

# OCR

Oxford Cambridge and RSA

## Friday 20 May 2016 – Morning

### AS GCE MATHEMATICS (MEI)

4755/01 Further Concepts for Advanced Mathematics (FP1)

#### QUESTION PAPER

Candidates answer on the Printed Answer Book.

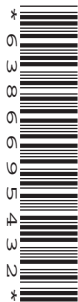
**OCR supplied materials:**

- Printed Answer Book 4755/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



#### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

#### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

#### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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## Section A (36 marks)

1 The matrix  $\mathbf{M}$  is given by  $\mathbf{M} = \begin{pmatrix} 8 & -2 \\ p & 1 \end{pmatrix}$ , where  $p \neq -4$ .

(i) Find the inverse of  $\mathbf{M}$  in terms of  $p$ .

[2]

(ii)

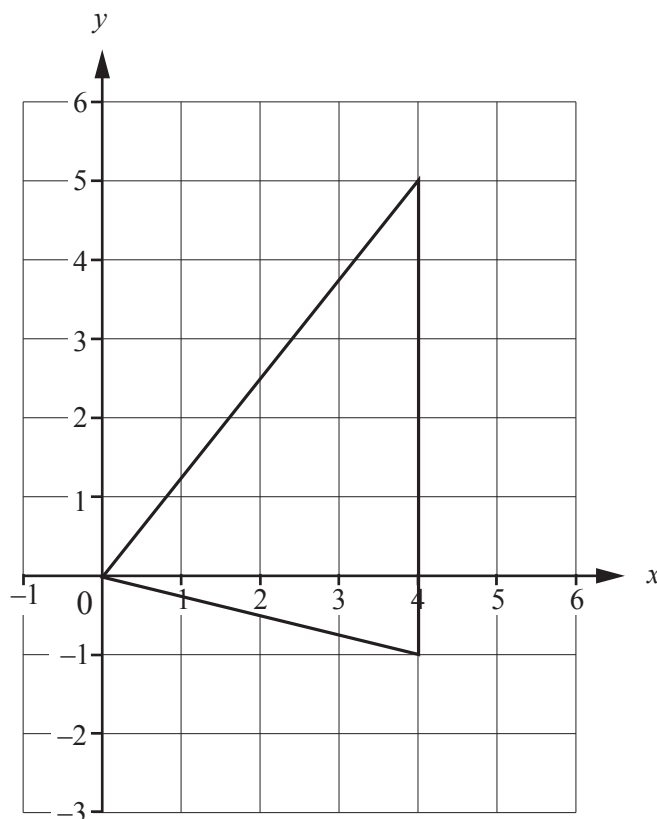


Fig. 1

The triangle shown in Fig. 1 undergoes the transformation represented by the matrix  $\begin{pmatrix} 8 & -2 \\ 3 & 1 \end{pmatrix}$ . Find the area of the image of the triangle following this transformation. [2]

2 The complex number  $z_1$  is  $2 - 5j$  and the complex number  $z_2$  is  $(a - 1) + (2 - b)j$ , where  $a$  and  $b$  are real.

(i) Express  $\frac{z_1^*}{z_1}$  in the form  $x + yj$ , giving  $x$  and  $y$  in exact form. You must show clearly how you obtain your answer. [4]

(ii) Given that  $\frac{z_1^*}{z_1} = z_2$ , find the exact values of  $a$  and  $b$ . [2]

- 3 You are given that  $\mathbf{A} = \begin{pmatrix} \lambda & 6 & -4 \\ 2 & 5 & -1 \\ -1 & 4 & 3 \end{pmatrix}$ ,  $\mathbf{B} = \begin{pmatrix} -19 & 34 & -14 \\ 5 & -5 & 5 \\ -13 & 18 & -3 \end{pmatrix}$  and  $\mathbf{AB} = \mu \mathbf{I}$ , where  $\mathbf{I}$  is the  $3 \times 3$  identity matrix.

(i) Find the values of  $\lambda$  and  $\mu$ . [4]

(ii) Hence find  $\mathbf{B}^{-1}$ . [2]

- 4 (i) Use standard series to show that

$$\sum_{r=1}^n r^2(2r-p) = \frac{1}{6}n(n+1)(3n^2 + (3-2p)n - p),$$

where  $p$  is a constant. [4]

(ii) Given that the coefficients of  $n^3$  and  $n^4$  in the expression for  $\sum_{r=1}^n r^2(2r-p)$  are equal, find the value of  $p$ . [2]

- 5 The loci  $C_1$  and  $C_2$  are given by  $|z+3-4j|=5$  and  $\arg(z+3-6j) = \frac{1}{2}\pi$  respectively.

(i) Sketch, on a single Argand diagram, the loci  $C_1$  and  $C_2$ . [5]

(ii) Write down the complex number represented by the point of intersection of  $C_1$  and  $C_2$ . [1]

(iii) Indicate, by shading on your sketch, the region satisfying

$$|z+3-4j| \geq 5 \quad \text{and} \quad \frac{1}{2}\pi \leq \arg(z+3-6j) \leq \frac{3}{4}\pi. \quad [2]$$

- 6 A sequence is defined by  $u_1 = 8$  and  $u_{n+1} = 3u_n + 2n + 5$ . Prove by induction that  $u_n = 4(3^n) - n - 3$ . [6]

## Section B (36 marks)

7 The function  $f(z) = 2z^4 - 9z^3 + Az^2 + Bz - 26$  has real coefficients. The equation  $f(z) = 0$  has two real roots,  $\alpha$  and  $\beta$ , where  $\alpha > \beta$ , and two complex roots,  $\gamma$  and  $\delta$ , where  $\gamma = 3 + 2j$ .

(i) Show that  $\alpha + \beta = -\frac{3}{2}$  and find the value of  $\alpha\beta$ . [5]

(ii) Hence find the two real roots  $\alpha$  and  $\beta$ . [3]

(iii) Find the values of  $A$  and  $B$ . [3]

(iv) Write down the roots of the equation  $f\left(\frac{w}{j}\right) = 0$ . [2]

8 A curve has equation  $y = \frac{3x^2 - 9}{x^2 + 3x - 4}$ .

(i) Find the equations of the two vertical asymptotes and the one horizontal asymptote of this curve. [3]

(ii) State, with justification, how the curve approaches the horizontal asymptote for large positive and large negative values of  $x$ . [3]

(iii) Sketch the curve. [3]

(iv) Solve the inequality  $\frac{3x^2 - 9}{x^2 + 3x - 4} \geq 0$ . [3]

9 You are given that  $\frac{3}{4(2r-1)} - \frac{1}{2r+1} + \frac{1}{4(2r+3)} = \frac{2r+5}{(2r-1)(2r+1)(2r+3)}$ .

(i) Use the method of differences to show that

$$\sum_{r=1}^n \frac{2r+5}{(2r-1)(2r+1)(2r+3)} = \frac{2}{3} - \frac{3}{4(2n+1)} + \frac{1}{4(2n+3)}. \quad [6]$$

(ii) Write down the limit to which  $\sum_{r=1}^n \frac{2r+5}{(2r-1)(2r+1)(2r+3)}$  converges as  $n$  tends to infinity. [1]

(iii) Find the sum of the finite series

$$\frac{45}{39 \times 41 \times 43} + \frac{47}{41 \times 43 \times 45} + \frac{49}{43 \times 45 \times 47} + \dots + \frac{105}{99 \times 101 \times 103},$$

giving your answer to 3 significant figures. [4]

## END OF QUESTION PAPER

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