OCR Maths FP1

Past Paper Pack

2005-2013

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4725

Further Pure Mathematics 1

Tuesday **7 JUNE 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

1 Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n}(6r^2 + 2r + 1) = n(2n^2 + 4n + 3).$$ [6]

2 The matrices **A** and **I** are given by $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$ and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ respectively.

(i) Find $\mathbf{A}^2$ and verify that $\mathbf{A}^2 = 4\mathbf{A} - \mathbf{I}$. [4]

(ii) Hence, or otherwise, show that $\mathbf{A}^{-1} = 4\mathbf{I} - \mathbf{A}$. [2]

3 The complex numbers $2 + 3\text{i}$ and $4 - \text{i}$ are denoted by z and w respectively. Express each of the following in the form $x + \text{i}y$, showing clearly how you obtain your answers.

(i) $z + 5w$, [2]

(ii) z^*w, where z^* is the complex conjugate of z, [3]

(iii) $\frac{1}{w}$. [2]

4 Use an algebraic method to find the square roots of the complex number $21 - 20\text{i}$. [6]

5 **(i)** Show that

$$\frac{r+1}{r+2} - \frac{r}{r+1} = \frac{1}{(r+1)(r+2)}.$$ [2]

(ii) Hence find an expression, in terms of n, for

$$\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{(n+1)(n+2)}.$$ [4]

(iii) Hence write down the value of $\sum_{r=1}^{\infty} \frac{1}{(r+1)(r+2)}$. [1]

6 The loci C_1 and C_2 are given by

$$|z - 2\text{i}| = 2 \qquad \text{and} \qquad |z + 1| = |z + \text{i}|$$

respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [5]

(ii) Hence write down the complex numbers represented by the points of intersection of C_1 and C_2. [2]

7 The matrix **B** is given by $\mathbf{B} = \begin{pmatrix} a & 1 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}$.

(i) Given that **B** is singular, show that $a = -\frac{2}{3}$. [3]

(ii) Given instead that **B** is non-singular, find the inverse matrix $\mathbf{B}^{-1}$. [4]

(iii) Hence, or otherwise, solve the equations

$$\begin{aligned} -x + y + 3z &= 1, \\ 2x + y - z &= 4, \\ y + 2z &= -1. \end{aligned}$$ [3]

8 **(a)** The quadratic equation $x^2 - 2x + 4 = 0$ has roots α and β.

(i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [2]

(ii) Show that $\alpha^2 + \beta^2 = -4$. [2]

(iii) Hence find a quadratic equation which has roots α^2 and β^2. [3]

(b) The cubic equation $x^3 - 12x^2 + ax - 48 = 0$ has roots p, $2p$ and $3p$.

(i) Find the value of p. [2]

(ii) Hence find the value of a. [2]

9 **(i)** Write down the matrix **C** which represents a stretch, scale factor 2, in the x-direction. [2]

(ii) The matrix **D** is given by $\mathbf{D} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$. Describe fully the geometrical transformation represented by **D**. [2]

(iii) The matrix **M** represents the combined effect of the transformation represented by **C** followed by the transformation represented by **D**. Show that

$$\mathbf{M} = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}.$$ [2]

(iv) Prove by induction that $\mathbf{M}^n = \begin{pmatrix} 2^n & 3(2^n - 1) \\ 0 & 1 \end{pmatrix}$, for all positive integers n. [6]

RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4725

Further Pure Mathematics 1

Wednesday **18 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

1 (i) Express $(1 + 8\mathrm{i})(2 - \mathrm{i})$ in the form $x + \mathrm{i}y$, showing clearly how you obtain your answer. [2]

(ii) Hence express $\dfrac{1 + 8\mathrm{i}}{2 + \mathrm{i}}$ in the form $x + \mathrm{i}y$. [3]

2 Prove by induction that, for $n \geqslant 1$, $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$. [5]

3 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$.

(i) Find the value of the determinant of $\mathbf{M}$. [3]

(ii) State, giving a brief reason, whether $\mathbf{M}$ is singular or non-singular. [1]

4 Use the substitution $x = u + 2$ to find the exact value of the real root of the equation

$$x^3 - 6x^2 + 12x - 13 = 0.$$ [5]

5 Use the standard results for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} (8r^3 - 6r^2 + 2r) = 2n^3(n+1).$$ [6]

6 The matrix $\mathbf{C}$ is given by $\mathbf{C} = \begin{pmatrix} 1 & 2 \\ 3 & 8 \end{pmatrix}$.

(i) Find $\mathbf{C}^{-1}$. [2]

(ii) Given that $\mathbf{C} = \mathbf{AB}$, where $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, find $\mathbf{B}^{-1}$. [5]

7 (a) The complex number $3 + 2\mathrm{i}$ is denoted by w and the complex conjugate of w is denoted by w^*. Find

(i) the modulus of w, [1]

(ii) the argument of w^*, giving your answer in radians, correct to 2 decimal places. [3]

(b) Find the complex number u given that $u + 2u^* = 3 + 2\mathrm{i}$. [4]

(c) Sketch, on an Argand diagram, the locus given by $|z + 1| = |z|$. [2]

8 The matrix $\mathbf{T}$ is given by $\mathbf{T} = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$.

(i) Draw a diagram showing the unit square and its image under the transformation represented by $\mathbf{T}$. [3]

(ii) The transformation represented by matrix $\mathbf{T}$ is equivalent to a transformation A, followed by a transformation B. Give geometrical descriptions of possible transformations A and B, and state the matrices that represent them. [6]

9 **(i)** Show that $\dfrac{1}{r} - \dfrac{1}{r+2} = \dfrac{2}{r(r+2)}$. [2]

(ii) Hence find an expression, in terms of n, for

$$\frac{2}{1 \times 3} + \frac{2}{2 \times 4} + \ldots + \frac{2}{n(n+2)}.$$ [5]

(iii) Hence find the value of

(a) $\displaystyle\sum_{r=1}^{\infty} \frac{2}{r(r+2)}$, [1]

(b) $\displaystyle\sum_{r=n+1}^{\infty} \frac{2}{r(r+2)}$. [2]

10 The roots of the equation

$$x^3 - 9x^2 + 27x - 29 = 0$$

are denoted by α, β and γ, where α is real and β and γ are complex.

(i) Write down the value of $\alpha + \beta + \gamma$. [1]

(ii) It is given that $\beta = p + \mathrm{i}q$, where $q > 0$. Find the value of p, in terms of α. [4]

(iii) Write down the value of $\alpha\beta\gamma$. [1]

(iv) Find the value of q, in terms of α only. [5]

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4725

Further Pure Mathematics 1

Thursday **8 JUNE 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

 [Turn over

1 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 0 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$.

(i) Find $\mathbf{A} + 3\mathbf{B}$. [2]

(ii) Show that $\mathbf{A} - \mathbf{B} = k\mathbf{I}$, where $\mathbf{I}$ is the identity matrix and k is a constant whose value should be stated. [2]

2 The transformation S is a shear parallel to the x-axis in which the image of the point (1, 1) is the point (0, 1).

(i) Draw a diagram showing the image of the unit square under S. [2]

(ii) Write down the matrix that represents S. [2]

3 One root of the quadratic equation $x^2 + px + q = 0$, where p and q are real, is the complex number $2 - 3\mathrm{i}$.

(i) Write down the other root. [1]

(ii) Find the values of p and q. [4]

4 Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} (r^3 + r^2) = \tfrac{1}{12}n(n+1)(n+2)(3n+1).$$ [5]

5 The complex numbers $3 - 2\mathrm{i}$ and $2 + \mathrm{i}$ are denoted by z and w respectively. Find, giving your answers in the form $x + \mathrm{i}y$ and showing clearly how you obtain these answers,

(i) $2z - 3w$, [2]

(ii) $(\mathrm{i}z)^2$, [3]

(iii) $\dfrac{z}{w}$. [3]

6 In an Argand diagram the loci C_1 and C_2 are given by

$$|z| = 2 \qquad \text{and} \qquad \arg z = \tfrac{1}{3}\pi$$

respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [5]

(ii) Hence find, in the form $x + \mathrm{i}y$, the complex number representing the point of intersection of C_1 and C_2. [2]

7 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$.

(i) Find $\mathbf{A}^2$ and $\mathbf{A}^3$. [3]

(ii) Hence suggest a suitable form for the matrix $\mathbf{A}^n$. [1]

(iii) Use induction to prove that your answer to part **(ii)** is correct. [4]

8 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} a & 4 & 2 \\ 1 & a & 0 \\ 1 & 2 & 1 \end{pmatrix}$.

(i) Find, in terms of a, the determinant of **M**. [3]

(ii) Hence find the values of a for which **M** is singular. [3]

(iii) State, giving a brief reason in each case, whether the simultaneous equations

$$\begin{aligned} ax + 4y + 2z &= 3a, \\ x + ay &= 1, \\ x + 2y + z &= 3, \end{aligned}$$

have any solutions when

(a) $a = 3$,

(b) $a = 2$.

[4]

9 **(i)** Use the method of differences to show that

$$\sum_{r=1}^{n} \{(r+1)^3 - r^3\} = (n+1)^3 - 1.$$ [2]

(ii) Show that $(r+1)^3 - r^3 \equiv 3r^2 + 3r + 1$. [2]

(iii) Use the results in parts **(i)** and **(ii)** and the standard result for $\sum_{r=1}^{n} r$ to show that

$$3\sum_{r=1}^{n} r^2 = \tfrac{1}{2}n(n+1)(2n+1).$$ [6]

10 The cubic equation $x^3 - 2x^2 + 3x + 4 = 0$ has roots α, β and γ.

(i) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. [3]

The cubic equation $x^3 + px^2 + 10x + q = 0$, where p and q are constants, has roots $\alpha + 1$, $\beta + 1$ and $\gamma + 1$.

(ii) Find the value of p. [3]

(iii) Find the value of q. [5]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4725/01**

MATHEMATICS

Further Pure Mathematics 1

THURSDAY 18 JANUARY 2007 Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} a & -1 \\ -3 & -2 \end{pmatrix}$.

(i) Given that $2\mathbf{A} + \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$, write down the value of a. [1]

(ii) Given instead that $\mathbf{AB} = \begin{pmatrix} 7 & -4 \\ 9 & -7 \end{pmatrix}$, find the value of a. [2]

2 Use an algebraic method to find the square roots of the complex number $15 + 8\mathrm{i}$. [6]

3 Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$ to find

$$\sum_{r=1}^{n} r(r-1)(r+1),$$

expressing your answer in a fully factorised form. [6]

4 **(i)** Sketch, on an Argand diagram, the locus given by $|z - 1 + \mathrm{i}| = \sqrt{2}$. [3]

(ii) Shade on your diagram the region given by $1 \leqslant |z - 1 + \mathrm{i}| \leqslant \sqrt{2}$. [3]

5 **(i)** Verify that $z^3 - 8 = (z-2)(z^2 + 2z + 4)$. [1]

(ii) Solve the quadratic equation $z^2 + 2z + 4 = 0$, giving your answers exactly in the form $x + \mathrm{i}y$. Show clearly how you obtain your answers. [3]

(iii) Show on an Argand diagram the roots of the cubic equation $z^3 - 8 = 0$. [3]

6 The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_n = n^2 + 3n$, for all positive integers n.

(i) Show that $u_{n+1} - u_n = 2n + 4$. [3]

(ii) Hence prove by induction that each term of the sequence is divisible by 2. [5]

7 The quadratic equation $x^2 + 5x + 10 = 0$ has roots α and β.

(i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [2]

(ii) Show that $\alpha^2 + \beta^2 = 5$. [2]

(iii) Hence find a quadratic equation which has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [4]

8 **(i)** Show that $(r+2)! - (r+1)! = (r+1)^2 \times r!$. [3]

(ii) Hence find an expression, in terms of n, for

$$2^2 \times 1! + 3^2 \times 2! + 4^2 \times 3! + \ldots + (n+1)^2 \times n!.$$ [4]

(iii) State, giving a brief reason, whether the series

$$2^2 \times 1! + 3^2 \times 2! + 4^2 \times 3! + \ldots$$

converges. [1]

9 The matrix $\mathbf{C}$ is given by $\mathbf{C} = \begin{pmatrix} 0 & 3 \\ -1 & 0 \end{pmatrix}$.

(i) Draw a diagram showing the unit square and its image under the transformation represented by $\mathbf{C}$. [2]

The transformation represented by $\mathbf{C}$ is equivalent to a rotation, R, followed by another transformation, S.

(ii) Describe fully the rotation R and write down the matrix that represents R. [3]

(iii) Describe fully the transformation S and write down the matrix that represents S. [4]

10 The matrix $\mathbf{D}$ is given by $\mathbf{D} = \begin{pmatrix} a & 2 & 0 \\ 3 & 1 & 2 \\ 0 & -1 & 1 \end{pmatrix}$, where $a \neq 2$.

(i) Find $\mathbf{D}^{-1}$. [7]

(ii) Hence, or otherwise, solve the equations

$$\begin{aligned} ax + 2y \quad\quad &= 3, \\ 3x + y + 2z &= 4, \\ -y + z &= 1. \end{aligned}$$ [4]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4725/01**

MATHEMATICS

Further Pure Mathematics 1

MONDAY 11 JUNE 2007 Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 The complex number $a + \mathrm{i}b$ is denoted by z. Given that $|z| = 4$ and $\arg z = \frac{1}{3}\pi$, find a and b. [4]

2 Prove by induction that, for $n \geqslant 1$, $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$. [5]

3 Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n}(3r^2 - 3r + 1) = n^3.$$ [6]

4 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 3 & 5 \end{pmatrix}$.

(i) Find $\mathbf{A}^{-1}$. [2]

The matrix $\mathbf{B}^{-1}$ is given by $\mathbf{B}^{-1} = \begin{pmatrix} 1 & 1 \\ 4 & -1 \end{pmatrix}$.

(ii) Find $(\mathbf{AB})^{-1}$. [4]

5 **(i)** Show that

$$\frac{1}{r} - \frac{1}{r+1} = \frac{1}{r(r+1)}.$$ [1]

(ii) Hence find an expression, in terms of n, for

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots + \frac{1}{n(n+1)}.$$ [3]

(iii) Hence find the value of $\sum_{r=n+1}^{\infty} \frac{1}{r(r+1)}$. [3]

6 The cubic equation $3x^3 - 9x^2 + 6x + 2 = 0$ has roots α, β and γ.

(i) (a) Write down the values of $\alpha + \beta + \gamma$ and $\alpha\beta + \beta\gamma + \gamma\alpha$. [2]

(b) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [2]

(ii) (a) Use the substitution $x = \frac{1}{u}$ to find a cubic equation in u with integer coefficients. [2]

(b) Use your answer to part **(ii) (a)** to find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. [2]

7 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} a & 4 & 0 \\ 0 & a & 4 \\ 2 & 3 & 1 \end{pmatrix}$.

(i) Find, in terms of a, the determinant of $\mathbf{M}$. [3]

(ii) In the case when $a = 2$, state whether $\mathbf{M}$ is singular or non-singular, justifying your answer. [2]

(iii) In the case when $a = 4$, determine whether the simultaneous equations

$$\begin{aligned} ax + 4y \quad\quad &= 6, \\ ay + 4z &= 8, \\ 2x + 3y + z &= 1, \end{aligned}$$

have any solutions. [3]

8 The loci C_1 and C_2 are given by $|z - 3| = 3$ and $\arg(z - 1) = \frac{1}{4}\pi$ respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [6]

(ii) Indicate, by shading, the region of the Argand diagram for which

$$|z - 3| \leqslant 3 \text{ and } 0 \leqslant \arg(z - 1) \leqslant \tfrac{1}{4}\pi. \quad [2]$$

9 **(i)** Write down the matrix, $\mathbf{A}$, that represents an enlargement, centre (0, 0), with scale factor $\sqrt{2}$. [1]

(ii) The matrix $\mathbf{B}$ is given by $\mathbf{B} = \begin{pmatrix} \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix}$. Describe fully the geometrical transformation represented by $\mathbf{B}$. [3]

(iii) Given that $\mathbf{C} = \mathbf{AB}$, show that $\mathbf{C} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$. [1]

(iv) Draw a diagram showing the unit square and its image under the transformation represented by $\mathbf{C}$. [2]

(v) Write down the determinant of $\mathbf{C}$ and explain briefly how this value relates to the transformation represented by $\mathbf{C}$. [2]

10 **(i)** Use an algebraic method to find the square roots of the complex number $16 + 30\mathrm{i}$. [6]

(ii) Use your answers to part **(i)** to solve the equation $z^2 - 2z - (15 + 30\mathrm{i}) = 0$, giving your answers in the form $x + \mathrm{i}y$. [5]

RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4725/01**

MATHEMATICS

Further Pure Mathematics 1

FRIDAY 11 JANUARY 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 The transformation S is a shear with the y-axis invariant (i.e. a shear parallel to the y-axis). It is given that the image of the point (1, 1) is the point (1, 0).

(i) Draw a diagram showing the image of the unit square under the transformation S. [2]

(ii) Write down the matrix that represents S. [2]

2 Given that $\sum_{r=1}^{n}(ar^2 + b) \equiv n(2n^2 + 3n - 2)$, find the values of the constants a and b. [5]

3 The cubic equation $2x^3 - 3x^2 + 24x + 7 = 0$ has roots α, β and γ.

(i) Use the substitution $x = \frac{1}{u}$ to find a cubic equation in u with integer coefficients. [2]

(ii) Hence, or otherwise, find the value of $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$. [2]

4 The complex number $3 - 4\text{i}$ is denoted by z. Giving your answers in the form $x + \text{i}y$, and showing clearly how you obtain them, find

(i) $2z + 5z^*$, [2]

(ii) $(z - \text{i})^2$, [3]

(iii) $\frac{3}{z}$. [3]

5 The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$ and $\mathbf{C} = (2 \quad 4 \quad -1)$. Find

(i) $\mathbf{A} - 4\mathbf{B}$, [2]

(ii) $\mathbf{BC}$, [4]

(iii) $\mathbf{CA}$. [2]

6 The loci C_1 and C_2 are given by

$$|z| = |z - 4\text{i}| \qquad \text{and} \qquad \arg z = \tfrac{1}{6}\pi$$

respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [5]

(ii) Hence find, in the form $x + \text{i}y$, the complex number represented by the point of intersection of C_1 and C_2. [3]

7 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 3 \\ -2 & 1 \end{pmatrix}$.

(i) Given that **A** is singular, find a. [2]

(ii) Given instead that **A** is non-singular, find $\mathbf{A}^{-1}$ and hence solve the simultaneous equations

$$\begin{aligned} ax + 3y &= 1, \\ -2x + \ \ y &= -1. \end{aligned}$$ [5]

8 The sequence $u_1, u_2, u_3, \dots$ is defined by $u_1 = 1$ and $u_{n+1} = u_n + 2n + 1$.

(i) Show that $u_4 = 16$. [2]

(ii) Hence suggest an expression for u_n. [1]

(iii) Use induction to prove that your answer to part **(ii)** is correct. [4]

9 **(i)** Show that $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$. [2]

(ii) The quadratic equation $x^2 - 5x + 7 = 0$ has roots α and β. Find a quadratic equation with roots α^3 and β^3. [6]

10 **(i)** Show that $\dfrac{2}{r} - \dfrac{1}{r+1} - \dfrac{1}{r+2} = \dfrac{3r+4}{r(r+1)(r+2)}$. [2]

(ii) Hence find an expression, in terms of n, for

$$\sum_{r=1}^{n} \frac{3r+4}{r(r+1)(r+2)}.$$ [6]

(iii) Hence write down the value of $\displaystyle\sum_{r=1}^{\infty} \frac{3r+4}{r(r+1)(r+2)}$. [1]

(iv) Given that $\displaystyle\sum_{r=N+1}^{\infty} \frac{3r+4}{r(r+1)(r+2)} = \frac{7}{10}$, find the value of N. [4]

RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4725/01**

MATHEMATICS

Further Pure Mathematics 1

MONDAY 2 JUNE 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

 [Turn over

1 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 5 & 2 \end{pmatrix}$ and **I** is the 2×2 identity matrix. Find

(i) $\mathbf{A} - 3\mathbf{I}$, [2]

(ii) $\mathbf{A}^{-1}$. [2]

2 The complex number $3 + 4\mathrm{i}$ is denoted by a.

(i) Find $|a|$ and $\arg a$. [2]

(ii) Sketch on a single Argand diagram the loci given by

(a) $|z - a| = |a|$, [2]

(b) $\arg(z - 3) = \arg a$. [3]

3 **(i)** Show that $\dfrac{1}{r!} - \dfrac{1}{(r+1)!} = \dfrac{r}{(r+1)!}$. [2]

(ii) Hence find an expression, in terms of n, for

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{n}{(n+1)!}.$$ [4]

4 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$. Prove by induction that, for $n \geqslant 1$,

$$\mathbf{A}^n = \begin{pmatrix} 3^n & \frac{1}{2}(3^n - 1) \\ 0 & 1 \end{pmatrix}.$$ [6]

5 Find $\sum_{r=1}^{n} r^2(r-1)$, expressing your answer in a fully factorised form. [6]

6 The cubic equation $x^3 + ax^2 + bx + c = 0$, where a, b and c are real, has roots $(3 + \mathrm{i})$ and 2.

(i) Write down the other root of the equation. [1]

(ii) Find the values of a, b and c. [6]

7 Describe fully the geometrical transformation represented by each of the following matrices:

(i) $\begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$, [1]

(ii) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, [2]

(iii) $\begin{pmatrix} 1 & 0 \\ 0 & 6 \end{pmatrix}$, [2]

(iv) $\begin{pmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix}$. [2]

8 The quadratic equation $x^2 + kx + 2k = 0$, where k is a non-zero constant, has roots α and β. Find a quadratic equation with roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [7]

9 **(i)** Use an algebraic method to find the square roots of the complex number $5 + 12\mathrm{i}$. [5]

(ii) Find $(3 - 2\mathrm{i})^2$. [2]

(iii) Hence solve the quartic equation $x^4 - 10x^2 + 169 = 0$. [4]

10 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} a & 8 & 10 \\ 2 & 1 & 2 \\ 4 & 3 & 6 \end{pmatrix}$. The matrix $\mathbf{B}$ is such that $\mathbf{AB} = \begin{pmatrix} a & 6 & 1 \\ 1 & 1 & 0 \\ 1 & 3 & 0 \end{pmatrix}$.

(i) Show that $\mathbf{AB}$ is non-singular. [2]

(ii) Find $(\mathbf{AB})^{-1}$. [4]

(iii) Find $\mathbf{B}^{-1}$. [5]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS

4725

Further Pure Mathematics 1

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Thursday 15 January 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Express $\dfrac{2+3\mathrm{i}}{5-\mathrm{i}}$ in the form $x + \mathrm{i}y$, showing clearly how you obtain your answer. **[4]**

2 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ a & 5 \end{pmatrix}$. Find

(i) $\mathbf{A}^{-1}$, **[2]**

(ii) $2\mathbf{A} - \begin{pmatrix} 1 & 2 \\ 0 & 4 \end{pmatrix}$. **[2]**

3 Find $\displaystyle\sum_{r=1}^{n}(4r^3 + 6r^2 + 2r)$, expressing your answer in a fully factorised form. **[6]**

4 Given that **A** and **B** are 2×2 non-singular matrices and **I** is the 2×2 identity matrix, simplify

$$\mathbf{B}(\mathbf{AB})^{-1}\mathbf{A} - \mathbf{I}.$$ **[4]**

5 By using the determinant of an appropriate matrix, or otherwise, find the value of k for which the simultaneous equations

$$\begin{aligned} 2x - y + z &= 7, \\ 3y + z &= 4, \\ x + ky + kz &= 5, \end{aligned}$$

do not have a unique solution for x, y and z. **[5]**

6 **(i)** The transformation P is represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Give a geometrical description of transformation P. **[2]**

(ii) The transformation Q is represented by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$. Give a geometrical description of transformation Q. **[2]**

(iii) The transformation R is equivalent to transformation P followed by transformation Q. Find the matrix that represents R. **[2]**

(iv) Give a geometrical description of the **single** transformation that is represented by your answer to part **(iii)**. **[3]**

7 It is given that $u_n = 13^n + 6^{n-1}$, where n is a positive integer.

(i) Show that $u_n + u_{n+1} = 14 \times 13^n + 7 \times 6^{n-1}$. **[3]**

(ii) Prove by induction that u_n is a multiple of 7. **[4]**

8 **(i)** Show that $(\alpha - \beta)^2 \equiv (\alpha + \beta)^2 - 4\alpha\beta$. **[2]**

The quadratic equation $x^2 - 6kx + k^2 = 0$, where k is a positive constant, has roots α and β, with $\alpha > \beta$.

(ii) Show that $\alpha - \beta = 4\sqrt{2}k$. **[4]**

(iii) Hence find a quadratic equation with roots $\alpha + 1$ and $\beta - 1$. **[4]**

9 **(i)** Show that $\dfrac{1}{2r-3} - \dfrac{1}{2r+1} = \dfrac{4}{4r^2 - 4r - 3}$. **[2]**

(ii) Hence find an expression, in terms of n, for

$$\sum_{r=2}^{n} \frac{4}{4r^2 - 4r - 3}.$$ **[6]**

(iii) Show that $\displaystyle\sum_{r=2}^{\infty} \frac{4}{4r^2 - 4r - 3} = \frac{4}{3}$. **[1]**

10 **(i)** Use an algebraic method to find the square roots of the complex number $2 + i\sqrt{5}$. Give your answers in the form $x + iy$, where x and y are exact real numbers. **[6]**

(ii) Hence find, in the form $x + iy$ where x and y are exact real numbers, the roots of the equation

$$z^4 - 4z^2 + 9 = 0.$$ **[4]**

(iii) Show, on an Argand diagram, the roots of the equation in part **(ii)**. **[1]**

(iv) Given that α is the root of the equation in part **(ii)** such that $0 < \arg \alpha < \frac{1}{2}\pi$, sketch on the same Argand diagram the locus given by $|z - \alpha| = |z|$. **[3]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS

4725

Further Pure Mathematics 1

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Friday 5 June 2009
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Evaluate $\sum_{r=101}^{250} r^3$. **[3]**

2 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}$ and **I** is the 2×2 identity matrix. Find the values of the constants a and b for which $a\mathbf{A} + b\mathbf{B} = \mathbf{I}$. **[4]**

3 The complex numbers z and w are given by $z = 5 - 2\text{i}$ and $w = 3 + 7\text{i}$. Giving your answers in the form $x + \text{i}y$ and showing clearly how you obtain them, find

(i) $4z - 3w$, **[2]**

(ii) z^*w. **[2]**

4 The roots of the quadratic equation $x^2 + x - 8 = 0$ are p and q. Find the value of $p + q + \frac{1}{p} + \frac{1}{q}$. **[4]**

5 The cubic equation $x^3 + 5x^2 + 7 = 0$ has roots α, β and γ.

(i) Use the substitution $x = \sqrt{u}$ to find a cubic equation in u with integer coefficients. **[3]**

(ii) Hence find the value of $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$. **[2]**

6 The complex number $3 - 3\text{i}$ is denoted by a.

(i) Find $|a|$ and $\arg a$. **[2]**

(ii) Sketch on a single Argand diagram the loci given by

(a) $|z - a| = 3\sqrt{2}$, **[3]**

(b) $\arg(z - a) = \frac{1}{4}\pi$. **[3]**

(iii) Indicate, by shading, the region of the Argand diagram for which

$$|z - a| \geqslant 3\sqrt{2} \quad \text{and} \quad 0 \leqslant \arg(z - a) \leqslant \tfrac{1}{4}\pi.$$ **[3]**

7 **(i)** Use the method of differences to show that

$$\sum_{r=1}^{n} \{(r+1)^4 - r^4\} = (n+1)^4 - 1.$$ **[2]**

(ii) Show that $(r+1)^4 - r^4 \equiv 4r^3 + 6r^2 + 4r + 1$. **[2]**

(iii) Hence show that

$$4\sum_{r=1}^{n} r^3 = n^2(n+1)^2.$$ **[6]**

8 The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$.

(i) Draw a diagram showing the image of the unit square under the transformation represented by **C**. **[3]**

The transformation represented by **C** is equivalent to a transformation S followed by another transformation T.

(ii) Given that S is a shear with the y-axis invariant in which the image of the point (1, 1) is (1, 2), write down the matrix that represents S. **[2]**

(iii) Find the matrix that represents transformation T and describe fully the transformation T. **[6]**

9 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & 2 \end{pmatrix}$.

(i) Find, in terms of a, the determinant of **A**. **[3]**

(ii) Hence find the values of a for which **A** is singular. **[3]**

(iii) State, giving a brief reason in each case, whether the simultaneous equations

$$\begin{aligned} ax + y + z &= 2a, \\ x + ay + z &= -1, \\ x + y + 2z &= -1, \end{aligned}$$

have any solutions when

(a) $a = 0$,

(b) $a = 1$.

[4]

10 The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 3$ and $u_{n+1} = 3u_n - 2$.

(i) Find u_2 and u_3 and verify that $\frac{1}{2}(u_4 - 1) = 27$. **[3]**

(ii) Hence suggest an expression for u_n. **[2]**

(iii) Use induction to prove that your answer to part **(ii)** is correct. **[5]**

RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS **4725**

Further Pure Mathematics 1

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Wednesday 20 January 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 2 \\ 3 & 4 \end{pmatrix}$ and **I** is the 2×2 identity matrix.

(i) Find $\mathbf{A} - 4\mathbf{I}$. **[2]**

(ii) Given that **A** is singular, find the value of a. **[3]**

2 The cubic equation $2x^3 + 3x - 3 = 0$ has roots α, β and γ.

(i) Use the substitution $x = u - 1$ to find a cubic equation in u with integer coefficients. **[3]**

(ii) Hence find the value of $(\alpha + 1)(\beta + 1)(\gamma + 1)$. **[2]**

3 The complex number z satisfies the equation $z + 2\mathrm{i}z^* = 12 + 9\mathrm{i}$. Find z, giving your answer in the form $x + \mathrm{i}y$. **[5]**

4 Find $\sum_{r=1}^{n} r(r+1)(r-2)$, expressing your answer in a fully factorised form. **[6]**

5 **(i)** The transformation T is represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Give a geometrical description of T. **[2]**

(ii) The transformation T is equivalent to a reflection in the line $y = -x$ followed by another transformation S. Give a geometrical description of S and find the matrix that represents S. **[4]**

6 One root of the cubic equation $x^3 + px^2 + 6x + q = 0$, where p and q are real, is the complex number $5 - \mathrm{i}$.

(i) Find the real root of the cubic equation. **[3]**

(ii) Find the values of p and q. **[4]**

7 **(i)** Show that $\frac{1}{r^2} - \frac{1}{(r+1)^2} \equiv \frac{2r+1}{r^2(r+1)^2}$. **[1]**

(ii) Hence find an expression, in terms of n, for $\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2}$. **[4]**

(iii) Find $\sum_{r=2}^{\infty} \frac{2r+1}{r^2(r+1)^2}$. **[2]**

8 The complex number a is such that $a^2 = 5 - 12\mathrm{i}$.

(i) Use an algebraic method to find the two possible values of a. **[5]**

(ii) Sketch on a single Argand diagram the two possible loci given by $|z - a| = |a|$. **[4]**

9 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 3 & 1 \\ 1 & 1 & a \end{pmatrix}$, where $a \neq 1$.

(i) Find $\mathbf{A}^{-1}$. **[7]**

(ii) Hence, or otherwise, solve the equations

$$\begin{aligned} 2x - y + z &= 1, \\ 3y + z &= 2, \\ x + y + az &= 2. \end{aligned}$$ **[4]**

10 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

(i) Find $\mathbf{M}^2$ and $\mathbf{M}^3$. **[3]**

(ii) Hence suggest a suitable form for the matrix $\mathbf{M}^n$. **[1]**

(iii) Use induction to prove that your answer to part **(ii)** is correct. **[4]**

(iv) Describe fully the single geometrical transformation represented by $\mathbf{M}^{10}$. **[3]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS

4725

Further Pure Mathematics 1

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:

- Scientific or graphical calculator

Friday 11 June 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

2R–9K03

Turn over

1 Prove by induction that, for $n \geqslant 1$, $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$. **[5]**

2 The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 1 & -4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 & 0 \\ -2 & 2 \end{pmatrix}$. Find

(i) **AB**, **[2]**

(ii) **BA** – 4**C**. **[4]**

3 Find $\sum_{r=1}^{n} (2r-1)^2$, expressing your answer in a fully factorised form. **[6]**

4 The complex numbers a and b are given by $a = 7 + 6\mathrm{i}$ and $b = 1 - 3\mathrm{i}$. Showing clearly how you obtain your answers, find

(i) $|a - 2b|$ and $\arg(a - 2b)$, **[4]**

(ii) $\frac{b}{a}$, giving your answer in the form $x + \mathrm{i}y$. **[3]**

5 **(a)** Write down the matrix that represents a reflection in the line $y = x$. **[2]**

(b) Describe fully the geometrical transformation represented by each of the following matrices:

(i) $\begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}$, **[2]**

(ii) $\begin{pmatrix} \frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$. **[2]**

6 **(i)** Sketch on a single Argand diagram the loci given by

(a) $|z - 3 + 4\mathrm{i}| = 5$, **[2]**

(b) $|z| = |z - 6|$. **[2]**

(ii) Indicate, by shading, the region of the Argand diagram for which

$$|z - 3 + 4\mathrm{i}| \leqslant 5 \quad \text{and} \quad |z| \geqslant |z - 6|.$$ **[2]**

7 The quadratic equation $x^2 + 2kx + k = 0$, where k is a non-zero constant, has roots α and β. Find a quadratic equation with roots $\frac{\alpha + \beta}{\alpha}$ and $\frac{\alpha + \beta}{\beta}$. **[7]**

8 **(i)** Show that $\dfrac{1}{\sqrt{r+2}+\sqrt{r}} \equiv \dfrac{\sqrt{r+2}-\sqrt{r}}{2}$. **[2]**

(ii) Hence find an expression, in terms of n, for

$$\sum_{r=1}^{n} \frac{1}{\sqrt{r+2}+\sqrt{r}}.$$ **[6]**

(iii) State, giving a brief reason, whether the series $\displaystyle\sum_{r=1}^{\infty} \frac{1}{\sqrt{r+2}+\sqrt{r}}$ converges. **[1]**

9 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} a & a & -1 \\ 0 & a & 2 \\ 1 & 2 & 1 \end{pmatrix}$.

(i) Find, in terms of a, the determinant of $\mathbf{A}$. **[3]**

(ii) Three simultaneous equations are shown below.

$$\begin{aligned} ax + ay - z &= -1 \\ ay + 2z &= 2a \\ x + 2y + z &= 1 \end{aligned}$$

For each of the following values of a, determine whether the equations are consistent or inconsistent. If the equations are consistent, determine whether or not there is a unique solution.

(a) $a = 0$

(b) $a = 1$

(c) $a = 2$

[6]

10 The complex number z, where $0 < \arg z < \frac{1}{2}\pi$, is such that $z^2 = 3 + 4\text{i}$.

(i) Use an algebraic method to find z. **[5]**

(ii) Show that $z^3 = 2 + 11\text{i}$. **[1]**

The complex number w is the root of the equation

$$w^6 - 4w^3 + 125 = 0$$

for which $-\frac{1}{2}\pi < \arg w < 0$.

(iii) Find w. **[5]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS 4725

Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4725
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Wednesday 19 January 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 2 & 5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 & -1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$. Find

(i) $2\mathbf{A} + \mathbf{B}$, **[2]**

(ii) $\mathbf{AC}$, **[2]**

(iii) $\mathbf{CB}$. **[3]**

2 The complex numbers z and w are given by $z = 4 + 3\mathrm{i}$ and $w = 6 - \mathrm{i}$. Giving your answers in the form $x + \mathrm{i}y$ and showing clearly how you obtain them, find

(i) $3z - 4w$, **[2]**

(ii) $\dfrac{z^*}{w}$. **[4]**

3 The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 2$, and $u_{n+1} = 2u_n - 1$ for $n \geqslant 1$. Prove by induction that $u_n = 2^{n-1} + 1$. **[4]**

4 Given that $\sum_{r=1}^{n}(ar^3 + br) \equiv n(n-1)(n+1)(n+2)$, find the values of the constants a and b. **[6]**

5 Given that **A** and **B** are non-singular square matrices, simplify

$$\mathbf{AB}(\mathbf{A}^{-1}\mathbf{B})^{-1}.$$ **[3]**

6 **(i)** Sketch on a single Argand diagram the loci given by

(a) $|z| = |z - 8|$, **[2]**

(b) $\arg(z + 2\mathrm{i}) = \frac{1}{4}\pi$. **[3]**

(ii) Indicate by shading the region of the Argand diagram for which

$$|z| \leqslant |z - 8| \quad \text{and} \quad 0 \leqslant \arg(z + 2\mathrm{i}) \leqslant \tfrac{1}{4}\pi.$$ **[3]**

7 **(i)** Write down the matrix, **A**, that represents a shear with x-axis invariant in which the image of the point (1, 1) is (4, 1). **[2]**

(ii) The matrix **B** is given by $\mathbf{B} = \begin{pmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{3} \end{pmatrix}$. Describe fully the geometrical transformation represented by **B**. **[2]**

(iii) The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 2 & 6 \\ 0 & 2 \end{pmatrix}$.

(a) Draw a diagram showing the unit square and its image under the transformation represented by **C**. **[3]**

(b) Write down the determinant of **C** and explain briefly how this value relates to the transformation represented by **C**. **[2]**

8 The quadratic equation $2x^2 - x + 3 = 0$ has roots α and β, and the quadratic equation $x^2 - px + q = 0$ has roots $\alpha + \frac{1}{\alpha}$ and $\beta + \frac{1}{\beta}$.

(i) Show that $p = \frac{5}{6}$. **[4]**

(ii) Find the value of q. **[5]**

9 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} a & -a & 1 \\ 3 & a & 1 \\ 4 & 2 & 1 \end{pmatrix}$.

(i) Find, in terms of a, the determinant of **M**. **[3]**

(ii) Hence find the values of a for which $\mathbf{M}^{-1}$ does not exist. **[3]**

(iii) Determine whether the simultaneous equations

$$6x - 6y + z = 3k,$$
$$3x + 6y + z = 0,$$
$$4x + 2y + z = k,$$

where k is a non-zero constant, have a unique solution, no solution or an infinite number of solutions, justifying your answer. **[3]**

10 **(i)** Show that $\frac{1}{r} - \frac{2}{r+1} + \frac{1}{r+2} \equiv \frac{2}{r(r+1)(r+2)}$. **[2]**

(ii) Hence find an expression, in terms of n, for

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)}.$$ **[6]**

(iii) Show that $\sum_{r=n+1}^{\infty} \frac{2}{r(r+1)(r+2)} = \frac{1}{(n+1)(n+2)}$. **[3]**

OCR RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS 4725

Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4725
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Thursday 16 June 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 2 & a \\ 0 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & a \\ 4 & 1 \end{pmatrix}$. **I** denotes the 2×2 identity matrix. Find

(i) $\mathbf{A} + 3\mathbf{B} - 4\mathbf{I}$, **[3]**

(ii) $\mathbf{AB}$. **[2]**

2 Prove by induction that, for $n \geqslant 1$, $\displaystyle\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$. **[5]**

3 By using the determinant of an appropriate matrix, find the values of k for which the simultaneous equations

$$kx + 8y = 1,$$
$$2x + ky = 3,$$

do not have a unique solution. **[3]**

4 Find $\displaystyle\sum_{r=1}^{2n} \left(3r^2 - \tfrac{1}{2}\right)$, expressing your answer in a fully factorised form. **[6]**

5 The complex number $1 + i\sqrt{3}$ is denoted by a.

(i) Find $|a|$ and $\arg a$. **[2]**

(ii) Sketch on a single Argand diagram the loci given by $|z - a| = |a|$ and $\arg(z - a) = \frac{1}{2}\pi$. **[6]**

6 The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} a & 1 & 0 \\ 1 & 2 & 1 \\ -1 & 3 & 4 \end{pmatrix}$, where $a \neq 1$. Find $\mathbf{C}^{-1}$. **[7]**

7 **(i)** Show that $\dfrac{1}{r-1} - \dfrac{1}{r+1} \equiv \dfrac{2}{r^2 - 1}$. **[1]**

(ii) Hence find an expression, in terms of n, for $\displaystyle\sum_{r=2}^{n} \frac{2}{r^2 - 1}$. **[5]**

(iii) Find the value of $\displaystyle\sum_{r=1000}^{\infty} \frac{2}{r^2 - 1}$. **[3]**

8 The matrix $\mathbf{X}$ is given by $\mathbf{X} = \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix}$.

(i) The diagram in the printed answer book shows the unit square $OABC$. The image of the unit square under the transformation represented by $\mathbf{X}$ is $OA'B'C'$. Draw and label $OA'B'C'$. **[3]**

(ii) The transformation represented by $\mathbf{X}$ is equivalent to a transformation A, followed by a transformation B. Give geometrical descriptions of possible transformations A and B and state the matrices that represent them. **[4]**

9 One root of the quadratic equation $x^2 + ax + b = 0$, where a and b are real, is $16 - 30\text{i}$.

(i) Write down the other root of the quadratic equation. **[1]**

(ii) Find the values of a and b. **[4]**

(iii) Use an algebraic method to solve the quartic equation $y^4 + ay^2 + b = 0$. **[7]**

10 The cubic equation $x^3 + 3x^2 + 2 = 0$ has roots α, β and γ.

(i) Use the substitution $x = \frac{1}{\sqrt{u}}$ to show that $4u^3 + 12u^2 + 9u - 1 = 0$. **[5]**

(ii) Hence find the values of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$ and $\frac{1}{\alpha^2\beta^2} + \frac{1}{\beta^2\gamma^2} + \frac{1}{\gamma^2\alpha^2}$. **[5]**

OCR RECOGNISING ACHIEVEMENT

Friday 20 January 2012 – Afternoon

AS GCE MATHEMATICS

4725 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4725
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

4732840112

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The complex number $a + 5\mathrm{i}$, where a is positive, is denoted by z. Given that $|z| = 13$, find the value of a and hence find $\arg z$. **[4]**

2 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 3 & 4 \\ 2 & -3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 4 & 6 \\ 3 & -5 \end{pmatrix}$, and **I** is the 2×2 identity matrix. Given that $p\mathbf{A} + q\mathbf{B} = \mathbf{I}$, find the values of the constants p and q. **[5]**

3 Use an algebraic method to find the square roots of $3 + (6\sqrt{2})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are exact real numbers. **[6]**

4 Find $\sum_{r=1}^{n} r(r^2 - 3)$, expressing your answer in a fully factorised form. **[6]**

5 **(a)** Find the matrix that represents a reflection in the line $y = -x$. **[2]**

(b) The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$.

(i) Describe fully the geometrical transformation represented by **C**. **[2]**

(ii) State the value of the determinant of **C** and describe briefly how this value relates to the transformation represented by **C**. **[2]**

6 Sketch, on a single Argand diagram, the loci given by $|z - \sqrt{3} - \mathrm{i}| = 2$ and $\arg z = \frac{1}{6}\pi$. **[6]**

7 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix}$.

(i) Show that $\mathbf{M}^4 = \begin{pmatrix} 81 & 0 \\ 80 & 1 \end{pmatrix}$. **[3]**

(ii) Hence suggest a suitable form for the matrix $\mathbf{M}^n$, where n is a positive integer. **[2]**

(iii) Use induction to prove that your answer to part **(ii)** is correct. **[4]**

8 **(i)** Show that $\frac{r}{r+1} - \frac{r-1}{r} \equiv \frac{1}{r(r+1)}$. **[2]**

(ii) Hence find an expression, in terms of n, for

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots + \frac{1}{n(n+1)}.$$ **[4]**

(iii) Hence find $\sum_{r=n+1}^{\infty} \frac{1}{r(r+1)}$. **[2]**

9 The matrix $\mathbf{X}$ is given by $\mathbf{X} = \begin{pmatrix} a & 2 & 9 \\ 2 & a & 3 \\ 1 & 0 & -1 \end{pmatrix}$.

(i) Find the determinant of $\mathbf{X}$ in terms of a. **[3]**

(ii) Hence find the values of a for which $\mathbf{X}$ is singular. **[3]**

(iii) Given that $\mathbf{X}$ is non-singular, find $\mathbf{X}^{-1}$ in terms of a. **[4]**

10 The cubic equation $3x^3 - 9x^2 + 6x + 2 = 0$ has roots α, β and γ.

(i) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. **[3]**

The cubic equation $x^3 + ax^2 + bx + c = 0$ has roots α^2, β^2 and γ^2.

(ii) Show that $c = -\frac{4}{9}$ and find the values of a and b. **[9]**

OCR
RECOGNISING ACHIEVEMENT

Friday 1 June 2012 – Morning

AS GCE MATHEMATICS

4725 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4725
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The complex numbers z and w are given by $z = 6 - \mathrm{i}$ and $w = 5 + 4\mathrm{i}$. Giving your answers in the form $x + \mathrm{i}y$ and showing clearly how you obtain them, find

(i) $z + 3w$, **[2]**

(ii) $\dfrac{z}{w}$. **[3]**

2 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$. Find

(i) **AB**, **[2]**

(ii) $\mathbf{B}^{-1}\mathbf{A}^{-1}$. **[3]**

3 One root of the quadratic equation $x^2 + ax + b = 0$, where a and b are real, is the complex number $4 - 3\mathrm{i}$. Find the values of a and b. **[4]**

4 Find $\displaystyle\sum_{r=1}^{n} (3r^2 - 3r + 2)$, expressing your answer in a fully factorised form. **[7]**

5 Prove by induction that, for $n \geqslant 1$, $\displaystyle\sum_{r=1}^{n} 4 \times 3^r = 6(3^n - 1)$. **[5]**

6 The quadratic equation $2x^2 + x + 5 = 0$ has roots α and β.

(i) Use the substitution $x = \dfrac{1}{u+1}$ to obtain a quadratic equation in u with integer coefficients. **[3]**

(ii) Hence, or otherwise, find the value of $\left(\dfrac{1}{\alpha} - 1\right)\left(\dfrac{1}{\beta} - 1\right)$. **[3]**

7 The loci C_1 and C_2 are given by $|z - 3 - 4\mathrm{i}| = 4$ and $|z| = |z - 8\mathrm{i}|$ respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. **[6]**

(ii) Hence find the complex numbers represented by the points of intersection of C_1 and C_2. **[2]**

(iii) Indicate, by shading, the region of the Argand diagram for which

$$|z - 3 - 4\mathrm{i}| \leqslant 4 \text{ and } |z| \geqslant |z - 8\mathrm{i}|.$$ **[2]**

8 **(i)** Show that $\dfrac{1}{r} - \dfrac{1}{r+2} \equiv \dfrac{2}{r(r+2)}$. **[1]**

(ii) Hence find an expression, in terms of n, for $\displaystyle\sum_{r=1}^{n} \frac{2}{r(r+2)}$. **[6]**

(iii) Given that $\displaystyle\sum_{r=N+1}^{\infty} \frac{2}{r(r+2)} = \frac{11}{30}$, find the value of N. **[4]**

9 **(i)** The matrix **X** is given by $\mathbf{X} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. Describe fully the geometrical transformation represented by **X**. **[2]**

(ii) The matrix **Z** is given by $\mathbf{Z} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2}(2+\sqrt{3}) \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2}(1-2\sqrt{3}) \end{pmatrix}$. The transformation represented by **Z** is equivalent to the transformation represented by **X**, followed by another transformation represented by the matrix **Y**. Find **Y**. **[5]**

(iii) Describe fully the geometrical transformation represented by **Y**. **[2]**

10 The matrix **D** is given by $\mathbf{D} = \begin{pmatrix} a & 2 & -1 \\ 2 & a & 1 \\ 1 & 1 & a \end{pmatrix}$.

(i) Find the determinant of **D** in terms of a. **[3]**

(ii) Three simultaneous equations are shown below.

$$\begin{aligned} ax + 2y - z &= 0 \\ 2x + ay + z &= a \\ x + y + az &= a \end{aligned}$$

For each of the following values of a, determine whether or not there is a unique solution. If the solution is not unique, determine whether the equations are consistent or inconsistent.

(a) $a = 3$

(b) $a = 2$

(c) $a = 0$ **[7]**

OCR
RECOGNISING ACHIEVEMENT

Wednesday 23 January 2013 – Morning

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 1 \\ 1 & 4 \end{pmatrix}$, where $a \neq \frac{1}{4}$, and **I** denotes the 2×2 identity matrix. Find

(i) $2\mathbf{A} - 3\mathbf{I}$, **[3]**

(ii) $\mathbf{A}^{-1}$. **[2]**

2 Find $\sum_{r=1}^{n} (r-1)(r+1)$, giving your answer in a fully factorised form. **[6]**

3 The complex number $2 - \mathrm{i}$ is denoted by z.

(i) Find $|z|$ and $\arg z$. **[2]**

(ii) Given that $az + bz^* = 4 - 8\mathrm{i}$, find the values of the real constants a and b. **[5]**

4 The quadratic equation $x^2 + x + k = 0$ has roots α and β.

(i) Use the substitution $x = 2u + 1$ to obtain a quadratic equation in u. **[2]**

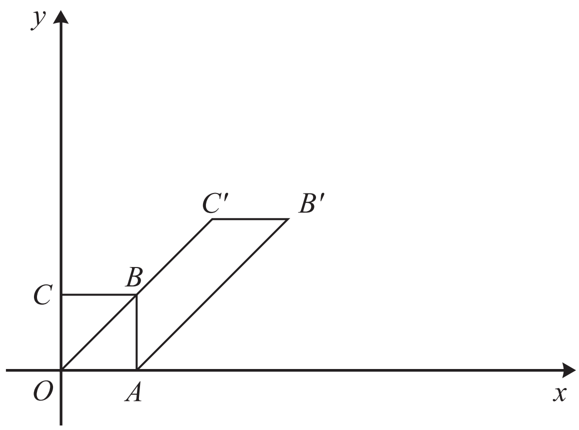
(ii) Hence, or otherwise, find the value of $\left(\frac{\alpha - 1}{2}\right)\left(\frac{\beta - 1}{2}\right)$ in terms of k. **[2]**

5 By using the determinant of an appropriate matrix, find the values of λ for which the simultaneous equations

$$\begin{aligned} 3x + 2y + 4z &= 5, \\ \lambda y + z &= 1, \\ x + \lambda y + \lambda z &= 4, \end{aligned}$$

do not have a unique solution for x, y and z. **[6]**

6



The diagram shows the unit square $OABC$, and its image $OAB'C'$ after a transformation. The points have the following coordinates: $A(1, 0)$, $B(1, 1)$, $C(0, 1)$, $B'(3, 2)$ and $C'(2, 2)$.

(i) Write down the matrix, $\mathbf{X}$, for this transformation. **[2]**

(ii) The transformation represented by $\mathbf{X}$ is equivalent to a transformation P followed by a transformation Q. Give geometrical descriptions of a pair of possible transformations P and Q and state the matrices that represent them. **[6]**

(iii) Find the matrix that represents transformation Q followed by transformation P. **[2]**

7 **(i)** Sketch on a single Argand diagram the loci given by

(a) $|z| = 2$, **[2]**

(b) $\arg(z - 3 - \mathrm{i}) = \pi$. **[3]**

(ii) Indicate, by shading, the region of the Argand diagram for which

$$|z| \leqslant 2 \text{ and } 0 \leqslant \arg(z - 3 - \mathrm{i}) \leqslant \pi.$$ **[2]**

8 **(i)** Show that $\dfrac{1}{r} - \dfrac{3}{r+1} + \dfrac{2}{r+2} \equiv \dfrac{2-r}{r(r+1)(r+2)}$. **[2]**

(ii) Hence show that $\displaystyle\sum_{r=1}^{n} \frac{2-r}{r(r+1)(r+2)} = \frac{n}{(n+1)(n+2)}$. **[5]**

(iii) Find the value of $\displaystyle\sum_{r=2}^{\infty} \frac{2-r}{r(r+1)(r+2)}$. **[2]**

9 **(i)** Show that $(\alpha\beta + \beta\gamma + \gamma\alpha)^2 \equiv \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 + 2\alpha\beta\gamma(\alpha + \beta + \gamma)$. **[3]**

(ii) It is given that α, β and γ are the roots of the cubic equation $x^3 + px^2 - 4x + 3 = 0$, where p is a constant. Find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$ in terms of p. **[5]**

10 The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 2$ and $u_{n+1} = \frac{u_n}{1 + u_n}$ for $n \geqslant 1$.

(i) Find u_2 and u_3, and show that $u_4 = \frac{2}{7}$. **[3]**

(ii) Hence suggest an expression for u_n. **[2]**

(iii) Use induction to prove that your answer to part **(ii)** is correct. **[5]**

OCR RECOGNISING ACHIEVEMENT

Monday 10 June 2013 – Morning

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

4733206613

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The complex number $3 + a\mathrm{i}$, where a is real, is denoted by z. Given that $\arg z = \frac{1}{6}\pi$, find the value of a and hence find $|z|$ and $z^* - 3$. **[6]**

2 The matrices **A**, **B** and **C** are given by $\mathbf{A} = (5 \quad 1)$, $\mathbf{B} = (2 \quad -5)$ and $\mathbf{C} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

(i) Find $3\mathbf{A} - 4\mathbf{B}$. **[2]**

(ii) Find **CB**. Determine whether **CB** is singular or non-singular, giving a reason for your answer. **[5]**

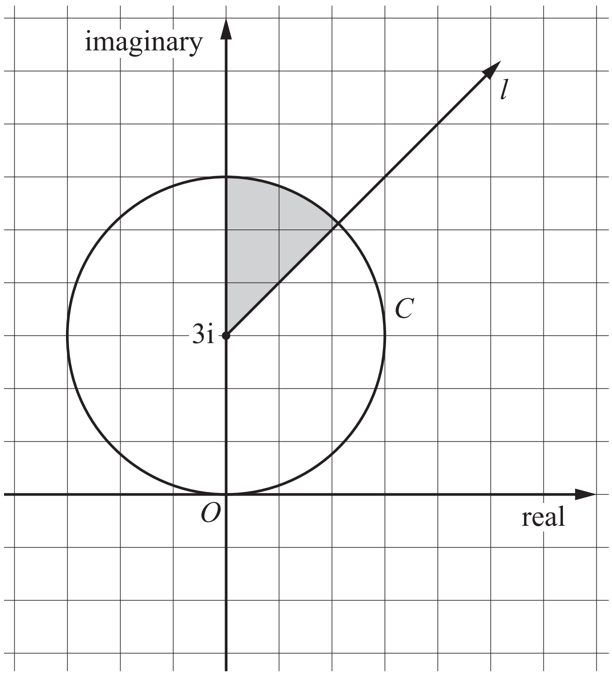
3 Use an algebraic method to find the square roots of $11 + (12\sqrt{5})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are exact real numbers. **[6]**

4 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}$. Prove by induction that, for $n \geqslant 1$,

$$\mathbf{M}^n = \begin{pmatrix} 2^n & 2^{n+1} - 2 \\ 0 & 1 \end{pmatrix}.$$ **[6]**

5 Find $\sum_{r=1}^{n} (4r^3 - 3r^2 + r)$, giving your answer in a fully factorised form. **[6]**

6



The Argand diagram above shows a half-line l and a circle C. The circle has centre $3\mathrm{i}$ and passes through the origin.

(i) Write down, in complex number form, the equations of l and C. **[4]**

(ii) Write down inequalities that define the region shaded in the diagram. [The shaded region includes the boundaries.] **[3]**

7 **(i)** Find the matrix that represents a rotation through 90° clockwise about the origin. **[2]**

(ii) Find the matrix that represents a reflection in the x-axis. **[2]**

(iii) Hence find the matrix that represents a rotation through 90° clockwise about the origin, followed by a reflection in the x-axis. **[2]**

(iv) Describe a **single** transformation that is represented by your answer to part **(iii)**. **[2]**

8 The cubic equation $kx^3 + 6x^2 + x - 3 = 0$, where k is a non-zero constant, has roots α, β and γ.

Find the value of $(\alpha + 1)(\beta + 1) + (\beta + 1)(\gamma + 1) + (\gamma + 1)(\alpha + 1)$ in terms of k. **[6]**

9 **(i)** Show that $\dfrac{1}{3r-1} - \dfrac{1}{3r+2} \equiv \dfrac{3}{(3r-1)(3r+2)}$. **[2]**

(ii) Hence show that $\displaystyle\sum_{r=1}^{2n} \frac{1}{(3r-1)(3r+2)} = \frac{n}{2(3n+1)}$. **[6]**

10 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} a & 2 & 1 \\ 1 & 3 & 2 \\ 4 & 1 & 1 \end{pmatrix}$.

(i) Find the value of a for which $\mathbf{A}$ is singular. **[5]**

(ii) Given that $\mathbf{A}$ is non-singular, find $\mathbf{A}^{-1}$ and hence solve the equations

$$\begin{aligned} ax + 2y + z &= 1, \\ x + 3y + 2z &= 2, \\ 4x + y + z &= 3. \end{aligned}$$

[7]