OCR
RECOGNISING ACHIEVEMENT

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Advanced Subsidiary General Certificate of Education**
**Advanced General Certificate of Education**

# MATHEMATICS 4725

Further Pure Mathematics 1

Wednesday **18 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

**TIME** 1 hour 30 minutes

**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

**This question paper consists of 3 printed pages and 1 blank page.**


Registered Charity Number: 1066969
**[Turn over**

1 (i) Express $(1 + 8i)(2 - i)$ in the form $x + iy$, showing clearly how you obtain your answer. [2]

(ii) Hence express $\dfrac{1 + 8i}{2 + i}$ in the form $x + iy$. [3]

2 Prove by induction that, for $n \geqslant 1$, $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n + 1)(2n + 1)$. [5]

3 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$.

(i) Find the value of the determinant of $\mathbf{M}$. [3]

(ii) State, giving a brief reason, whether $\mathbf{M}$ is singular or non-singular. [1]

4 Use the substitution $x = u + 2$ to find the exact value of the real root of the equation

$$x^3 - 6x^2 + 12x - 13 = 0.$$ [5]

5 Use the standard results for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that, for all positive integers $n$,

$$\sum_{r=1}^{n} (8r^3 - 6r^2 + 2r) = 2n^3(n + 1).$$ [6]

6 The matrix $\mathbf{C}$ is given by $\mathbf{C} = \begin{pmatrix} 1 & 2 \\ 3 & 8 \end{pmatrix}$.

(i) Find $\mathbf{C}^{-1}$. [2]

(ii) Given that $\mathbf{C} = \mathbf{AB}$, where $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, find $\mathbf{B}^{-1}$. [5]

7 (a) The complex number $3 + 2i$ is denoted by $w$ and the complex conjugate of $w$ is denoted by $w^*$. Find

(i) the modulus of $w$, [1]

(ii) the argument of $w^*$, giving your answer in radians, correct to 2 decimal places. [3]

(b) Find the complex number $u$ given that $u + 2u^* = 3 + 2i$. [4]

(c) Sketch, on an Argand diagram, the locus given by $|z + 1| = |z|$. [2]

**8** The matrix **T** is given by $\mathbf{T} = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$.

**(i)** Draw a diagram showing the unit square and its image under the transformation represented by **T**. [3]

**(ii)** The transformation represented by matrix **T** is equivalent to a transformation A, followed by a transformation B. Give geometrical descriptions of possible transformations A and B, and state the matrices that represent them. [6]

**9** **(i)** Show that $\frac{1}{r} - \frac{1}{r+2} = \frac{2}{r(r+2)}$. [2]

**(ii)** Hence find an expression, in terms of $n$, for

$$\frac{2}{1\times 3} + \frac{2}{2\times 4} + \ldots + \frac{2}{n(n+2)}.$$ [5]

**(iii)** Hence find the value of

**(a)** $\sum_{r=1}^{\infty} \frac{2}{r(r+2)}$, [1]

**(b)** $\sum_{r=n+1}^{\infty} \frac{2}{r(r+2)}$. [2]

**10** The roots of the equation

$$x^3 - 9x^2 + 27x - 29 = 0$$

are denoted by $\alpha$, $\beta$ and $\gamma$, where $\alpha$ is real and $\beta$ and $\gamma$ are complex.

**(i)** Write down the value of $\alpha + \beta + \gamma$. [1]

**(ii)** It is given that $\beta = p + iq$, where $q > 0$. Find the value of $p$, in terms of $\alpha$. [4]

**(iii)** Write down the value of $\alpha\beta\gamma$. [1]

**(iv)** Find the value of $q$, in terms of $\alpha$ only. [5]