

**ADVANCED SUBSIDIARY GCE**  
**MATHEMATICS**  
Further Pure Mathematics 1

**4725**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- List of Formulae (MF1)

**Other Materials Required:**

None

**Wednesday 20 January 2010**  
**Afternoon**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

- 1 The matrix  $\mathbf{A}$  is given by  $\mathbf{A} = \begin{pmatrix} a & 2 \\ 3 & 4 \end{pmatrix}$  and  $\mathbf{I}$  is the  $2 \times 2$  identity matrix.
- (i) Find  $\mathbf{A} - 4\mathbf{I}$ . [2]
- (ii) Given that  $\mathbf{A}$  is singular, find the value of  $a$ . [3]
- 2 The cubic equation  $2x^3 + 3x - 3 = 0$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ .
- (i) Use the substitution  $x = u - 1$  to find a cubic equation in  $u$  with integer coefficients. [3]
- (ii) Hence find the value of  $(\alpha + 1)(\beta + 1)(\gamma + 1)$ . [2]
- 3 The complex number  $z$  satisfies the equation  $z + 2iz^* = 12 + 9i$ . Find  $z$ , giving your answer in the form  $x + iy$ . [5]
- 4 Find  $\sum_{r=1}^n r(r+1)(r-2)$ , expressing your answer in a fully factorised form. [6]
- 5 (i) The transformation  $T$  is represented by the matrix  $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ . Give a geometrical description of  $T$ . [2]
- (ii) The transformation  $T$  is equivalent to a reflection in the line  $y = -x$  followed by another transformation  $S$ . Give a geometrical description of  $S$  and find the matrix that represents  $S$ . [4]
- 6 One root of the cubic equation  $x^3 + px^2 + 6x + q = 0$ , where  $p$  and  $q$  are real, is the complex number  $5 - i$ .
- (i) Find the real root of the cubic equation. [3]
- (ii) Find the values of  $p$  and  $q$ . [4]
- 7 (i) Show that  $\frac{1}{r^2} - \frac{1}{(r+1)^2} \equiv \frac{2r+1}{r^2(r+1)^2}$ . [1]
- (ii) Hence find an expression, in terms of  $n$ , for  $\sum_{r=1}^n \frac{2r+1}{r^2(r+1)^2}$ . [4]
- (iii) Find  $\sum_{r=2}^{\infty} \frac{2r+1}{r^2(r+1)^2}$ . [2]
- 8 The complex number  $a$  is such that  $a^2 = 5 - 12i$ .
- (i) Use an algebraic method to find the two possible values of  $a$ . [5]
- (ii) Sketch on a single Argand diagram the two possible loci given by  $|z - a| = |a|$ . [4]

9 The matrix  $\mathbf{A}$  is given by  $\mathbf{A} = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 3 & 1 \\ 1 & 1 & a \end{pmatrix}$ , where  $a \neq 1$ .

(i) Find  $\mathbf{A}^{-1}$ . [7]

(ii) Hence, or otherwise, solve the equations

$$2x - y + z = 1,$$

$$3y + z = 2,$$

$$x + y + az = 2.$$

[4]

10 The matrix  $\mathbf{M}$  is given by  $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ .

(i) Find  $\mathbf{M}^2$  and  $\mathbf{M}^3$ . [3]

(ii) Hence suggest a suitable form for the matrix  $\mathbf{M}^n$ . [1]

(iii) Use induction to prove that your answer to part (ii) is correct. [4]

(iv) Describe fully the single geometrical transformation represented by  $\mathbf{M}^{10}$ . [3]

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