OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS 4725

Further Pure Mathematics 1

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Thursday 15 January 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Express $\dfrac{2+3\mathrm{i}}{5-\mathrm{i}}$ in the form $x + \mathrm{i}y$, showing clearly how you obtain your answer. **[4]**

2 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ a & 5 \end{pmatrix}$. Find

(i) $\mathbf{A}^{-1}$, **[2]**

(ii) $2\mathbf{A} - \begin{pmatrix} 1 & 2 \\ 0 & 4 \end{pmatrix}$. **[2]**

3 Find $\displaystyle\sum_{r=1}^{n}(4r^3 + 6r^2 + 2r)$, expressing your answer in a fully factorised form. **[6]**

4 Given that **A** and **B** are 2×2 non-singular matrices and **I** is the 2×2 identity matrix, simplify

$$\mathbf{B}(\mathbf{AB})^{-1}\mathbf{A} - \mathbf{I}.$$ **[4]**

5 By using the determinant of an appropriate matrix, or otherwise, find the value of k for which the simultaneous equations

$$\begin{aligned} 2x - y + z &= 7, \\ 3y + z &= 4, \\ x + ky + kz &= 5, \end{aligned}$$

do not have a unique solution for x, y and z. **[5]**

6 **(i)** The transformation P is represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Give a geometrical description of transformation P. **[2]**

(ii) The transformation Q is represented by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$. Give a geometrical description of transformation Q. **[2]**

(iii) The transformation R is equivalent to transformation P followed by transformation Q. Find the matrix that represents R. **[2]**

(iv) Give a geometrical description of the **single** transformation that is represented by your answer to part **(iii)**. **[3]**

7 It is given that $u_n = 13^n + 6^{n-1}$, where n is a positive integer.

(i) Show that $u_n + u_{n+1} = 14 \times 13^n + 7 \times 6^{n-1}$. **[3]**

(ii) Prove by induction that u_n is a multiple of 7. **[4]**

8 **(i)** Show that $(\alpha - \beta)^2 \equiv (\alpha + \beta)^2 - 4\alpha\beta$. **[2]**

The quadratic equation $x^2 - 6kx + k^2 = 0$, where k is a positive constant, has roots α and β, with $\alpha > \beta$.

(ii) Show that $\alpha - \beta = 4\sqrt{2}k$. **[4]**

(iii) Hence find a quadratic equation with roots $\alpha + 1$ and $\beta - 1$. **[4]**

9 **(i)** Show that $\dfrac{1}{2r-3} - \dfrac{1}{2r+1} = \dfrac{4}{4r^2 - 4r - 3}$. **[2]**

(ii) Hence find an expression, in terms of n, for

$$\sum_{r=2}^{n} \frac{4}{4r^2 - 4r - 3}.$$ **[6]**

(iii) Show that $\displaystyle\sum_{r=2}^{\infty} \frac{4}{4r^2 - 4r - 3} = \frac{4}{3}$. **[1]**

10 **(i)** Use an algebraic method to find the square roots of the complex number $2 + i\sqrt{5}$. Give your answers in the form $x + iy$, where x and y are exact real numbers. **[6]**

(ii) Hence find, in the form $x + iy$ where x and y are exact real numbers, the roots of the equation

$$z^4 - 4z^2 + 9 = 0.$$ **[4]**

(iii) Show, on an Argand diagram, the roots of the equation in part **(ii)**. **[1]**

(iv) Given that α is the root of the equation in part **(ii)** such that $0 < \arg \alpha < \frac{1}{2}\pi$, sketch on the same Argand diagram the locus given by $|z - \alpha| = |z|$. **[3]**