OCR Maths FP1

Mark Scheme Pack

2005-2013

Mark Scheme 4725
June 2005

1.	$6\Sigma r^2 + 2\Sigma r + \Sigma 1$	M1		Consider the sum of three separate terms
	$6\Sigma r^2 = n(n+1)(2n+1)$	A1		Correct formula stated
	$2\Sigma r = n(n+1)$	A1		Correct formula stated
	$\Sigma 1 = n$	A1		Correct term seen
	$n(2n^2 + 4n + 3)$	M1	6	Correct algebraic processes including factorisation and simplification
		A1	**6**	Obtain given answer correctly
2.	(i) $A^2 = \begin{pmatrix} 3 & 8 \\ 4 & 11 \end{pmatrix}$	M1		Attempt to find A^2, 2 elements correct
		A1		All elements correct
	$4A = \begin{pmatrix} 4 & 8 \\ 4 & 12 \end{pmatrix}$	M1		Use correct matrix $4\mathbf{A}$
	$\mathbf{A}^2 = 4\mathbf{A} - \mathbf{I}$	A1	4	Obtain given answer correctly
	(ii) $\mathbf{A}^{-1} = 4\mathbf{I} - \mathbf{A}$	M1	2	Multiply answer to (i) by $\mathbf{A}^{-1}$ or obtain $\mathbf{A}^{-1}$ or factorise $\mathbf{A}^2 - 4\mathbf{A}$
		A1	**6**	Obtain given answer correctly
3.	(i) $22 - 2i$	B1B1	2	Correct real and imaginary parts
	(ii) $z^* = 2 - 3i$	B1		Correct conjugate seen or implied
	$5 - 14i$	B1B1	3	Correct real and imaginary parts
	(iii) $\frac{4}{17} + \frac{1}{17}i$	M1		Attempt to use w^*
		A1	2	Obtain correct answer in any form
			7	

4.	$x^2 - y^2 = 21$ and $xy = -10$	M1		Attempt to equate real and imaginary parts of $(x + iy)^2$ and 21 –20i
		A1A1		Obtain each result
		M1		Eliminate to obtain a quadratic in x^2 or y^2
		M1		Solve to obtain $x = (\pm)\,5$ or $y = (\pm)\,2$
	$\pm(5 - 2\mathrm{i})$	A1	6	Obtain correct answers as complex numbers
			6	
5.	(i) $\dfrac{(r+1)^2 - r(r+2)}{(r+2)(r+1)}$	M1		Show correct process for subtracting fractions
	$\dfrac{1}{(r+1)(r+2)}$	A1	2	Obtain given answer correctly
	(ii) *EITHER*			
	$\dfrac{2}{3} - \dfrac{1}{2} + \dfrac{3}{4} - \dfrac{2}{3} \ldots \dfrac{n+1}{n+2} - \dfrac{n}{n+1}$	M1		Express terms as differences using (i)
		A1		At least first two and last term correct
		M1		Show or imply that pairs of terms cancel
	$\dfrac{n+1}{n+2} - \dfrac{1}{2}$	A1	4	Obtain correct answer in any form
	OR	M2		State that $\sum_{r=1}^{n} u_r = f(n+1) - f(1)$
		A1A1		Each term correct
	(iii) $\dfrac{1}{2}$	B1 ft	1	Obtain value from their sum to *n* terms
			7	
6.	(i) Circle	B1		Sketch(s) showing correct features, each mark independent
	Centre (0, 2)	B1		
	Radius 2	B1		
	Straight line	B1		
	Through origin with positive slope	B1	5	
	(ii) 0 or 0 +0i and 2 + 2i	B1ftB1ft	2	Obtain intersections as complex numbers
			7	
8.	(a) (i) $\alpha + \beta = 2$ $\alpha\beta = 4$	B1B1	2	Values stated
	(ii) *EITHER*			
	$\alpha^2 + \beta^2 = -4$	M1		Use $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
		A1	2	Obtain given answer correctly
	OR	M1		Find numeric values of roots, square and add
		A1		Obtain given answer correctly
	(iii)			
	$x^2 + 4x + 16 = 0$	B1		State or use $\alpha^2\beta^2 = 16$

				Or use substitution $u = x^2$
		M1		Write down a quadratic equation of correct form or rearrange and square
		A1	3	Obtain $x^2 + 4x + 16 = 0$
	(b) (i) $p = 2$	M1		Use sum or product of roots to obtain $6p = 12$ Or $6p^3 = 48$
		A1	2	Obtain $p = 2$
	(ii) $a = 44$	M1		Attempt to find $\Sigma\alpha\beta$ numerically or in terms of p or substitute their 2, 4 or 6 in equation
		A1ft	2	Obtain $11p^2$
			11	
9.	(i) $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$	B1B1	2	Each column correct
	(ii) Shear, e.g. (0,1) transforms to (3,1)	B1B1	2	One example or sensible explanation
	(iii) $\mathbf{M} = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$	M1		Attempt to find **DC** (not **CD**)
		A1	2	Obtain given answer
	(iv)	B1		Explicit check for n = 1or n = 2
	$\mathbf{M}^k = \begin{pmatrix} 2^k & 3(2^k - 1) \\ 0 & 1 \end{pmatrix}$.	M1		Induction hypothesis that result is true for $\mathbf{M}^k$
		M1		Attempt to multiply $\mathbf{MM}^k$ or vice versa
	$\begin{pmatrix} 2^{k+1} & 3(2^{k+1} - 1) \\ 0 & 1 \end{pmatrix}$.	A1		Element $3(2^{k+1} - 1)$ derived correctly
		A1		All other elements correct
		A1	6	Explicit statement of induction conclusion
			12	

Mark Scheme 4725
January 2006

Mark Total

1.	(i) $2 + 16i - i - 8i^2$ $10 + 15i$ (ii) $\frac{1}{5}(10 + 15i)$ or $2 + 3i$	M1 A1 M1 A1 A1ft	2 3 **5**	Attempt to multiply correctly Obtain correct answer Multiply numerator & denominator by conjugate Obtain denominator 5 Their part (i) or 10 + 15i derived again / 5
2.	$1^2 = \frac{1}{6} \times 1 \times 2 \times 3$ $\frac{1}{6}n(n+1)(2n+1) + (n+1)^2$ $\frac{1}{6}(n+1)(n+2)\{2(n+1)+1\}$	B1 M1 DM1 A1 A1	5 **5**	Show result true for n = 1or 2 Add next term to given sum formula, any letter OK Attempt to factorise or expand and simplify Correct expression obtained Specific statement of induction conclusion, with no errors seen
3.	(i) $2\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} - 1\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ 2 x 5 – 1 x 2 +3 x -1 5 (ii)	M1 A1 A1 B1ft	3 1 **4**	Show correct expansion process, allow sign slips Obtain correct (unsimplified) expression Obtain correct answer State that **M** is non-singular as det **M** non-zero, ft their determinant
4.	$u^2 + 4u + 4$ $u^3 + 6u^2 + 12u + 8$ $u = \sqrt[3]{5}$ $x = 2 + \sqrt[3]{5}$	B1 M1 A1 A1ft A1ft	5 **5**	$u + 2$ squared and cubed correctly Substitute these and attempt to simplify Obtain $u^3 - 5 = 0$ or equivalent Correct solution to their equation Obtain 2 + their answer [Decimals score 0/2 of final A marks]

5.	$8\Sigma r^3 - 6\Sigma r^2 + 2\Sigma r$	M1		Consider the sum of three separate terms
	$8\Sigma r^3 = 2n^2(n+1)^2$	A1		Correct formula stated or used a.e.f.
	$6\Sigma r^2 = n(n+1)(2n+1)$	A1		Correct formula stated or used a.e.f.
	$2\Sigma r = n(n+1)$	A1		Correct term seen
	$2n^3(n+1)$ **AG**	M1 A1	6 **6**	Attempt to factorise or expand and simplify Obtain given answer correctly

6.	(i) $\frac{1}{2}\begin{pmatrix} 8 & -2 \\ -3 & 1 \end{pmatrix}$	B1 B1	2	Transpose leading diagonal and negate other diagonal Divide by determinant
	(ii) Either	B1 M1A1		State or imply $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ Use this result and obtain $\mathbf{B}^{-1} = \mathbf{C}^{-1}\mathbf{A}$, or equivalent matrix algebra
	$\frac{1}{2}\begin{pmatrix} 14 & 2 \\ -5 & 0 \end{pmatrix}$	M1 A1ft	5	Matrix multn., two elements correct, for any pair All elements correct ft their (i)
	Or			
	$\frac{1}{5}\begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$	B1		Find $\mathbf{A}^{-1}$
	$\mathbf{B} = \mathbf{A}^{-1}\mathbf{C}$	M1		Premultiply by $\mathbf{A}^{-1}$ stated or implied
	$\mathbf{B} = \frac{1}{5}\begin{pmatrix} 0 & -2 \\ 5 & 14 \end{pmatrix}$	M1 A1ft		Matrix multn. Two elements correct All elements correct
	$\frac{1}{2}\begin{pmatrix} 14 & 2 \\ -5 & 0 \end{pmatrix}$	A1		Correct $\mathbf{B}^{-1}$
	Or			
	$\mathbf{AB} = \begin{pmatrix} 2a+c & 2b+d \\ a+3c & b+3d \end{pmatrix}$	B1		Find **AB**
	$a = 0, c = 1, b = -0.4, d = 2.8$	M1		Solve one pair of simultaneous equations
		A1A1		Each pair of answers
	$\frac{1}{2}\begin{pmatrix} 14 & 2 \\ -5 & 0 \end{pmatrix}$	A1	**7**	Correct $\mathbf{B}^{-1}$

7.	(a) (i) $\sqrt{13}$	B1	1	Obtain correct answer, decimals OK
	(ii)			
		M1		Using $\tan^{-1} {}^{b}/_{a}$, or equivalent trig allow + or -
	- 0.59	A1		Obtain 0.59
		A1	3	Obtain correct answer
	(b)	M1		Express LHS in Cartesian form & equate real and imaginary parts
	1 – 2i	A1A1		Obtain x = 1 and y = -2
		A1	4	Correct answer written as a complex number
	(c)	B1		Sketch of vertical straight line
		B1	2	Through (- 0.5, 0)
			10	
8.	(i)	B1		For correct vertex (2, -2)
	$\begin{pmatrix}0\\0\end{pmatrix}$ $\begin{pmatrix}2\\0\end{pmatrix}$ $\begin{pmatrix}2\\-2\end{pmatrix}$ $\begin{pmatrix}0\\-2\end{pmatrix}$	B1		For all vertices correct
		B1	3	For correct diagram
	(ii) Either $\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$	B1,B1		Reflection, in x-axis
		B1		Correct matrix
	$\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}$	B1,B1		Enlargement, centre O s.f.2
		B1	6	Correct matrix
	Or $\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$	B1,B1		Reflection, in the y-axis
		B1		Correct matrix
	$\begin{pmatrix}-2 & 0\\0 & -2\end{pmatrix}$	B1,B1		Enlargement, centre O s.f. –2
		B1		Correct matrix
	Or $\begin{pmatrix}2 & 0\\0 & 1\end{pmatrix}$	B1,B1		Stretch, in x-direction s.f. 2
		B1		Correct matrix
	$\begin{pmatrix}1 & 0\\0 & -2\end{pmatrix}$	B1,B1		Stretch, in y-direction s.f. -2
		B1		Correct matrix
			9	

9.	(i) $\frac{r+2-r}{r(r+2)}$	M1		Show correct process for subtracting fractions
	$\frac{2}{r(r+2)}$ **AG**	A1	2	Obtain given answer correctly
	(ii)	M1		Express terms as differences using (i)
		M1		Express 1st 3 (or last 3) terms so that cancelling occurs
		A1		Obtain $1+\frac{1}{2}$
		A1		Obtain $-\frac{1}{n+2}$, $-\frac{1}{n+1}$
	$\frac{3}{2}-\frac{1}{n+1}-\frac{1}{n+2}$	A1	5	Obtain correct answer in any form
	(iii) (a) $\frac{3}{2}$	B1ft	1	Obtain value from their sum to n terms
	(b) $\frac{1}{n+1}+\frac{1}{n+2}$	M1		Using (iii) (a) – (ii) or method of differences again [$n \to \infty$ is a method error]
		A1 ft	2	Obtain answer in any form
			10	
10.	(i) $\alpha+\beta+\gamma=9$	B1	1	
	(ii)	B1		State or use other root is p - iq
		M1		Substitute into (i)
	$p=\frac{9-\alpha}{2}$	A1		Obtain $2p+\alpha=9$
		A1	4	Obtain correct answer a.e.f.
	(iii) $\alpha\beta\gamma=29$	B1	1	
	(iv) $\alpha(p^2+q^2)=29$	M1		Substitute into (iii)
		A1ft		Obtain unsimplified expression with no i's
		M1		Rearrange to obtain q or q^2
		M1		Substitute their expression for p a.e.f.
	$q=\sqrt{\frac{29}{\alpha}-\frac{(9-\alpha)^2}{4}}$	A1	5	Obtain correct answer a.e.f.
			11	
	(iv) Alternative method $2p\alpha+p^2+q^2=27$	M1		Substitute into $\alpha\beta+\beta\gamma+\gamma\alpha=27$
		A1		Obtain unsimplified expression with no i's
		M1		Rearrange to obtain q or q^2
		M1		Substitute their expression for p a.e.f.
	$q=\sqrt{27-\frac{(9-\alpha)^2}{4}-\alpha(9-\alpha)}$	A1		Obtain correct answer a.e.f.

Mark Scheme 4725
June 2006

1.	i) $\begin{pmatrix} 7 & 4 \\ 0 & -1 \end{pmatrix}$	B1 B1	 2	Two elements correct All four elements correct
	(ii) $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ $k = 3$	B1 B1	 2 $\boxed{4}$	**A** – **B** correctly found Find k
2	(i) 	M1 A1	 2	For 2 other correct vertices For completely correct diagram
	(ii) $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$	B1 B1	2 $\boxed{4}$	Each column correct
3.	(i) 2 + 3i	B1	1	Conjugate seen
	(ii) $p = -4$ $q = 13$	M1 A1 M1 A1	 4 $\boxed{5}$	Attempt to sum roots or consider x terms in expansion or substitute 2 – 3i into equation and equate imaginary parts Correct answer Attempt at product of roots or consider last term in expansion or consider real parts Correct answer

4.	$\Sigma r^3 + \Sigma r^2$	M1		Consider the sum as two separate parts
	$\Sigma r^2 = \frac{1}{6}n(n+1)(2n+1)$	A1		Correct formula stated
	$\Sigma r^3 = \frac{1}{4}n^2(n+1)^2$	A1		Correct formula stated
	$\frac{1}{12}n(n+1)(n+2)(3n+1)$	M1		Attempt to factorise and simplify or expand both expressions
		A1	5	Obtain given answer correctly or complete verification
			$\boxed{5}$	
5.	(i) -7i	B1		Real part correct
		B1	2	Imaginary part correct
	(ii) 2 + 3i	B1		iz stated or implied or $i^2 = -1$ seen
		B1		Real part correct
	-5 + 12i	B1	3	Imaginary part correct
	(iii) $\frac{1}{5}(4 - 7i)$ or equivalent	M1		Multiply by conjugate
		A1		Real part correct
		A1	3	Imaginary part correct
			$\boxed{8}$	**N.B. Working must be shown**
6..	(i) Circle , Centre *O* radius 2	B1 B1		Sketch showing correct features
	One straight line	B1		
	Through *O* with +ve slope	B1		
	In 1st quadrant only	B1	5	
	(ii) $1 + i\sqrt{3}$	M1		Attempt to find intersections by trig, solving equations or from graph
		A1	2	Correct answer stated as complex number
			$\boxed{7}$	

7.	(i)	M1		Attempt at matrix multiplication
	$\mathbf{A}^2 = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$ $\mathbf{A}^3 = \begin{pmatrix} 8 & 0 \\ 0 & 1 \end{pmatrix}$	A1 A1	 3	Correct $\mathbf{A}^2$ Correct $\mathbf{A}^3$
	(ii) $\mathbf{A}^n = \begin{pmatrix} 2^n & 0 \\ 0 & 1 \end{pmatrix}$	B1	1	Sensible conjecture made
	(iii)	B1 M1 A1 A1	 4 $\boxed{8}$	State that conjecture is true for $n = 1$ or 2 Attempt to multiply $\mathbf{A}^n$ and $\mathbf{A}$ or vice versa Obtain correct matrix Statement of induction conclusion
8.	(i)	M1		Correct expansion process shown
	$a\begin{bmatrix} a & 0 \\ 2 & 1 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + 2\begin{bmatrix} 1 & a \\ 1 & 2 \end{bmatrix}$	A1		Obtain correct unsimplified expression
	$a^2 - 2a$	A1	3	Obtain correct answer
	(ii)	M1		Solve their det $\mathbf{M} = 0$
	$a = 0$ or $a = 2$	A1A1ft	3	Obtain correct answers
	(iii) (a)	B1 B1		Solution, as inverse matrix exists or $\mathbf{M}$ non-singular or det$\mathbf{M} \neq 0$
	(b)	B1 B1	4 $\boxed{10}$	Solutions, eqn. 1 is multiple of eqn 3

9.	(i)	M1 A1	2	Show that terms cancel in pairs Obtain given answer correctly
	(ii)	M1 A1	2	Attempt to expand and simplify Obtain given answer correctly
	(iii) $(n+1)^3 - 1 - \frac{3}{2}n(n+1) - n$ $\frac{1}{2}n(n+1)(2n+1)$	B1 B1 M1 M1 A1 A1	6 $\boxed{10}$	Correct Σr stated $\Sigma 1 = n$ Consider sum of three separate terms on RHS Required sum is LHS – two terms Correct unsimplified expression Obtain given answer correctly

10	(i) $\alpha + \beta + \gamma = 2$ $\alpha\beta\gamma = -4$	B1 B1		Write down correct values
.	$\alpha\beta + \beta\gamma + \gamma\alpha = 3$	B1	3	
	(ii)	M1		Sum new roots
	$\alpha + 1 + \beta + 1 + \gamma + 1 = 5$	A1ft		Obtain numeric value using their (i)
	$p = -5$	A1ft	3	p is negative of their answer
	(iii)	M1*		Expand three brackets
		A1		$\alpha\beta\gamma + \alpha\beta + \beta\gamma + \gamma\alpha + \alpha + \beta + \gamma + 1$
		DM1		Use their (i) results
		A1ft		Obtain 2
	$q = -2$	A1ft	5	q is negative of their answer
			$\boxed{11}$	**Alternative for (ii) & (iii)**
		M2		Substitute $x = u - 1$ in given equation
		A1		Obtain correct unsimplified equation for u
		M1		Expand
		A2		Obtain $u^3 - 5u^2 + 10u - 2 = 0$
		A1 A1		State correct values of p and q.

Mark Scheme 4725
January 2007

1.	(i) $a = -3$ (ii) $2a - 3 = 7$ or $3a - 6 = 9$ $a = 5$	B1 M1 A1	1 2 **3**	State correct value Sensible attempt at multiplication Obtain correct answer
2.	 $x^2 - y^2 = 15$ and $xy = 4$ $\pm(4 + i)$	M1 A1 A1 M1 DM1 A1	 6 **6**	Attempt to equate real and imaginary parts of $(x + iy)^2$ and 15 +8i Obtain each result Eliminate to obtain a quadratic in x^2 or y^2 Solve to obtain $x = (\pm)4$, or $y = (\pm)1$ Obtain only correct two answers as complex numbers
3.	 $\frac{1}{4}n^2(n+1)^2 - \frac{1}{2}n(n+1)$ $\frac{1}{4}n(n-1)(n+1)(n+2)$	M1 M1 A1 M1 A1 A1	 6 **6**	Expand to obtain $r^3 - r$ Consider difference of two standard results Obtain correct unfactorised answer Attempt to factorise Obtain factor of $\frac{1}{4}n(n+1)$ Obtain correct answer
4.	(i) (ii)	B1 B1 B1 B1 B1 B1	 3 3	Circle Centre (1, -1) Passing through (0, 0) Sketch a concentric circle Inside (i) and touching axes Shade between the circles
5.	(i)	B1	1	Show given answer correctly

	(ii) $-1 \pm i\sqrt{3}$ (iii)	M1 A1 A1 B1 B1 B1	 3 3 **7**	Attempt to solve quadratic equation or substitute $x + iy$ and equate real and imaginary parts Obtain answers as complex numbers Obtain correct answers, simplified Correct root on x axis, co-ords. shown Other roots in 2nd and 3rd quadrants Correct lengths and angles or co-ordinates or complex numbers shown
6.	(i) $u_{n+1} - u_n = 2n + 4$ (ii)	B1 M1 A1 B1 M1 M1 A1 A1	 3 5 **8**	Correct expression for u_{n+1} Attempt to expand and simplify Obtain given answer correctly State $u_1 = 4$ (or $u_2 = 10$)and is divisible by 2 State induction hypothesis true for u_n Attempt to use result in (ii) Correct conclusion reached for u_{n+1} Clear,explicit statement of induction conclusion
7.	(i) $\alpha + \beta = -5 \quad \alpha\beta = 10$ (ii) $\alpha^2 + \beta^2 = 5$ (iii) $x^2 - \frac{1}{2}x + 1 = 0$	B1 B1 M1 A1 B1 M1 A1 B1ft	2 2 4 **8**	State correct values Use $(\alpha + \beta)^2 - 2\alpha\beta$ Obtain given answer correctly, using value of -5 Product of roots = 1 Attempt to find sum of roots Obtain $\frac{5}{10}$ or equivalent Write down required quadratic equation, or any multiple.

8.	(i)	M1		Factor of $r!$ or $(r + 1)!$ seen
		A1		Factor of $(r + 1)$ found
	$(r + 1)^2 r!$	A1	3	Obtain given answer correctly
	(ii)	M1		Express terms as differences using
		A1		(i)
		M1		At least 1st two and last term correct
	$(n + 2)! - 2!$	A1	4	Show that pairs of terms cancel
	(iii)	B1ft	1	Obtain correct answer in any form
			8	Convincing statement for non-converging, ft their (ii)
9.	(i) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$	M1		For at least two correct images
		A1	2	For correct diagram, co-ords.clearly written down
	(ii) 90^0 clockwise, centre origin	B1 B1		Or equivalent correct description
	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	B1	3	Correct matrix, not in trig form
	(iii) Stretch parallel to x-axis, s.f. 3	B1 B1		Or equivalent correct description, but must be a stretch for 2nd B1
	$\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$	B1 B1	4	Each correct column
			9	

10.	(i)	M1		Show correct expansion process for 3 x 3
		M1		Correct evaluation of any 2 x 2 det
	$\Delta = \det \mathbf{D} = 3a - 6$	A1		Obtain correct answer
		M1		Show correct process for adjoint entries
		A1		Obtain at least 4 correct entries in adjoint
		B1		Divide by their determinant
	$\mathbf{D}^{-1} = \frac{1}{\Delta}\begin{pmatrix} 3 & -2 & 4 \\ -3 & a & -2a \\ -3 & a & a-6 \end{pmatrix}$	A1	7	Obtain completely correct answer
	(ii) $\frac{1}{\Delta}\begin{pmatrix} 5 \\ 2a-9 \\ 5a-15 \end{pmatrix}$	M1		Attempt product of form $\mathbf{D}^{-1}\mathbf{C}$, or eliminate to get 2 equations and solve
		A1A1A1 ft all 3	4	Obtain correct answers, ft their inverse
			11	

Mark Scheme 4725
June 2007

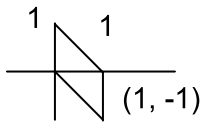
1	*EITHER* $a = 2$ $b = 2\sqrt{3},$ *OR* $a = 2 \quad b = 2\sqrt{3}$	M1 A1 M1 A1 M1 M1 A1 A1	 4 **4**	Use trig to find an expression for a (or b) Obtain correct answer Attempt to find other value Obtain correct answer a.e.f. (Allow 3.46) State 2 equations for a and b Attempt to solve these equations Obtain correct answers a.e.f. SR $\pm$ scores A1 only
2	$(1^3 =)\frac{1}{4} \times 1^2 \times 2^2$ $\frac{1}{4}n^2(n+1)^2 + (n+1)^3$ $\frac{1}{4}(n+1)^2(n+2)^2$	B1 M1 M1(indep) A1 A1	 5 **5**	Show result true for $n = 1$ Add next term to given sum formula Attempt to factorise and simplify Correct expression obtained convincingly Specific statement of induction conclusion
3	$3\Sigma r^2 - 3\Sigma r + \Sigma 1$ $3\Sigma r^2 = \frac{1}{2}n(n+1)(2n+1)$ $3\Sigma r = \frac{3}{2}n(n+1)$ $\Sigma 1 = n$ n^3	M1 A1 A1 A1 M1 A1	 6 **6**	Consider the sum of three separate terms Correct formula stated Correct formula stated Correct term seen Attempt to simplify Obtain given answer correctly
4	(i) $\frac{1}{2}\begin{pmatrix} 5 & -1 \\ -3 & 1 \end{pmatrix}$ (ii) $\frac{1}{2}\begin{pmatrix} 2 & 0 \\ 23 & -5 \end{pmatrix}$	B1 B1 M1 M1(indep) A1ft A1ft	 2 4 **6**	Transpose leading diagonal and negate other diagonal or solve sim. eqns. to get 1st column Divide by the determinant or solve 2nd pair to get 2nd column Attempt to use $B^{-1}A^{-1}$ or find B Attempt at matrix multiplication One element correct, a.e.f, All elements correct, a.e.f. NB ft consistent with their (i)

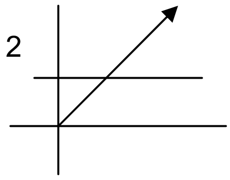
5	(i) $\frac{1}{r(r+1)}$	B1	1	Show correct process to obtain given result
	(ii) $1 - \frac{1}{n+1}$	M1 M1 A1	3	Express terms as differences using (i) Show that terms cancel Obtain correct answer, must be n not any other letter
	(iii) $S_\infty = 1$ $\frac{1}{n+1}$	B1ft M1 A1 c.a.o.	3 **7**	State correct value of sum to infinity Ft their (ii) Use sum to infinity – their (ii) Obtain correct answer a.e.f.
6	(i) (a) $\alpha+\beta+\gamma=3, \alpha\beta+\beta\gamma+\gamma\alpha=2$	B1 B1	2	State correct values
	(b) $\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$ $=9-4=5$	M1 A1 ft	2	State or imply the result and use their values Obtain correct answer
	(ii) (a) $\frac{3}{u^3} - \frac{9}{u^2} + \frac{6}{u} + 2 = 0$ $2u^3+6u^2-9u+3=0$	M1 A1	2	Use given substitution to obtain an equation Obtain correct answer
	(b) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = -3$	M1 A1ft	2 **8**	Required expression is related to new cubic stated or implied -(their "b" / their "a")

7	(i) $a(a - 12) + 32$ (ii) det **M** = 12 non-singular (iii) *EITHER* *OR*	M1 M1 A1 M1 A1ft B1 M1 A1 M1 A1 A1	 3 2 3 **8**	Show correct expansion process Show evaluation of a 2 x 2 determinant Obtain correct answer a.e.f. Substitute $a = 2$ in their determinant Obtain correct answer and state a consistent conclusion det M = 0 so non-unique solutions Attempt to solve and obtain 2 inconsistent equations Deduce that there are no solutions Substitute $a = 4$ and attempt to solve Obtain 2 correct inconsistent equations Deduce no solutions
8	(i) Circle, centre (3, 0), y-axis a tangent at origin Straight line, through (1, 0) with +ve slope In 1st quadrant only (ii) Inside circle, below line, above x-axis	B1B1 B1 B1 B1 B1 B2ft	 6 2 **8**	Sketch showing correct features N.B. treat 2 diagrams asa MR Sketch showing correct region SR: B1ft for any 2 correct features

9	(i) $\begin{pmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix}$	B1	1	Correct matrix
	(ii) Rotation (centre O), 45^0, clockwise	B1B1B1	3	Sensible alternatives OK, must be a single transformation
	(iii)	B1	1	Matrix multiplication or combination of transformations
	(iv) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$	M1 A1	2	For at least two correct images For correct diagram
	(v) det $\mathbf{C} = 2$	B1		State correct value
	area of square has been doubled	B1	2	State correct relation a.e.f.
			9	
10	(i)	M1		Attempt to equate real and imaginary parts of $(x + iy)^2$ and 16+30i
	$x^2 - y^2 = 16$ and $xy = 15$	A1A1		Obtain each result
		M1		Eliminate to obtain a quadratic in x^2 or y^2
	$\pm(5 + 3i)$	M1		Solve to obtain $x = (\pm)\,5$ or $y = (\pm)\,3$
	(ii)	A1	6	Obtain correct answers as complex numbers
	$z = 1 \pm \sqrt{16 + 30i}$	M1*		Use quadratic formula or complete the square
	6 + 3i, -4 - 3i	A1 *M1dep A1 A1ft	5	Simplify to this stage Use answers from (i) Obtain correct answers
			11	

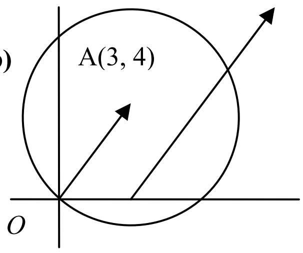
4725 Further Pure Mathematics 1

1	(i)  (ii) $\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$	M1 A1 B1 B1	 2 2 **4**	For 2 other correct vertices seen, correct direction of shear seen For completely correct diagram, must include scales Each column correct
2	$\frac{a}{6}n(n+1)(2n+1)+bn$ $a = 6 \;\; b = \text{-}3$	M1 A1 M1 A1 A1	 5 **5**	Consider sum as two separate parts Correct answer a.e.f. Compare co-efficients Obtain correct answers
3	(i) $7u^3 + 24u^2 - 3u + 2 = 0$ (ii) *EITHER* correct value is $-\frac{3}{7}$ *OR* correct value is $-\frac{3}{7}$	M1 A1 M1 A1ft M1 A1	 2 2 **4**	Use given substitution Obtain correct equation a.e.f. Required expression related to new cubic Their c / their a Use $\dfrac{\alpha+\beta+\gamma}{\alpha\beta\gamma}$ or equivalent Obtain correct answer
4	(i) $z^* = 3 + 4\text{i}$ 21 +12i (ii) 3 – 5i -16 – 30i (iii) $\frac{9}{25} + \frac{12}{25}\text{i}$	B1 B1 B1 B1ft B1ft M1 A1 A1	 2 3 3 **8**	Conjugate seen or implied Obtain correct answer Correct $z - \text{i}$ or expansion of $(z - \text{I})^2$ seen Real part correct Imaginary part correct Multiply by conjugate Numerator correct Denominator correct
5	(i) $\begin{pmatrix} -13 \\ 1 \\ -10 \end{pmatrix}$ (ii) $\begin{pmatrix} 8 & 16 & -4 \\ 0 & 0 & 0 \\ 6 & 12 & -3 \end{pmatrix}$ (iii) (8)	B1 B1 M1 A1A1A1 M1 A1	 2 4 2 **8**	4**B** seen or implied or 2 elements correct Obtain correct answer Obtain a 3 x 3 matrix Each row (or column) correct Obtain a single value Obtain correct answer, must have matrix

6	(i)  (ii) $2\sqrt{3}+2\mathrm{i}$	B1 B1 B1 B1 B1 B1 M1 A1	 5 3 **8**	Horizontal straight line in 2 quadrants Through (0, 2) Straight line Through O with positive slope In 1st quadrant only State or obtain algebraically that $y = 2$ Use suitable trigonometry Obtain correct answer a.e.f. decimals OK must be a complex number
7	(i) $a = -6$ (ii) $\mathbf{A}^{-1} = \frac{1}{a+6}\begin{pmatrix} 1 & -3 \\ 2 & a \end{pmatrix}$ $x = \frac{4}{a+6}$, $y = \frac{2-a}{a+6}$	M1 A1 B1 B1ft M1 A1ft A1ft	 2 5 **7**	Use det $\mathbf{A} = 0$ Obtain correct answer Both diagonals correct Divide by det $\mathbf{A}$ Premultiply column by $\mathbf{A}^{-1}$, no other method Obtain correct answers from their $\mathbf{A}^{-1}$
8	(i) $u_2 = 4,\ u_3 = 9,\ u_4 = 16$ (ii) $u_n = n^2$ (iii)	M1 A1 B1 B1 M1 A1 A1	 2 1 4 **7**	Obtain next terms All terms correct Sensible conjecture made State that conjecture is true for $n = 1$ or 2 Find u_{n+1} in terms of n Obtain $(n+1)^2$ Statement of Induction conclusion
9	(i) $\alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$ (ii) *Either* $\alpha+\beta = 5, \alpha\beta = 7$ $\alpha^3 + \beta^3 = 20$ $x^2 - 20x + 343 = 0$ *Or* $u^{\frac{2}{3}} - 5u^{\frac{1}{3}} + 7 = 0$ $u^3 - 20u + 343 = 0$	M1 A1 B1 B1 M1 A1 M1 A1ft M1 A1 M2 A2	 2 6 **8**	Correct binomial expansion seen Obtain given answer with no errors seen State or use correct values Find numeric value for $\alpha^3 + \beta^3$ Obtain correct answer Use new sum and product correctly in quadratic expression Obtain correct equation Substitute $x = u^{\frac{1}{3}}$ Obtain correct answer Complete method for removing fractional powers Obtain correct answer

10	(i)		M1 A1	2	Attempt to combine 3 fractions Obtain given answer correctly
	(ii)		M1 A1 M1 A1 M1 A1	6	Express at least first 3 terms using (i) All terms correct Express at least last 2 terms using (i) All terms correct in terms of n Show that correct terms cancel Obtain unsimplified correct answer
		$2+1-\frac{1}{2}-\frac{2}{n+1}-\frac{1}{n+2}$			
	(iii)	$\frac{5}{2}$	B1ft	1	Obtain correct answer from their (ii)
	(iv)	$\frac{2}{N+1}+\frac{1}{N+2}=\frac{7}{10}$	B1ft		Their (iii) – their (ii)
		$7N^2-9N-36=0$	M1		Attempt to clear fractions & solve equation, Obtain correct simplified equation
		$N=3$	A1 A1	4 **13**	Obtain only the correct answer

4725 Further Pure Mathematics 1

1	**(i)**	$\begin{pmatrix} 1 & 1 \\ 5 & -1 \end{pmatrix}$	**B1**	Two elements correct
			B1	All four elements correct
			$\boxed{2}$	
	(ii)	*EITHER*		
			B1	Both diagonals correct
		$\frac{1}{3}\begin{pmatrix} 2 & -1 \\ -5 & 4 \end{pmatrix}$	**B1**	Divide by determinant
			$\boxed{2}$	
		OR		
			B1	Solve sim. eqns. 1st column correct
			B1	2nd column correct
2	**(i)**	5	**B1**	Correct modulus
		0.927 or 53.1°	**B1**	Correct argument, any equivalent form
			2	
	(ii)(a)		**B1**	Circle centre A (3, 4)
			B1	Through O, allow if centre is (4, 3)
			$\boxed{2}$	
	(b)		**B1**	Half line with +ve slope
			B1	Starting at (3, 0)
			B1	Parallel to OA, (implied by correct arg shown)
			$\boxed{3}$	
3	**(i)**	$\frac{r}{(r+1)!}$	**M1**	Common denominator of $(r + 1)!$ or $r!(r + 1)!$
			A1	Obtain given answer correctly
			$\boxed{2}$	
	(ii)	$1-\frac{1}{(n+1)!}$	**M1**	Express terms as differences using (i)
			A1	At least 1st two and last term correct
			M1	Show pairs cancelling
			A1	Correct answer a.e.f.
			$\boxed{4}$	
4			**B1**	Establish result is true, for n = 1 (or 2 or 3)
			M1	Attempt to multiply **A** and $\mathbf{A}^n$, or vice versa
			M1	Correct process for matrix multiplication
			A1	Obtain 3^{n+1}, 0 and 1
			A1	Obtain ½(3^{n+1} – 1)
			A1	Statement of Induction conclusion, only if 5 marks earned, but may be in body of working
			$\boxed{6}$	

5			M1	Express as difference of two series
			M1	Use standard results
		$\frac{1}{4}n^2(n+1)^2 - \frac{1}{6}n(n+1)(2n+1)$	A1	Correct unsimplified answer
			M1	Attempt to factorise
			A1	At least factor of $n(n+1)$
		$\frac{1}{12}n(n+1)(3n+2)(n-1)$	A1	Obtain correct answer
			$\boxed{6}$	
6	(i)	3 – i	B1	Conjugate stated
			$\boxed{1}$	
	(ii)	*EITHER*	M1	Use sum of roots
			A1	Obtain correct answer
			M1	Use sum of pairs of roots
			A1	Obtain correct answer
			M1	Use product of roots
		$a = -8$, $b = 22$, $c = -20$	A1	Obtain correct answers
			$\boxed{6}$	
		OR	M1	Attempt to find a quadratic factor
			A1	Obtain correct factor
			M1	Expand linear and quadratic factors
		$a = -8$, $b = 22$, $c = -20$	A1A1A1	Obtain correct answers
		OR		
			M1	Substitute 1 imaginary & the real root into eqn
			M1	Equate real and imaginary parts
			M1	Attempt to solve 3 eqns.
		$a = -8$, $b = 22$, $c = -20$	A1A1A1	Obtain correct answers
7	(i)		B1	Enlargement (centre O) scale factor 6
			$\boxed{1}$	
	(ii)		B1	Reflection
			B1	Mirror line is $y = x$
			$\boxed{2}$	
	(iii)		B1	Stretch in y direction
			B1	Scale factor 6, must be a stretch
			$\boxed{2}$	
	(iv)		B1	Rotation
			B1	36.9° clockwise or equivalent
			$\boxed{2}$	

8	$\alpha+\beta=-k$	B1	State or use correct value
	$\alpha\beta=2k$	B1	State or use correct value
		M1	Attempt to express sum of new roots in terms of $\alpha+\beta,\quad \alpha\beta$
	$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$	A1	Obtain correct expression
	$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{1}{2}(k-4)$	A1	Obtain correct answer a.e.f.
	$\alpha'\beta'=1$	B1	Correct product of new roots seen
	$x^2-\frac{1}{2}(k-4)x+1=0$	B1ft	Obtain correct answer, must be an eqn.
		$\boxed{7}$	
			Alternative for last 5 marks
		M1	Obtain expression for $u=\frac{\alpha}{\beta}$ in terms of k and α or k and β
		A1	Obtain a correct expression
		A1	rearrange to get α in terms of u
		M1	Substitute into given equation
		A1	Obtain correct answer
9 (i)		M1	Attempt to equate real and imaginary parts of $(x+\mathrm{i}y)^2$ and 5 + 12i
	$x^2-y^2=5$ and $xy=6$	A1	Obtain both results
		M1	Eliminate to obtain a quadratic in x^2 or y^2
	±(3 + 2i)	M1	Solve a 3 term quadratic & obtain x or y
		A1	Obtain correct answers as complex nos.
		$\boxed{5}$	
(ii)	5 – 12i	B1B1	Correct real and imaginary parts
		$\boxed{2}$	
(iii)		M1	Attempt to solve a quadratic equation
	$x^2=5\pm 12\mathrm{i}$	A1	Obtain correct answers
	$x=\pm(3\pm 2\mathrm{i})$	A1A1	Each pair of correct answers a.e.f.
		$\boxed{4}$	

10	**(i)**	**M1**	Find value of det **AB**
		A1	Correct value 2 seen
		$\boxed{2}$	
	(ii)	**M1**	Show correct process for adjoint entries
		A1	Obtain at least 4 correct entries in adjoint
		B1	Divide by their determinant
	$(\mathbf{AB})^{-1} = \frac{1}{2}\begin{pmatrix} 0 & 3 & -1 \\ 0 & -1 & 1 \\ 2 & 6-3a & a-6 \end{pmatrix}$	**A1**	Obtain completely correct answer
		$\boxed{4}$	
	(iii) *EITHER*	**M1**	State or imply $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$
		A1	Obtain $\mathbf{B}^{-1} = (\mathbf{AB})^{-1} \times \mathbf{A}$
		M1	Correct multiplication process seen
		A1	Obtain three correct elements
	$\mathbf{B}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ -6 & 2 & -2 \end{pmatrix}$	**A1**	All elements correct
		$\boxed{5}$	
	OR	**M1**	Attempt to find elements of **B**
		A1	All correct
		M1	Correct process for $\mathbf{B}^{-1}$
		A1	3 elements correct
		A1	All elements correct

4725 Further Pure Mathematics 1

1	$\frac{7}{26}+\frac{17}{26}\text{i}.$	M1 A1 A1 A1	 4 **4**	Multiply by conjugate of denominator Obtain correct numerator Obtain correct denominator
2	**(i)** $\frac{1}{10}\begin{pmatrix}5 & 0\\ -a & 2\end{pmatrix}$ **(ii)** $\begin{pmatrix}3 & -2\\ 2a & 6\end{pmatrix}$	B1 B1 B1 B1	 2 2 **4**	Both diagonals correct Divide by correct determinant Two elements correct Remaining elements correct
3	$n^2(n+1)^2+n(n+1)(2n+1)+n(n+1)$ $n(n+1)^2(n+2)$	M1 A1 A1 M1 A1ft A1	 6 **6**	Express as sum of 3 terms 2 correct unsimplified terms 3rd correct unsimplified term Attempt to factorise Two factors found, ft their quartic Correct final answer a.e.f.
4	$\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$	B1 M1 A1 A1	 4 **4**	State or use correct result Combine matrix and its inverse Obtain **I** or $\mathbf{I}^2$ but not 1 Obtain zero **matrix** but not 0 **S.C. If 0/4, B1 for $\mathbf{AA}^{-1}=\mathbf{I}$**
5	*Either* $4k-4$ $k=1$ *Or*	M1 M1 A1 M1 A1ft M1 A1 M1 A1 A1	 5 **5**	Consider determinant of coefficients of LHS Sensible attempt at evaluating any 3×3 det Obtain correct answer a.e.f. unsimplified Equate det to 0 Obtain $k=1$, ft provided all M's awarded Eliminate either x or y Obtain correct equation Eliminate 2nd variable Obtain correct linear equation Deduce that $k=1$
6	**(i)** *Either* *Or* **(ii)** **(iii)** $\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$ **(iv)**	B1 DB1 B1 DB1 B1 DB1 B1 B1 B1B1B1	2 2 2 3 **9**	Reflection, in x-axis Stretch parallel to y-axis, s.f. –1 Reflection, in $y = -x$ Each column correct Rotation, 90° ,clockwise about O **S.C. If (iii) incorrect, B1 for identifying their transformation, B1 all details correct**

7	**(i)** $13^n + 6^{n-1} + 13^{n+1} + 6^n$	B1		Correct expression seen
	(ii)	M1		Attempt to factorise both terms in (i)
		A1	3	Obtain correct expression
		B1		Check that result is true for n =1 (or 2)
		B1		Recognise that (i) is divisible by 7
		B1		Deduce that u_{n+1} is divisible by 7
		B1	4	Clear statement of Induction conclusion
			7	
8	**(i)**	M1		Expand at least 1 of the brackets
		A1	2	Derive given answer correctly
	(ii) $\alpha+\beta=6k, \alpha\beta=k^2$	B1 B1		State or use correct values
	$\alpha-\beta=(4\sqrt{2})k$	M1		Find value of $\alpha-\beta$ using (i)
		A1		Obtain given value correctly (allow if $-6k$ used)
			4	
	(iii) $\sum\alpha'=6k$	B1ft		Sum of new roots stated or used
	$\alpha'\beta'=\alpha\beta-(\alpha-\beta)-1$	M1		Express new product in terms of old roots
	$\alpha'\beta'=k^2-(4\sqrt{2})k-1$	A1ft		Obtain correct value for new product
	$x^2-6kx+k^2-(4\sqrt{2})k-1=0$	B1ft	4	Write down correct quadratic **equation**
			10	
9	**(i)**	M1		Use correct denominator
		A1	2	Obtain given answer correctly
	(ii)	M1		Express terms as differences using (i)
		M1		Do this for at least 1st 3 terms
		A1		First 3 terms all correct
		A1		Last 3 terms all correct (in terms or n or r)
	$1+\frac{1}{3}-\frac{1}{2n-1}-\frac{1}{2n+1}$	M1		Show pairs cancelling
		A1	6	Obtain correct answer, a.e.f.(in terms of n)
	(iii) $\frac{4}{3}$	B1ft	1	Given answer deduced correctly, ft their (ii)
			9	

10	**(i)** $x^2 - y^2 = 2, 2xy = \sqrt{5}$	M1 A1		Attempt to equate real and imaginary parts Obtain both results a.e.f.
	$4x^4 - 8x^2 - 5 = 0$	M1 M1		Eliminate to obtain quadratic in x^2 or y^2 Solve to obtain x (or y) values
	$x = \pm\frac{\sqrt{10}}{2}, y = \pm\frac{\sqrt{2}}{2}$	A1		Correct values for both x & y obtained a.e.f.
	$\pm(\frac{\sqrt{10}}{2} + \mathrm{i}\frac{\sqrt{2}}{2})$	A1	6	Correct answers as complex numbers
	(ii) $z^2 = 2 \pm \mathrm{i}\sqrt{5}$	M1 A1		Solve quadratic in z^2 Obtain correct answers
	$z = \pm(\frac{\sqrt{10}}{2} \pm \mathrm{i}\frac{\sqrt{2}}{2})$	M1 A1ft	 4	Use results of (i) Obtain correct answers, ft must include root from conjugate
	(iii)	B1ft	1	Sketch showing roots correctly
	(iv)	B1 B1ft B1ft	 3 **14**	Sketch of straight line, $\perp$ to α Bisector

4725 Further Pure Mathematics 1

1.	 984390625 – 25502500 = 958888125	B1 M1 A1	 3 **3**	State correct value of S_{250} or S_{100} Subtract $S_{250} - S_{100}$ (or S_{101} or S_{99}) Obtain correct exact answer
2.	$3a + 5b = 1,\ a + 2b = 1$ $a = -3,\ b = 2$	M1 M1 A1 A1	 4 **4**	Obtain a pair of simultaneous equations Attempt to solve Obtain correct answers.
3.	(i) 11 – 29i (ii) 1 + 41i	B1 B1 B1 B1	2 2 **4**	Correct real and imaginary parts Correct real and imaginary parts
4.	Either $p+q=-1, pq=-8$ $\frac{p+q}{pq}$ $-\frac{7}{8}$ Or $\frac{1}{p}+\frac{1}{q}=8$ $p+q=1$ $-\frac{7}{8}$ Or $\frac{-1\pm\sqrt{33}}{2}$ $-\frac{7}{8}$	B1 B1 M1 A1 B1 B1 M1 A1 M1 A1 M1 A1	 4 **4**	Both values stated or used Correct expression seen Use their values in their expression Obtain correct answer Substitute $x = \frac{1}{u}$ and use new quadratic Correct value stated Use their values in given expression Obtain correct answer Find roots of given quadratic equation Correct values seen Use their values in given expression Obtain correct answer
5.	(i) $u^3 = \{(-)(5u+7)\}^2$ $u^3 - 25u^2 - 70u - 49 = 0$ (ii) -70	M1 A1 A1 M1 A1 ft	 3 2 **5**	Use given substitution and rearrange Obtain correct expression, or equivalent Obtain correct final answer Use coefficient of u of their cubic or identity connecting the symmetric functions and substitute values from given equation Obtain correct answer

6.	(i) $3\sqrt{2}, -\frac{\pi}{4}$ or -45° AEF	B1 B1	2	State correct answers
	(ii)(a)	B1B1 B1 ft	3	Circle, centre (3, -3), through *O* ft for $(\pm 3, \pm 3)$ only
	(ii)(b)	B1 B1 B1	3	Straight line with +ve slope, through (3, -3) or their centre Half line only starting at centre
	(iii)	B1ft B1ft B1ft	3 **11**	Area above horizontal through *a*, below (ii) (b) Outside circle
7.	(i)	M1 A1	2	Show that terms cancel in pairs Obtain given answer correctly
	(ii)	M1 A1	2	Attempt to expand and simplify Obtain given answer correctly
	(iii)	B1 B1		Correct $\sum r$ stated $\sum 1 = n$
		M1* *DM1		Consider sum of 4 separate terms on RHS Required sum is LHS – 3 terms
	$(n+1)^4 - 1 - n(n+1)(2n+1) - 2n(n+1) - n$	A1		Correct unsimplified expression
	$4\sum_{r=1}^{n} r^3 = n^2(n+1)^2$	A1	6 **10**	Obtain given answer correctly
8.	(i)	B1 B1 B1	3	Find coordinates (0, 0) (3, 1) (2, 1) (5, 2) found Accurate diagram sketched
	(ii) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$	B1 B1	2	Each column correct
	(iii) *Either* $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$	B1 M1 A1ft		Correct inverse for their (ii) stated Post multiply **C** by inverse of (ii) Correct answer found
	Or	M1 A2ft		Set up 4 equations for elements from correct matrix multiplication All elements correct, -1 each error
		B1 B1 B1	6 **11**	Shear, *x* axis invariant or parallel to *x*-axis eg image of (1, 1) is (3, 1) **SR** allow s.f. 2 or shearing angle of correct angle to appropriate axis

9.	(i) $a\begin{vmatrix} a & 1 \\ 1 & 2 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & a \\ 1 & 1 \end{vmatrix}$	M1 A1		Correct expansion process shown Obtain correct unsimplified expression
	$2a^2 - 2a$	A1	3	Obtain correct answer
	(ii) $a = 0$ or 1	M1 A1ft A1ft	3	Equate their det to 0 Obtain correct answers, ft solving a quadratic
	(iii) (a)	B1 B1		Equations consistent, but non unique solutions
	(b)	B1 B1	4 **10**	Correct equations seen & inconsistent, no solutions
10.	i) $u_2 = 7$ $u_3 = 19$	M1 A1 A1	3	Attempt to find next 2 terms Obtain correct answers Show given result correctly
	(ii) $u_n = 2(3^{n-1}) + 1$	M1 A1	2	Expression involving a power of 3 Obtain correct answer
	(iii) $u_{n+1} = 3(2(3^{n-1})+1) - 2$ $u_{n+1} = 2(3^n) + 1$	B1ft M1 A1 A1 B1	5 **10**	Verify result true when $n = 1$ or $n = 2$ Expression for u_{n+1} using recurrence relation Correct unsimplified answer Correct answer in correct form Statement of induction conclusion

4725 Further Pure Mathematics 1

1 (i)	$\begin{pmatrix} a-4 & 2 \\ 3 & 0 \end{pmatrix}$	B1		Two elements correct
		B1	**2**	Remaining elements correct
(ii)	$4a-6$	B1		Correct determinant
		M1		Equate det A to 0 and solve
	$a=\frac{3}{2}$	A1	**3**	Obtain correct answer a. e. f.
		$\boxed{5}$		
2 (i)	u^3-3u^2+3u-1	B1		Correct unsimplified expansion of $(u-1)^3$
		M1		Substitute for x
	$2u^3-6u^2+9u-8=0$	A1	**3**	Obtain correct **equation**
(ii)		M1		Use $(\pm)\frac{\mathrm{d}}{\mathrm{a}}$ of new equation
	4	A1ft	**2**	Obtain correct answer from their equation
		$\boxed{5}$		
3	$x-\mathrm{i}y$	B1		Conjugate known
		M1		Equate real and imaginary parts
	$x+2y=12 \quad 2x+y=9$	A1		Obtain both equations, OK with factor of i
		M1		Solve pair of equations
	$z=2+5\mathrm{i}$	A1	**5**	Obtain correct answer as a complex number **S.C.** Solving $z+2\mathrm{i}z=12+9\mathrm{i}$ can get max 4/5, not first B1
		$\boxed{5}$		
4		M1		Express as sum of three series
		M1		Use standard results
	$\frac{1}{4}n^2(n+1)^2-\frac{1}{6}n(n+1)(2n+1)-n(n+1)$	A1		Obtain correct unsimplified answer
		M1		Attempt to factorise
		A1		Obtain at least factor of $n(n+1)$
	$\frac{1}{12}n(n+1)(n+2)(3n-7)$	A1	**6**	Obtain fully factorised correct answer
		$\boxed{6}$		

5	**(i)**	B1		Rotation 90° (about origin)
		B1	**2**	Anticlockwise
	(ii) ***Either***	M1		Show image of unit square after reflection in $y = -x$
	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	A1		Deduce reflection in x-axis
		B1ft		Each column correct
		B1ft	**4**	ft for matrix of their transformation
	Or	M1		Post multiply by correct reflection matrix
		A1		Obtain correct answer
		B1B1		State reflection, in x-axis
			S.C.	If pre-multiplication, M0 but B1 B1 Available for correct description of their matrix
		$\boxed{6}$		
6	**(i)**	B1		State or use 5 + i as a root
		M1		Use $\sum \alpha\beta = 6$
	$x = -2$	A1	**3**	Obtain correct answer
	(ii) ***Either***	M1		Use $p = -\sum \alpha$
	$p = -8$	A1ft		Obtain correct answer, from their root
		M1		Use $q = -\alpha\beta\gamma$
	$q = 52$	A1ft	**4**	Obtain correct answer, from their root
	Or	M1		Attempt to find quadratic factor
		M1		Attempt to expand quadratic and linear
		A1A1		Obtain correct answers
	Or	M1		Substitute (5 – i)into equation
		M1		Equate real and imaginary parts
		A1		Obtain correct answer for p
		A1ft		Obtain correct answer for q, ft their p
		$\boxed{7}$		
7	**(i)**	B1	**1**	Obtain **given** answer correctly
	(ii)	M1		Express at least 1st two and last term using **(i)**
		A1		All terms correct
		M1		Show that correct terms cancel
	$1 - \frac{1}{(n+1)^2}$	A1	**4**	Obtain correct answer, in terms of n
	(iii) $\frac{1}{4}$	B1		Sum to infinity seen or implied
		B1	**2**	Obtain correct answer
				S.C. -¾ scores B1
		$\boxed{7}$		

8 (i)		M1		Attempt to equate real and imaginary parts of $(x + \text{i}y)^2$ & $5 - 12\text{i}$
	$x^2 - y^2 = 5$ and $xy = -6$	A1		Obtain both results, a.e.f
		M1		Obtain quadratic in x^2 or y^2
		M1		Solve to obtain $x = (\pm)3$ or $y = (\pm)2$
	$\pm(3 - 2\text{i})$	A1	**5**	Obtain correct answers as complex nos
(ii)		B1ft		Circle with centre at their square root
		B1		Circle passing through origin
		B1ft		2nd circle centre correct relative to 1st
		B1	**4**	Circle passing through origin
		$\boxed{9}$		
9 (i)		M1		Show correct expansion process for 3×3 or multiply adjoint by **A**
		M1		Correct evaluation of any 2×2 at any stage
	det **A** = $\Delta = 6a - 6$	A1		Obtain correct answer
	$\mathbf{A}^{-1} = \frac{1}{\Delta}\begin{pmatrix} 3a-1 & a+1 & -4 \\ 1 & 2a-1 & -2 \\ -3 & -3 & 6 \end{pmatrix}$	M1		Show correct process for adjoint entries
		A1		Obtain at least 4 correct entries in adjoint
		B1		Divide by their determinant
		A1	**7**	Obtain completely correct answer
(ii)	$\frac{1}{\Delta}\begin{pmatrix} 5a-7 \\ 4a-5 \\ 3 \end{pmatrix}$	M1		Attempt product of form $\mathbf{A}^{-1}\mathbf{C}$ or eliminate to get 2 equations and solve
		A1A1A1 ft all 3		Obtain correct answer
			4	**S.C.** if det now omitted, allow max A2 ft
		$\boxed{11}$		
10 (i)		B1		Correct $\mathbf{M}^2$ seen
	$\mathbf{M}^2 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$ $\mathbf{M}^3 = \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$	M1		Convincing attempt at matrix multiplication for $\mathbf{M}^3$
		A1	**3**	Obtain correct answer
(ii)	$\mathbf{M}^n = \begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix}$	B1ft	**1**	State correct form, consistent with **(i)**

10 (iii)	M1		Correct attempt to multiply **M** & $\mathbf{M}^k$ or v.v.
	A1		Obtain element $2(k+1)$
	A1		Clear statement of induction step, from correct working
	B1	**4**	Clear statement of induction conclusion, following their working
(iv)	B1		Shear
	DB1		x-axis invariant
	DB1	**3**	e.g. $(1, 1) \rightarrow (21, 1)$ or equivalent using scale factor or angles
	$\boxed{11}$		

1			B1		Establish result true for $n = 1$or $n = 2$
			M1		Add next term to given sum formula
			M1		Attempt to factorise or expand and simplify to correct expression
			A1		Correct expression obtained
			A1	**5**	Specific statement of induction conclusion
				$\boxed{5}$	
2	**(i)**	(-7)	M1		Obtain a single value
			A1	**2**	Obtain correct answer as a matrix
	(ii)	BA = $\begin{pmatrix} 5 & -20 \\ 3 & -12 \end{pmatrix}$	M1		Obtain a 2 × 2 matrix
			A1		All elements correct
		$\begin{pmatrix} -7 & -20 \\ 11 & -20 \end{pmatrix}$	B1		4**C** seen or implied by correct answer
			B1ft	**4**	Obtain correct answer, ft for a slip in **BA**
				$\boxed{6}$	
3		Either	M1		Express as a sum of 3 terms
			M1		Use standard sum results
		$\frac{2}{3}n(n+1)(2n+1)-2n(n+1)+n$	A1		Correct unsimplified answer
			M1		Attempt to factorise
			A1		Obtain at least factor of n and a quadratic
		$\frac{1}{3}n(2n-1)(2n+1)$	A1	**6**	Obtain correct answer a.e.f.
		Or			
		$\sum_{r=1}^{2n} r^2 - 4\sum_{r=1}^{n} r^2$	M1		Express as difference of 2 $\sum r^2$ series
			M1		Use standard result
		$\frac{1}{6}\times 2n(2n+1)(4n+1)-4\times\frac{1}{6}n(n+1)(2n+1)$	A1		Correct unsimplified answer
			M1		Attempt to factorise
			A1		Obtain at least factor of n
		$\frac{1}{3}n(2n-1)(2n+1)$	A1		Obtain correct answer
				$\boxed{6}$	

4	(i)	5 + 12i	B1B1	Correct real and imaginary parts
		13	B1ft	Correct modulus
		67.4° or 1.18	B1ft **4**	Correct argument
	(ii)		M1	Multiply by conjugate
			A1	Obtain correct numerator
		$-\frac{11}{85}-\frac{27}{85}\text{i}$	A1 **3**	Obtain correct denominator
			$\boxed{7}$	
5	(a)	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	B1B1**2**	Each column correct **SC B2 use correct matrix from MF1** **Can be trig form**
	(b)	(i)	B1B1**2**	Stretch, in x-direction sf 5
		(ii)	B1B1**2**	Rotation, 60° clockwise
			$\boxed{6}$	
6	(i)	(a)	B1B1**2**	Circle centre (3, –4), through origin
		(b)	B1B1**2**	Vertical line, clearly $x = 3$
	(ii)		B1ft	Inside their circle
			B1ft **2**	And to right of their line, if vertical
			$\boxed{6}$	

7	*Either*	B1B1	State or use correct results
	$\alpha+\beta=-2k \quad \alpha\beta=k$	M1	Attempt to find sum of new roots
		A1	Obtain $4k$
		M1	Attempt to find product of new roots
		A1	Obtain $4k$
		B1ft **7**	Correct quadratic equation a.e.f.
	$y^2-4ky+4k=0$		
	Or		
	$\alpha+\beta=-2k$	B1	State or use correct result
	$\frac{-2k}{\alpha}$	B1	State or imply form of new roots
	$y=\frac{-2k}{x}$	B1	State correct substitution
		M1	Rearrange and substitute for x
		A1	Correct unsimplified equation
		M1	Attempt to clear fractions
	$y^2-4ky+4k=0$	A1	Correct quadratic equation a.e.f.
	Or		
	$-k\pm\sqrt{k^2-k}$	B1	Find roots of original equation
	$\frac{\alpha+\beta}{\alpha}=\frac{2k}{k+\sqrt{k^2-k}}, \frac{\alpha+\beta}{\beta}=\frac{2k}{k-\sqrt{k^2-k}}$	B1	Express both new roots in terms of k
		M1	Attempt to find sum of new roots
		A1	Obtain $4k$
		M1	Attempt to find product of new roots
	$y^2-4ky+4k=0$	A1	Obtain $4k$
		B1ft	Correct quadratic equation a.e.f.

$\boxed{7}$

8	(i)		M1		Attempt to rationalise denominator or cross multiply
			A1	2	Obtain **given** answer correctly
	(ii)		M1		Express terms as differences using (i)
			M1		Attempt this for at least 1st three terms
			A1		1st three terms all correct
			A1		Last two terms all correct
			M1		Show pairs cancelling
		$\frac{1}{2}(\sqrt{n+2}+\sqrt{n+1}-\sqrt{2}-1)$	A1	6	Obtain correct answer, in terms of n
	(iii)		B1	1	Sensible statement for divergence
				$\boxed{9}$	
9	(i)		M1		Show correct expansion process for 3 x 3
			M1		Correct evaluation of any 2 x 2
		det **A** = $a^2 - a$	A1	3	Obtain correct answer
	(ii)	(a)	M1		Find a pair of inconsistent equations
			A1		State inconsistent or no solutions
		(b)	M1		Find a repeated equation
			A1		State non unique solutions
		(c)	B1		State that det **A** is non-zero or find correct solution
			B1	6	State unique solution **SC if detA incorrect, can score 2 marks for correct deduction of a unique solution, but only once**
				$\boxed{9}$	
10	(i)		M1		Attempt to equate real and imaginary parts
		$x^2 - y^2 = 3 \quad xy = 2$	A1		Obtain both results
			M1		Eliminate to obtain quadratic in x^2 or y^2
			M1		Solve to obtain x or y value
		$z = 2 + \mathrm{i}$	A1	5	Obtain correct answer as a complex no.
	(ii)		B1	1	Obtain **given** answer correctly
	(iii)		M1		Attempt to solve quadratic equation
		$w^3 = 2 \pm 11\mathrm{i}$	A1		Obtain correct answers
			M1		Choose negative sign
			M1		Relate required value to conjugate of (i)
		$w = 2 - \mathrm{i}$	A1	5	Obtain correct answer
				$\boxed{11}$	

GCE

Mathematics

Advanced GCE

Unit **4725:** Further Pure Mathematics 1

Mark Scheme for January 2011

Oxford Cambridge and RSA Examinations

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

1 **(i)**	$\begin{pmatrix} 7 & 9 \end{pmatrix}$	B1B1	**2**	Each element correct **SC** (7,9) scores B1
(ii)	$\begin{pmatrix} 18 \end{pmatrix}$	B1* depB1	 **2**	Obtain correct value Clearly given as a matrix
(iii)	$\begin{pmatrix} 12 & -4 \\ 6 & -2 \end{pmatrix}$	M1 A1 A1	 **3**	Obtain 2×2 matrix Obtain 2 correct elements Obtain other 2 correct elements
		$\boxed{7}$		
2. **(i)**	$-12+13\text{i}$	B1B1	**2**	Real and imaginary parts correct
(ii)	 $\frac{27}{37}-\frac{14}{37}\text{i}$	B1 M1 A1 A1	 **4**	z^* seen Multiply by w^* Obtain correct real part or numerator Obtain correct imaginary part or denom. Sufficient working must be shown
		$\boxed{6}$		
3		B1* M1* A1* depA1	 **4**	Establish result true for n = 1 or 2 Use given result in recurrence relation in a relevant way Obtain $2^n + 1$ correctly Specific statement of induction conclusion
		$\boxed{4}$		
4	***Either*** $\frac{a}{4}n^2(n+1)^2+\frac{bn}{2}(n+1)$ $a=4 \quad b=-4$	B1 M1 A1 M1 A1 A1	 **6**	Correct value for $\sum r$ stated or used Express as sum of two series Obtain correct unsimplified answer Compare coefficients or substitute values for n Obtain correct answers
	Or $a+b=0 \quad 4a+b=12$ $a=4 \quad b=-4$	 M1 A1 A1 M1 A1 A1		 Use 2 values for n Obtain correct equations Solve simultaneous equations Obtain correct answers
		$\boxed{6}$		
5	 $\mathbf{A}^2$	B1 M1 A1cao	 **3**	$(\mathbf{A}^{-1})^{-1} = \mathbf{A}$ seen or implied Use product inverse correctly Obtain correct answer
		$\boxed{3}$		

6	**(i)**	**(a)**	B1*	Vertical line
			depB1 **2**	Clearly through (4, 0)
		(b)	B1	Sloping line with +ve slope
			B1	Through (0, -2)
			B1ft **3**	Half line starting on y-axis 45° shown convincingly
	(ii)		B1ft	Shaded to left of their **(i) (a)**
			B1ft	Shaded below their **(i) (b)** must be +ve slope
			B1ft **3**	Shaded above horizontal through their (0, -2)
				NB These 3 marks are independent, but 3/3 only for fully correct answer.
			$\boxed{8}$	
7	**(i)**	$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$	B1 B1 **2**	Each column correct
	(ii)		B1*	Enlargement or stretch in x **and** y axes
			depB1 **2**	Scale factor $\sqrt{3}$
	(iii) (a)		B1	$(2,0),(6,2)$ indicated
			B1	(8, 2) seen
			B1 **3**	Accurate diagram, including unit square
		(b) det**C** = 4	B1	Correct value found
			B1 **2**	Scale factor for area
			$\boxed{9}$	
8	**(i)** *Either*			
		$\alpha+\beta=\frac{1}{2}, \alpha\beta=\frac{3}{2}$	B1	State or use both correct results in (i) or (ii)
		$\alpha+\beta+\frac{\alpha+\beta}{\alpha\beta}$ or $\alpha+\beta+\frac{2}{3}(\alpha+\beta)$	M1	Express sum of new roots in terms of $\alpha+\beta$ and $\alpha\beta$
			M1	Substitute their values into their expression
		$p=\frac{5}{6}$	A1 **4**	Obtain **given** answer correctly
		Or		
		$3u^2-u+2(=0)$	B1	Substitute $x=\frac{1}{u}$ and obtain correct quadratic (equation)
			M1	Use sum of roots of new equation
			M1	Substitute their values into their expression
		$p=\frac{5}{6}$	A1	Obtain **given** answer correctly

Question	Answer	Mark	Total	Guidance
(ii)	$\alpha'\beta' = \alpha\beta + \frac{1}{\alpha\beta} + \frac{\beta}{\alpha} + \frac{\alpha}{\beta}$	B1		Correct expansion
	$\frac{\beta}{\alpha} + \frac{\alpha}{\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$	M1		Show how to deal with $\alpha^2 + \beta^2$
		A1		Obtain correct expression
	$q = \frac{1}{3}$	M1		Substitute their values into $\alpha'\beta'$
		A1	5	Obtain correct answer a.e.f.
		$\boxed{9}$		
9 (i)		M1		Show correct expansion process for 3 x 3
		M1		Correct evaluation of any 2 x 2
	det **M** = $a^2 - 7a + 6$	A1	3	correct answer
(ii)		M1		Solve det**M** = 0
	a = 1 or 6	A1A1	3	Obtain correct answer, ft their (i)
(iii)		M1		Attempt to eliminate one variable
		A1		Obtain 2 correct equations in 2 unknowns
		A1	3	Justify infinite number of solutions **SC** 3/3 if unique solution conclusion consistent with their (i) or (ii)
		$\boxed{9}$		
10 (i)		M1		Use correct denominator
		A1	2	Obtain **given** answer correctly
(ii)		M1		Express terms as differences using (i)
		M1		Do this for at least 3 terms
		A1		First 3 terms all correct
		A1		Last 2 terms all correct
	$\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$	M1		Show relevant cancelling
		A1	6	Obtain correct answer a.e.f.
(iii)	$\frac{1}{2}$	B1ft		S_∞ stated or start at n + 1 as in (ii)
	$\frac{1}{n+1} - \frac{1}{n+2}$	M1		S_∞ - their (ii) or show correct cancelling
	$\frac{1}{(n+1)(n+2)}$	A1	3	Obtain **given** answer correctly
		$\boxed{11}$		

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553

GCE

Mathematics

Advanced GCE

Unit **4725:** Further Pure Mathematics 1

Mark Scheme for June 2011

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

1	(i) $\begin{pmatrix} 4 & 4a \\ 12 & 0 \end{pmatrix}$	B1		3**B** seen or implied
		B1		2 elements correct
		B1	**3**	Other 2 elements correct, a.e.f., including brackets
	(ii) $\begin{pmatrix} 4+4a & 3a \\ 4 & 1 \end{pmatrix}$	M1		Sensible attempt at matrix multiplication for **AB** or **BA**
		A1	**2**	Obtain correct answer
		$\boxed{5}$		
2		B1		Establish result true for $n = 1$ or 2
		M1*		Add next term to given sum formula
		DM1		Combine with correct denominator
		A1		Obtain correct expression convincingly
		A1	**5**	Specific statement of induction conclusion, provided 1st 4 marks earned
		$\boxed{5}$		
3	$k^2 - 16$	B1		Obtain correct det
		M1		Equate their det to 0
	$k = \pm 4$	A1	**3**	Obtain correct answers
		$\boxed{3}$		
4		M1		Express as sum of two series
	$3 \times \frac{1}{6} \times 2n(2n+1)(4n+1) - \frac{1}{2} \times 2n$	A1 A1		Each term correct a.e.f.
		M1		Attempt to factorise
	$2n^2(4n+3)$	A2	**6**	Completely correct answer, (A1 if one factor not found)
		$\boxed{6}$		
5	(i) $\lvert a \rvert = 2$	B1		Correct modulus
	$\arg a = 60°, \frac{\pi}{3}, 1.05$	B1	**2**	Correct argument
	(ii)	B1		Circle
		B1		Centre $(1, \sqrt{3})$
		B1		Through origin, centre $(\pm 1, \pm\sqrt{3})$ and another y intercept
		B1		Vertical line
		B1*		Through a or their centre, with +ve gradient
		DB1		Correct half line
			6	
		$\boxed{8}$		

6		M1		Show correct expansion process for 3×3 or multiplication of **C** and adj**C**
		M1		Correct evaluation of any 2×2
	det **C** = $\Delta = 5a - 5$	A1		Obtain correct answer
		M1		Show correct process for adjoint entries
	$\frac{1}{\Delta}\begin{pmatrix} 5 & -4 & 1 \\ -5 & 4a & -a \\ 5 & -3a-1 & 2a-1 \end{pmatrix}$	A1		Obtain at least 4 correct entries in adjoint
		A1		Obtain completely correct adjoint
		B1	**7**	Divide their adjoint by their determinant
		$\boxed{7}$		
7 (i)		B1	**1**	Obtain **given** answer correctly
(ii)		M1		Express at least 1st two and last two terms using (i)
		A1		1st two terms correct
		A1		Last two terms correct
		M1		Show that correct terms cancel
	$\frac{3}{2} - \frac{1}{n} - \frac{1}{(n+1)}$	A1	**5**	Obtain correct answer, a.e.f. in terms of n
(iii)		B1ft		Sum to infinity stated or implied or start at 1000 as in (ii)
		M1		S_∞ – their (ii) with $n = 999$ or 1000 or show correct cancelling
	$\frac{1999}{999000}$	A1	**3**	Obtain correct answer, a.e.f.
				(condone 0.002)
		$\boxed{9}$		
8 (i)		B1		(0, 3) seen
		B1		(3, 0) seen
		B1	**3**	Square with A ' B' and C' positioned correctly
(ii)	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ **or** $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1*		Reflection in $y = x$ or $y = -x$
		DB1		Correct matrix, dep on stating reflection
	$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ **or** $\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$	B1*		Enlargement scale factor 3 or s.f. -3
		DB1	**4**	Correct matrix, dep on stating enlargement **S.C. B2 for a pair of transformations consistent with their diagram.**
		$\boxed{7}$		

9 (i)	16 + 30i	B1	**1**	State correct value
(ii)		M1		Use $a = -$ (sum of roots)
	$a = -32$	A1		Obtain correct answer
		M1		Use b = product of roots
	$b = 1156$	A1	**4**	Obtain correct answer
		M1		Substitute, expand and equate imag. parts
		A1		Obtain **a** = -32
		M1		Equate real parts
		A1		Obtain b = 1156
(iii)		M1		Attempt to equate real and imaginary parts of $(p+iq)^2$ & 16 – 30i or root from (ii)
	$p^2 - q^2 = 16$ and $pq = -15$	A1		Obtain both results cao
		M1		Obtain quadratic in p^2 or q^2
		M1		Solve to obtain $p = (\pm)5$ or $q = (\pm)3$
		A1		Obtain 2 correct answers as complex nos
		M1		Attempt at all 4 roots
	$\pm\ (5 \pm 3\text{i})$	A1	**7**	State other two roots as complex nos
		$\boxed{12}$		
10 (i)				
	$\frac{1}{u^{\frac{3}{2}}} + \frac{3}{u} + 2 = 0$	B1		Use substitution correctly
	EITHER	M1		Rearrange
		M1		Square
	$\frac{9}{u^2} + \frac{12}{u} + 4 = \frac{1}{u^3}$	A1		Obtain correct equation
	$4u^3 + 12u^2 + 9u - 1 = 0$	A1	**5**	Obtain **given** answer
	OR			
	e. g. $(2u^{3/2} + 3u^{1/2} + 1)(2u^{3/2} + 3u^{1/2} - 1) = 0$	M2		Multiply their equation in u by appropriate related expression
		A2		Obtain **given** answer
(ii)		B1		Stated or imply that $u = \frac{1}{x^2}$
		M1		Use $-\frac{b}{a}$
	-3	A1		Obtain correct answer
		M1		Use $\frac{c}{a}$
	$\frac{9}{4}$	A1	**5**	Obtain correct answer
		$\boxed{10}$		

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553

GCE

Mathematics

Advanced GCE

Unit **4725:** Further Pure Mathematics 1

Mark Scheme for January 2012

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

Annotations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last

(complete) attempt and ignore the others.
NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$a^2+5^2=13^2$ $a=12$ $\tan^{-1}\dfrac{5}{a}$ 0.395 or 22.6° or 0.126π	M1 A1 M1 A1FT **[4]**	Use formula for modulus Obtain correct answer Use formula for argument Obtain correct answer allow 0.39	
2			 $3p+4q=1,\quad -3p-5q=1,\quad 2p+3q=0$ $p=3$ and $q=-2$	B1 M1 A1 M1 A1 **[5]**	State identity matrix is $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ Find a pair of simultaneous equations Correct pair of distinct equations Attempt to solve Obtain correct answers	
3			 $x^2-y^2=3$ and $xy=3\sqrt{2}$ $x^4-3x^2-18=0$ or $y^4+3y^2-18=0$ $x=\pm\sqrt{6}$ or $y=\pm\sqrt{3}$ $\pm(\sqrt{6}+\mathrm{i}\sqrt{3})$	M1 A1 M1 M1 A1 A1 **[6]**	Attempt to equate real and imaginary parts Obtain both results Eliminate to obtain quadratic in x^2 or y^2 Solve to obtain x or y value Both values correct Correct answers as complex numbers	

Question			Answer	Marks	Guidance	
4				M1	Express as difference of two series	
				DM1	Use standard series results	
			$\frac{1}{4}n^2(n+1)^2 - \frac{3}{2}n(n+1)$	A1	Obtain correct unsimplified answer	
				M1	Attempt to factorise	
				A1	At least factor of $n(n+1)$	From their unsimplified answer
			$\frac{1}{4}n(n+1)(n+3)(n-2)$	A1	Obtain correct answer	
				[6]		
5	(a)		$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1	Each column correct	
				B1		
				[2]		
5	(b)	(i)		B1	Stretch	
				DB1	Scale factor 4 in the y direction	Not "in the y-axis"
				[2]		
5	(b)	(ii)	4	B1	Correct value of determinant	
				B1	Scale factor for area	Allow scale factor of stretch or eqiv.
				[2]		
6				B1	Circle	
				B1	Centre $(\sqrt{3},1)$	
				B1	Passing through O and crosses y-axis again	
				B1	Line, with correct slope shown	
				B1	$\frac{1}{2}$ line starting at O	
				B1	Completely correct diagram for both loci	Ignore shading
				[6]		

Question			Answer	Marks	Guidance	
7	(i)			M1 A1 A1 **[3]**	Attempt at matrix multiplication Obtain $\mathbf{M}^2$ correctly Obtain **given answer** correctly	
7	(ii)		$\begin{pmatrix} 3^n & 0 \\ 3^n - 1 & 1 \end{pmatrix}$	B1 B1 **[2]**	3 elements correct 4th element correct	
7	(iii)		$\begin{pmatrix} 3^{k+1} & 0 \\ 3^{k+1} - 1 & 1 \end{pmatrix}$	B1 M1 A1 B1 **[4]**	Show that their result is true for n = 1 or 2 Attempt to find $\mathbf{M}^k \cdot \mathbf{M}$ or vice versa Obtain correct answer Complete statement of induction conclusion	 Must have 1st 3 marks
8	(i)			M1 A1 **[2]**	Combine with a common denominator Obtain **given** answer correctly	
8	(ii)		$\frac{n}{n+1}$	M1 A1 M1 A1 **[4]**	Express terms using **(i)** At least 1st two and last two correct Show terms cancelling Obtain correct answer, in terms of n	

Question			Answer	Marks	Guidance	
8	(iii)		$1-\frac{n}{n+1}$	B1 B1FT **[2]**	$\lim_{n\to\infty}\frac{n}{n+1}=1$ This value – **(ii)**	
9	(i)		det $\mathbf{X}$ = $\Delta=10-9a-a^2$	M1 M1 A1 **[3]**	Show correct expansion process for 3×3 Correct evaluation of any 2×2 Obtain correct answer aef	
9	(ii)		$a = 1$ or -10	M1 A1FT A1FT **[3]**	Their det $\mathbf{X} = 0$ Obtain correct answers from their **(i)**	
9	(iii)		$\frac{1}{\Delta}\begin{pmatrix}-a & 2 & 6-9a\\ 5 & -a-9 & 18-3a\\ -a & 2 & a^2-4\end{pmatrix}$	M1 A1 A1 B1ft **[4]**	Show correct process for adjoint entries Obtain at least four correct entries in adjoint Obtain completely correct adjoint Divide by their determinant	
10	(i)		$\alpha+\beta+\gamma=3$ $\alpha\beta+\beta\gamma+\gamma\alpha=2$ $\alpha\beta\gamma=-\frac{2}{3}$	B1 B1 B1 **[3]**	State correct value State correct value State correct value	

Question			Answer	Marks	Guidance	
10	(ii)		EITHER			
				M1	$c = (\pm)\alpha^2\beta^2\gamma^2$	
			$c = -\frac{4}{9}$	A1FT	Obtain **given** correct answer	FT for sign error in **(i)**
			$\sum\alpha^2 = (\sum\alpha)^2 - 2\sum\alpha\beta$	M1	Use correct expression	
			5	A1FT	Obtain correct value	FT for sign error in **(i)**
			$a = -5$	A1FT	Obtain answer correctly	Sign change done correctly
				M1*	Attempt to find an identity	
			$\sum\alpha^2\beta^2 = (\sum\alpha\beta)^2 - 2\alpha\beta\gamma\sum\alpha$	A1	Obtain correct identity	
				DM1	Use appropriate values	
			$b = 8$	A1	Obtain correct answer cao	
				[9]		
			OR			
				B1	State or use correct substitution	
				M1	Rearrange, fractional indices isolated	
				DM1	Square both sides	
				DM1	Expand and simplify	
			$9y^3 - 45y^2 + 72y - 4 = 0$	A1	Obtain correct equation	
				M1	Use coefficients of their cubic	
			$c = -\frac{4}{9}$	A1	Obtain given answer correctly	
			$a = -5$	A1FT	Obtain correct answer	
			$b = 8$	A1FT	Obtain correct answer	
				[9]	**SC** mixture of methods only A1FT for *a* and *b*	

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4725:** Further Pure Mathematics 1

Mark Scheme for June 2012

Oxford Cambridge and RSA Examinations

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
Dep/D	mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
A2	Accuracy mark awarded 2

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.
NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		21 +11i	B1 B1 **[2]**	Real part correct Imaginary part correct	
1	(ii)		26 – 29i $\frac{26}{41}-\frac{29}{41}\mathrm{i}$	M1 A1 A1 **[3]**	Multiply by conjugate of denominator or find a pair of simultaneous equations Obtain correct numerator or real part Obtain correct denominator or imaginary part	
2	(i)		$\begin{pmatrix} 5 & 2 \\ 13 & 6 \end{pmatrix}$	M1 A1 **[2]**	Multiplication attempt, 2 elements correct All elements correct	
2	(ii)		EITHER $\mathbf{B}^{-1}\mathbf{A}^{-1} = (\mathbf{AB})^{-1}$ $\frac{1}{4}\begin{pmatrix} 6 & -2 \\ -13 & 5 \end{pmatrix}$ OR	B1 B1ft B1ft **[3]** B1 B1 B1	Stated or used Divide by correct determinant Both diagonals correct Either inverse correct Two elements correct in final answer , both inverses must be correct All elements correct	

Question			Answer	Marks	Guidance	
			EITHER			
3				M1	Use sum of root and conjugate	
			$a = -8$	A1	Obtain correct answer	
				M1	Use product of root and conjugate	
			$b = 25$	A1	Obtain correct answer	
				[4]		
			OR			
				M1	Substitute $4+3i$ or conjugate into equation	
				M1	Equate real and imaginary parts	
			$a = -8$	A1	Obtain correct answer	
			$b = 25$	A1	Obtain correct answer	
4				M1	Express as sum of 3 series	
				M1	Use standard series results, at least 1 correct	
				A1	Two terms correct	
			$\frac{1}{2}n(n+1)(2n+1) - \frac{3}{2}n(n+1) + 2n$	A1	Third term correct	
				M1	Obtain factor of n	
			$n(n^2+1)$	A2	Obtain correct answer c.a.o.	
					Allow A1 for $\dfrac{1}{2(2n^2+2)}$	
				[7]		
5				B1	Verify result true when $n = 1$	
				M1*	Add next term in series	
				DepM1	Attempt to obtain 3^{k+1} correctly	
				A1	Show sufficient working to justify correct expression	
				B1	Clear statements of Induction processes, but 1st 4 marks must all be earned.	
				[5]		

Question			Answer	Marks	Guidance	
6	(i)			M1	Attempt to clear fractions	
				M1	Attempt to expand and simplify to a quadratic	
			$5u^2 + 11u + 8 = 0$	A1	Obtain correct answer, must be an equation	
				[3]		
6	(ii)		EITHER			
			$u = \frac{1}{x} - 1$	B1	State or imply by using roots of new quadratic	
				M1	Use their *c*/*a*	
			$\frac{8}{5}$	A1 FT	Obtain correct answer	
				[3]		
			OR			
			$\frac{1}{\alpha\beta} - \frac{\alpha + \beta}{\alpha\beta} + 1$	B1	Express in terms of $\alpha + \beta$ and $\alpha\beta$	
				M1	Use values $-\frac{1}{2}$ and $\frac{5}{2}$ correctly	Must be values from original equation
			$\frac{8}{5}$	A1	Obtain correct answer	

Question			Answer	Marks	Guidance	
7	(i)			B1B1 B1ft B1ft B1B1 **[6]**	Circle, centre (3 , 4) Touching x-axis, ft for (3, −4) etc as centre Crossing y-axis twice Horizontal line, y intercept 4	
7	(ii)		−1 +4i 7 + 4i	B1B1 **[2]**	State correct answers	
7	(iii)			B1ft B1 **[2]**	Inside circle or above line Completely correct diagram	
8	(i)			B1 **[1]**	Show **given** answer correctly	
8	(ii)		$1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}$	M1 M1 A1 A1 M1 A1 **[6]**	Express terms as differences using (i) Attempt this for at least first 3 terms First 3 terms all correct Last 2 terms correct Show terms cancelling Obtain correct answer, must be in terms of n	
8	(iii)		$\frac{3}{2}$ $N = 4$	B1ft B1 M1 A1 **[4]**	State or use correct sum to infinity Their sum to infinity – their (ii) = $\frac{11}{30}$ Attempt to solve correct equation Obtain only $N = 4$	

Question			Answer	Marks	Guidance	
9	(i)			B1* depB1 **[2]**	Shear eg image of (0 , 1) is (2 , 1) or parallel to the *x*-axis	
9	(ii)		Either $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$ Or $\mathbf{Z} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} a & 2a+b \\ c & 2c+d \end{pmatrix}$ $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$	B1 B1 B1 M1 A1 **[5]** B1 B1 B1 M1 A1	State **Z** = **YX** Obtain **Y** = **ZX**$^{-1}$ State or use correct inverse Matrix multiplication, 2 elements correct Obtain completely correct simplified exact matrix Correct order for matrix multiplication Obtain 2correct elements Obtain other 2 correct elements Equate elements, 2 correct Obtain completely correct simplified exact matrix	
9	(iii)			B1* depB1 **[2]**	Rotation 60° clockwise	

Question			Answer	Marks	Guidance	
10	(i)		$a^3 - 4a$	M1 M1 A1 **[3]**	Show correct expansion process for 3×3 Correct evaluation of any 2×2 Obtain correct answer	
10	(ii)	(a)		B1 **[1]**	det **D = 15** so unique sol'n or solve to find correct solution (-2/5, 1, 4/5)	**SC** B1 once if unique solution following their incorrect det **D** non zero
10	(ii)	(b)		B1 M1 A1 **[3]**	Their det **D** = 0, so non-unique solutions Attempt to solve equations with $a = 2$ Explain inconsistency with correct working	
10	(ii)	(c)		B1 M1 A1 **[3]**	Their det **D** = 0, so non-unique solutions Attempt to solve equations with $a = 0$ Explain consistency with correct working	

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553

RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced Subsidiary GCE

Unit **4725:** Further Pure Mathematics 1

Mark Scheme for January 2013

Oxford Cambridge and RSA Examinations

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
DM1 or M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1	(i)		$\begin{pmatrix} 2a-3 & 2 \\ 2 & 5 \end{pmatrix}$	B1 B1 B1 **[3]**	**I** or 3**I** seen or used 2 elements correct Other 2 elements correct
1	(ii)		$\frac{1}{4a-1}\begin{pmatrix} 4 & -1 \\ -1 & a \end{pmatrix}$ or equivalent	B1 B1 **[2]**	Divide by correct determinant Both diagonals correct
2			$\frac{1}{6}n(n+1)(2n+1)-n$ $\frac{1}{6}n(2n+5)(n-1)$	M1* DM1 A1 DM1 A2 **[6]**	Attempt to expand $(r-1)(r+1)$ Use standard result for $\sum r$ Obtain correct unsimplified answer Attempt to factorise Obtain completely correct answer Allow A1 if one bracket still contains a common factor
3	(i)		$\lvert z\rvert = \sqrt{5}$ $\arg z = -26.6^\circ$ or -0.464	B1 B1 **[2]**	Allow 2.2 Allow -27° or -0.46(3)
3	(ii)		 $a+b=2, b-a=-8$ $a=5,\ b=-3$	B1 M1 A1 M1 A1 **[5]**	$z^* = 2 + i$ stated or used Obtain two equations from real and imaginary parts Obtain correct equations Attempt to solve 2 linear equations Obtain correct answers

Question			Answer	Marks	Guidance
4	(i)		$4u^2+6u+k+2=0$	M1 A1 **[2]**	Substitute and attempt to simplify Obtain correct answer, must be an **equation**
4	(ii)		*Either* $\frac{k+2}{4}$ *Or* $\frac{k+2}{4}$	M1 A1ft **[2]** M1 A1	Use products of roots of new quadratic i.e. use (±) *c*/*a* Obtain correct answer, from their quadratic Use sum and product of roots of original equation Obtain correct answer
5			$3\lambda^2-7\lambda+2$ $\frac{1}{3}$ or 2	M1 M1 A1 B1* DM1 A1 **[6]**	Show correct expansion process for correct 3 x 3 Correct evaluation of any 2 x 2 Obtain correct 3 term quadratic Equate their det to 0 Attempt to solve a quadratic equation Obtain correct answers
6	(i)		$\begin{pmatrix}1&2\\0&2\end{pmatrix}$	B1 B1 **[2]**	Each column correct
6	(ii)		*Either* P: $\begin{pmatrix}1&0\\0&2\end{pmatrix}$ *Or* $\begin{pmatrix}1&2\\0&1\end{pmatrix}$ Q: $\begin{pmatrix}1&1\\0&1\end{pmatrix}$ $\begin{pmatrix}1&0\\0&2\end{pmatrix}$	B1 DB1 B1 B1 DB1 B1 **[6]**	*Either* Stretch, s.f. 2 in *y* direction *Or* Shear, *x*-axis invariant e.g. (0,1) → (2,1) Correct matrix Shear, *x* axis invariant e.g. (0, 1) → (1, 1) Stretch, s.f.2 in *y* direction, Correct matrix N.B. **“in the *x/y* axis” is incorrect**
6	(iii)		PQ: $\begin{pmatrix}1&1\\0&2\end{pmatrix}$ $\begin{pmatrix}1&4\\0&2\end{pmatrix}$	M1 A1 **[2]**	Attempt at matrix multiplication of two 2 x 2 matrices from (ii) Obtain correct result cao

Question			Answer	Marks	Guidance
7	(i)	(a)		B1 B1 **[2]**	Circle Centre O and radius 2
7	(i)	(b)		B1 B1 B1 **[3]**	Horizontal line (3, 1) on their line ½ line to left i.e. horizontal
7	(ii)			B1 B1 **[2]**	Shade only inside their circle or above their horizontal line Completely correct diagram
8	(i)			M1 A1 **[2]**	Obtain correct numerator from addition or partial fractions Obtain **given** answer correctly
8	(ii)		$\frac{n}{(n+1)(n+2)}$	M1 A1 A1 M1 A1 **[5]**	Express at least three relevent terms using (i) 1st three terms correct Last two terms correct Show correct cancelling Obtain **given** answer correctly
8	(iii)		$-\frac{1}{6}$	M1 A1 **[2]**	Sum 1 to ∞ - 1st term or start process at $r = 2$ Obtain correct answer

Question			Answer	Marks	Guidance
9	(i)			M1 A1 A1 **[3]**	Attempt at complete expansion Obtain correct unsimplified answer Obtain **given** answer correctly
9	(ii)		*Either* $\sum\alpha = -p, \sum\alpha\beta = -4, \alpha\beta\gamma = -3$ $\frac{16-6p}{9}$ *Or* $9u^3 + (6p-16)u^2 + (8+p^2)u - 1 = 0$ $\frac{16-6p}{9}$	B1 M1 A1 M1 A1 **[5]** B1 M1 A1 M1 A1	State (anywhere) correct values for $\sum\alpha, \sum\alpha\beta, \sum\alpha\beta\gamma$ Express given expression as a single fraction Obtain correct expression using (i) Use their values for sum of roots etc. in their expression Obtain correct answer Use substitution $1/\sqrt{u}$ Rearrange appropriately and square out Obtain correct co-efficients of u^3 and u^2 Use (+/-)b/a from their cubic Obtain correct answer
10	(i)		$\frac{2}{3}, \quad \frac{2}{5}, \quad \frac{2}{7}$	B1 B1 B1 **[3]**	B1 x 3, Obtain 3 correct values Justify **given** answer
10	(ii)		$\frac{2}{2n-1}$	M1 A1 **[2]**	Fraction, in terms of n, with correct numerator or denominator Obtain correct answer a.e.f.
10	(iii)		$\frac{2}{2(n+1)-1}$	B1ft M1 A1 A1 B1 **[5]**	Verify result true when n = 1, for their (ii), or n = 2, 3 or 4 Expression for u_{n+1} using recurrence relation in terms of n using their (ii) Correct unsimplified answer Correct answer in correct form Specific statement of induction conclusion, previous 4 marks must be earned, n=1 must be verified

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553

PART OF THE CAMBRIDGE ASSESSMENT GROUP

UKAS MANAGEMENT SYSTEMS 001

GCE

Mathematics

Advanced Subsidiary GCE

Unit **4725:** Further Pure Mathematics 1

Mark Scheme for June 2013

Oxford Cambridge and RSA Examinations

Annotations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
aef	Any equivalent form

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1				M1	Use correct trig expression
			$\sqrt{3}$	A1	Obtain correct answer
				M1	Correct expression for modulus
			$2\sqrt{3}$	A1FT	Obtain correct answer aef
			$3 - \sqrt{3}i$	B1FT	Correct conjugate seen or implied
			$-\sqrt{3}\,i$	B1FT	Correct answer
				[6]	
2	(i)		$\begin{pmatrix} 7 & 23 \end{pmatrix}$	B1B1	Each element correct, missing brackets B1 only
				[2]	
2	(ii)		$\begin{pmatrix} 6 & -15 \\ 4 & -10 \end{pmatrix}$	M1	Obtain 2 × 2 matrix
				A1	Obtain 2 correct elements
				A1	Obtain other 2 correct elements
			det **CB** = 0	A1FT	Obtain their det **CB,** must be a 2 × 2 matrix
			singular	A1FT	Correct conclusion from their det **CB**
				[5]	
3			$x^2 - y^2 = 11$ and $xy = 6\sqrt{5}$	M1	Attempt to equate real and imaginary parts of $(x + iy)^2$ and $11 + 12\sqrt{5}$
				A1	Obtain both results cao
				M1*	Obtain a quadratic in x^2 or y^2
			$\pm(2\sqrt{5} + 3i)$	DM1	Solve a 3 term quadratic to obtain a value for x or y
				A1	Obtain 1 correct answer as complex number
				A1	Obtain only the other correct answer
				[6]	
4				B1	Establish result true for $n = 1$ or $n = 2$
				M1	Multiply **M** and $\mathbf{M}^k$, either order
			$2\left(2^{k+1} - 2\right) + 2$ or $2^{k+1} + 2^{k+1} - 2$	A1	Obtain correct element
				A1	Obtain other 3 correct elements
				A1	Obtain $2^{k+2} - 2$ convincingly
				B1	Specific statement of induction conclusion, provided 5/5 earned so far and verified for $n = 1$
				[6]	

Question			Answer	Marks	Guidance
5			$4\times\frac{1}{4}n^2(n+1)^2-3\times\frac{1}{6}n(n+1)(2n+1)+\frac{1}{2}n(n+1)$	M1 A1 A1	Express as sum of three series Obtain 2 correct (unsimplified) terms Obtain correct 3rd (unsimplified) term
			$n^3(n+1)$	M1 A2 **[6]**	Attempt to factorise, at least factor of n Obtain correct answer, A1 if not fully factorised
6	**(i)**		$\arg(z-3i)=\frac{1}{4}\pi$	M1 A1	Use arg $(z-\text{a}) = \theta$ in equation for l condone missing brackets Obtain correct answer
			$\lvert z-3i\rvert=3$	M1 A1 **[4]**	Use $\lvert z-a\rvert=k$ in equation for C, k must be real Obtain correct answer
	(ii)		$\lvert z-3i\rvert\le 3$ or e.g. $x^2+(y-3)^2\le 9$ $\frac{1}{4}\pi\le\arg(z-3i)\le\frac{1}{2}\pi$ or $y\ge x+3,\ x\ge 0$	B1 B1 B1 **[3]**	Obtain correct inequality, or answer consistent with sensible **(i)** Each correct single inequality, or answer consistent with sensible **(i)** **SC if < used consistently, but otherwise all correct, B2**
7	**(i)**		$\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$	B1B1 **[2]**	Each column correct
	(ii)		$\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$	B1B1 **[2]**	Each column correct
	(iii)		$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$	M1 A1FT **[2]**	Attempt at matrix multiplication in correct order Obtain correct answer from their **(i)** and **(ii)**
	(iv)		Reflection, in $y=x$	B1B1 **[2]**	Correct description of their **(iii) only**

Question			Answer	Marks	Guidance
8			*Either*		
			$\sum\alpha = -\frac{6}{k},\ \sum\alpha\beta = \frac{1}{k}$	B1B1	Correct values stated or used
			$\sum\alpha\beta + 2\sum\alpha + 3$	M1 A1	Expand brackets Obtain correct expression aef
			$3 - \frac{11}{k}$	M1 A1 **[6]**	Use their values, in terms of k, for $\sum\alpha$ and $\sum\alpha\beta$ Obtain correct answer aef
			Or		
				B1 M1	State or use substitution $x = u - 1$ Expand and attempt to simplify coefficients
			$ku^3 + (6-3k)u^2 + (3k-11)u + 2 - k = 0$	A1 A1	Obtain at least correct 1st and 3rd terms
				M1	Use their "$\frac{c}{a}$"
			$3 - \frac{11}{k}$	A1	Obtain correct answer a.e.f.
9	(i)			M1 A1 **[2]**	Use correct denominator or partial fractions Obtain **given** answer convincingly
	(ii)			M1 A1 M1	Express at least 1st two and last term using **(i)** All terms correct Show correct terms cancelling
			$\frac{1}{2} - \frac{1}{6n+2}$	A1	Obtain correct unsimplified answer
				M1 A1 **[6]**	Include $\frac{1}{3}$ and combine their sum as a single fraction Obtain **given** answer

Question			Answer	Marks	Guidance
10	(i)			M1	Show correct expansion process for 3 × 3
				M1	Correct evaluation of any 2 × 2
			$a + 3$	A1	Obtain correct answer
				M1	Use det **A** = 0
			$a = -3$	A1FT	Obtain correct answer from their det **A**
				[5]	
	(ii)		$\frac{1}{a+3}\begin{pmatrix} 1 & -1 & 1 \\ 7 & a-4 & 1-2a \\ -11 & 8-a & 3a-2 \end{pmatrix}$	M1	Show correct processes for adjoint entries
				A1	Obtain at least 4 correct entries in adjoint
				A1	Obtain completely correct adjoint
				B1	Divide adjoint by their det **A**
			$\frac{1}{a+3}\begin{pmatrix} 2 \\ 2-4a \\ 7a-1 \end{pmatrix}$	M1	Pre-multiply column matrix by their $\mathbf{A}^{-1}$
				A2	Obtain correct answer, A1 for 1 element correct
				[7]	

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553