OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4725

Further Pure Mathematics 1

Tuesday **7 JUNE 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

1 Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n}(6r^2 + 2r + 1) = n(2n^2 + 4n + 3).$$ [6]

2 The matrices **A** and **I** are given by $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$ and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ respectively.

(i) Find $\mathbf{A}^2$ and verify that $\mathbf{A}^2 = 4\mathbf{A} - \mathbf{I}$. [4]

(ii) Hence, or otherwise, show that $\mathbf{A}^{-1} = 4\mathbf{I} - \mathbf{A}$. [2]

3 The complex numbers $2 + 3\mathrm{i}$ and $4 - \mathrm{i}$ are denoted by z and w respectively. Express each of the following in the form $x + \mathrm{i}y$, showing clearly how you obtain your answers.

(i) $z + 5w$, [2]

(ii) z^*w, where z^* is the complex conjugate of z, [3]

(iii) $\dfrac{1}{w}$. [2]

4 Use an algebraic method to find the square roots of the complex number $21 - 20\mathrm{i}$. [6]

5 **(i)** Show that

$$\frac{r+1}{r+2} - \frac{r}{r+1} = \frac{1}{(r+1)(r+2)}.$$ [2]

(ii) Hence find an expression, in terms of n, for

$$\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{(n+1)(n+2)}.$$ [4]

(iii) Hence write down the value of $\sum_{r=1}^{\infty} \dfrac{1}{(r+1)(r+2)}$. [1]

6 The loci C_1 and C_2 are given by

$$|z - 2\mathrm{i}| = 2 \qquad \text{and} \qquad |z + 1| = |z + \mathrm{i}|$$

respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [5]

(ii) Hence write down the complex numbers represented by the points of intersection of C_1 and C_2. [2]

7 The matrix $\mathbf{B}$ is given by $\mathbf{B} = \begin{pmatrix} a & 1 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}$.

(i) Given that $\mathbf{B}$ is singular, show that $a = -\frac{2}{3}$. [3]

(ii) Given instead that $\mathbf{B}$ is non-singular, find the inverse matrix $\mathbf{B}^{-1}$. [4]

(iii) Hence, or otherwise, solve the equations

$$\begin{aligned} -x + y + 3z &= 1, \\ 2x + y - z &= 4, \\ y + 2z &= -1. \end{aligned}$$ [3]

8 **(a)** The quadratic equation $x^2 - 2x + 4 = 0$ has roots α and β.

(i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [2]

(ii) Show that $\alpha^2 + \beta^2 = -4$. [2]

(iii) Hence find a quadratic equation which has roots α^2 and β^2. [3]

(b) The cubic equation $x^3 - 12x^2 + ax - 48 = 0$ has roots p, $2p$ and $3p$.

(i) Find the value of p. [2]

(ii) Hence find the value of a. [2]

9 **(i)** Write down the matrix $\mathbf{C}$ which represents a stretch, scale factor 2, in the x-direction. [2]

(ii) The matrix $\mathbf{D}$ is given by $\mathbf{D} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$. Describe fully the geometrical transformation represented by $\mathbf{D}$. [2]

(iii) The matrix $\mathbf{M}$ represents the combined effect of the transformation represented by $\mathbf{C}$ followed by the transformation represented by $\mathbf{D}$. Show that

$$\mathbf{M} = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}.$$ [2]

(iv) Prove by induction that $\mathbf{M}^n = \begin{pmatrix} 2^n & 3(2^n - 1) \\ 0 & 1 \end{pmatrix}$, for all positive integers n. [6]

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