Mark Scheme 4725
June 2005

1.	$6\Sigma r^2 + 2\Sigma r + \Sigma 1$	M1		Consider the sum of three separate terms
	$6\Sigma r^2 = n(n+1)(2n+1)$	A1		Correct formula stated
	$2\Sigma r = n(n+1)$	A1		Correct formula stated
	$\Sigma 1 = n$	A1		Correct term seen
	$n(2n^2 + 4n + 3)$	M1	6	Correct algebraic processes including factorisation and simplification
		A1	**6**	Obtain given answer correctly
2.	(i) $A^2 = \begin{pmatrix} 3 & 8 \\ 4 & 11 \end{pmatrix}$	M1		Attempt to find A^2, 2 elements correct
		A1		All elements correct
	$\mathbf{4}A = \begin{pmatrix} 4 & 8 \\ 4 & 12 \end{pmatrix}$	M1		Use correct matrix 4**A**
	$\mathbf{A}^2 = 4\mathbf{A} - \mathbf{I}$	A1	4	Obtain given answer correctly
	(ii)	M1		Multiply answer to (i) by $\mathbf{A}^{-1}$ or obtain $\mathbf{A}^{-1}$ or factorise $\mathbf{A}^2 - 4\mathbf{A}$
	$\mathbf{A}^{-1} = 4\mathbf{I} - \mathbf{A}$	A1	2	Obtain given answer correctly
			6	
3.	(i) 22 – 2i	B1B1	2	Correct real and imaginary parts
	(ii) $z^* = 2 - 3i$	B1		Correct conjugate seen or implied
	5 – 14i	B1B1	3	Correct real and imaginary parts
	(iii) $\frac{4}{17} + \frac{1}{17}i$	M1		Attempt to use w^*
		A1	2	Obtain correct answer in any form
			7	

4.	$x^2 - y^2 = 21$ and $xy = -10$	M1		Attempt to equate real and imaginary parts of $(x + iy)^2$ and $21 - 20i$
		A1A1		Obtain each result
		M1		Eliminate to obtain a quadratic in x^2 or y^2
		M1		Solve to obtain $x = (\pm)\,5$ or $y = (\pm)\,2$
	$\pm(5 - 2i)$	A1	6	Obtain correct answers as complex numbers
			6	
5.	(i) $\dfrac{(r+1)^2 - r(r+2)}{(r+2)(r+1)}$	M1		Show correct process for subtracting fractions
	$\dfrac{1}{(r+1)(r+2)}$	A1	2	Obtain given answer correctly
	(ii) *EITHER*			
	$\frac{2}{3} - \frac{1}{2} + \frac{3}{4} - \frac{2}{3} \dots \frac{n+1}{n+2} - \frac{n}{n+1}$	M1		Express terms as differences using (i)
		A1		At least first two and last term correct
		M1		Show or imply that pairs of terms cancel
	$\frac{n+1}{n+2} - \frac{1}{2}$	A1	4	Obtain correct answer in any form
	OR	M2		State that $\sum_{r=1}^{n} u_r = f(n+1) - f(1)$
		A1A1		Each term correct
	(iii) $\frac{1}{2}$	B1 ft	1	Obtain value from their sum to n terms
			7	
6.	(i) Circle	B1		Sketch(s) showing correct features, each mark independent
	Centre (0, 2)	B1		
	Radius 2	B1		
	Straight line	B1		
	Through origin with positive slope	B1	5	
	(ii) 0 or 0 +0i and 2 + 2i	B1ftB1ft	2	Obtain intersections as complex numbers
			7	
8.	(a) (i) $\alpha + \beta = 2 \quad \alpha\beta = 4$	B1B1	2	Values stated
	(ii) *EITHER*			
	$\alpha^2 + \beta^2 = -4$	M1		Use $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
		A1	2	Obtain given answer correctly
	OR	M1		Find numeric values of roots, square and add
		A1		Obtain given answer correctly
	(iii)			
	$x^2 + 4x + 16 = 0$	B1		State or use $\alpha^2\beta^2 = 16$

	(b) (i) $p = 2$	M1 A1 M1 A1	 3 2	Or use substitution $u = x^2$ Write down a quadratic equation of correct form or rearrange and square Obtain $x^2 + 4x + 16 = 0$ Use sum or product of roots to obtain $6p = 12$ Or $6p^3 = 48$ Obtain $p = 2$
	(ii) $a = 44$	M1 A1ft	 2 **11**	Attempt to find $\Sigma\alpha\beta$ numerically or in terms of p or substitute their 2, 4 or 6 in equation Obtain $11p^2$
9.	(i) $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$	B1B1	2	Each column correct
	(ii) Shear, e.g. (0,1) transforms to (3,1)	B1B1	2	One example or sensible explanation
	(iii) $\mathbf{M} = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$	M1 A1	 2	Attempt to find **DC** (not **CD**) Obtain given answer
	(iv)	B1		Explicit check for n = 1or n = 2
	$\mathbf{M}^k = \begin{pmatrix} 2^k & 3(2^k - 1) \\ 0 & 1 \end{pmatrix}$.	M1 M1		Induction hypothesis that result is true for $\mathbf{M}^k$ Attempt to multiply $\mathbf{MM}^k$ or vice versa
	$\begin{pmatrix} 2^{k+1} & 3(2^{k+1} - 1) \\ 0 & 1 \end{pmatrix}$.	A1 A1		Element $3(2^{k+1} - 1)$ derived correctly All other elements correct
		A1	6 **12**	Explicit statement of induction conclusion