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1.	i) $\begin{pmatrix} 7 & 4 \\ 0 & -1 \end{pmatrix}$	B1 B1	 2	Two elements correct All four elements correct
	(ii) $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	B1		**A** – **B** correctly found
	$k = 3$	B1	2 4	Find k
2	(i)	M1 A1	 2	For 2 other correct vertices For completely correct diagram
	(ii) $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$	B1 B1	2 4	Each column correct
3.	(i) $2 + 3i$	B1	1	Conjugate seen
	(ii)	M1		Attempt to sum roots or consider x terms in expansion or substitute $2 - 3i$ into equation and equate imaginary parts
	$p = -4$	A1 M1 A1		Correct answer
	$q = 13$		4 5	Attempt at product of roots or consider last term in expansion or consider real parts Correct answer

4.	$\Sigma r^3 + \Sigma r^2$	M1		Consider the sum as two separate parts
	$\Sigma r^2 = \frac{1}{6}n(n+1)(2n+1)$	A1		Correct formula stated
	$\Sigma r^3 = \frac{1}{4}n^2(n+1)^2$	A1		Correct formula stated
	$\frac{1}{12}n(n+1)(n+2)(3n+1)$	M1		Attempt to factorise and simplify or expand both expressions
		A1	5	Obtain given answer correctly or complete verification
			5	
5.	(i) -7i	B1 B1	2	Real part correct Imaginary part correct
	(ii) 2 + 3i	B1		iz stated or implied or $i^2 = -1$ seen
		B1		Real part correct
	-5 + 12i	B1	3	Imaginary part correct
	(iii) $\frac{1}{5}(4 - 7i)$ or equivalent	M1		Multiply by conjugate
		A1		Real part correct
		A1	3	Imaginary part correct
			8	**N.B. Working must be shown**
6..	(i) Circle , Centre *O* radius 2	B1 B1		Sketch showing correct features
	One straight line	B1		
	Through *O* with +ve slope	B1		
	In 1st quadrant only	B1	5	
	(ii) $1 + i\sqrt{3}$	M1		Attempt to find intersections by trig, solving equations or from graph
		A1	2	Correct answer stated as complex number
			7	

7.	(i)	M1		Attempt at matrix multiplication
	$\mathbf{A}^2 = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$ $\mathbf{A}^3 = \begin{pmatrix} 8 & 0 \\ 0 & 1 \end{pmatrix}$	A1 A1	3	Correct $\mathbf{A}^2$ Correct $\mathbf{A}^3$
	(ii) $\mathbf{A}^n = \begin{pmatrix} 2^n & 0 \\ 0 & 1 \end{pmatrix}$	B1	1	Sensible conjecture made
	(iii)	B1 M1 A1 A1	4 **8**	State that conjecture is true for $n = 1$ or 2 Attempt to multiply $\mathbf{A}^n$ and $\mathbf{A}$ or vice versa Obtain correct matrix Statement of induction conclusion
8.	(i)	M1		Correct expansion process shown
	$a\begin{bmatrix} a & 0 \\ 2 & 1 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + 2\begin{bmatrix} 1 & a \\ 1 & 2 \end{bmatrix}$	A1		Obtain correct unsimplified expression
	$a^2 - 2a$	A1	3	Obtain correct answer
	(ii)	M1		Solve their det $\mathbf{M} = 0$
	$a = 0$ or $a = 2$	A1A1ft	3	Obtain correct answers
	(iii) (a)	B1 B1		Solution, as inverse matrix exists or $\mathbf{M}$ non-singular or det$\mathbf{M} \neq 0$
	(b)	B1 B1	4	Solutions, eqn. 1 is multiple of eqn 3
			10	

9.				
	(i)	M1 A1		Show that terms cancel in pairs Obtain given answer correctly
	(ii)	M1 A1		Attempt to expand and simplify Obtain given answer correctly
	(iii)	B1 B1 M1 M1 A1		Correct Σr stated $\Sigma\ 1 = n$ Consider sum of three separate terms on RHS Required sum is LHS – two terms Correct unsimplified expression
	$(n + 1)^3 - 1 - \frac{3}{2}n(n + 1) - n$			
	$\frac{1}{2}n(n + 1)(2n + 1)$	A1	2	Obtain given answer correctly
			2	
			6	
			10	

10	(i) $\alpha + \beta + \gamma = 2$ $\quad \alpha\beta\gamma = -4$	B1 B1		Write down correct values
.	$\alpha\beta + \beta\gamma + \gamma\alpha = 3$	B1	3	
	(ii)	M1		Sum new roots
	$\alpha + 1 + \beta + 1 + \gamma + 1 = 5$	A1ft		Obtain numeric value using their (i)
	$p = -5$	A1ft	3	p is negative of their answer
	(iii)	M1*		Expand three brackets
		A1		$\alpha\beta\gamma + \alpha\beta + \beta\gamma + \gamma\alpha + \alpha + \beta + \gamma + 1$
		DM1		Use their (i) results
		A1ft		Obtain 2
	$q = -2$	A1ft	5	q is negative of their answer
			11	**Alternative for (ii) & (iii)**
		M2		Substitute $x = u - 1$ in given equation
		A1		Obtain correct unsimplified equation for u
		M1		Expand
		A2		Obtain $u^3 - 5u^2 + 10u - 2 = 0$
		A1 A1		State correct values of p and q.