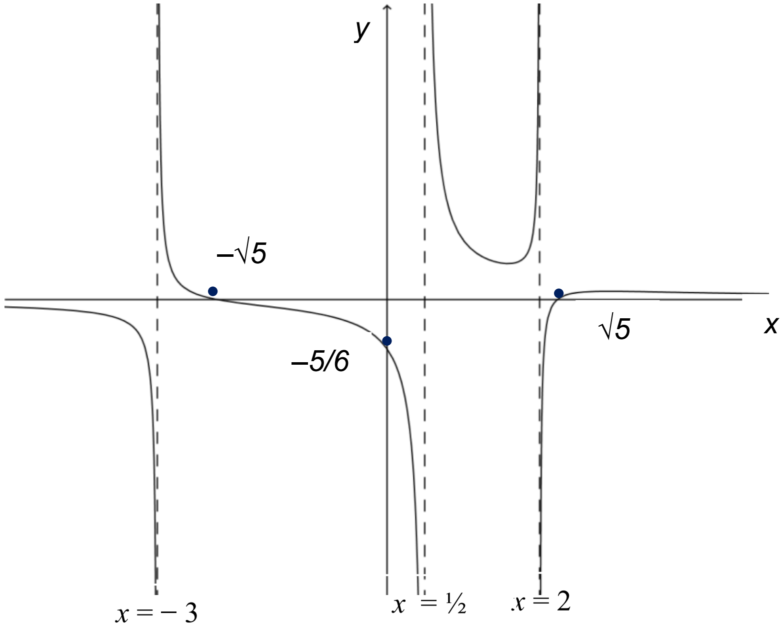
Question			Answer	Marks	Guidance
1			$\sum_{\gamma=1}^{n} r(r-2) = \sum_{\gamma=1}^{n} r^2 - 2\sum_{\gamma}^{n} r$	M1	Separate sum (may be implied)
			$= \frac{1}{6}n(n+1)(2n+1) - n(n+1)$	A1,A1	1 mark for each part oe
			$= \frac{1}{6}n(n+1)[(2n+1)-6]$	M1	$n(n+1)$(linear factor) seen
			$= \frac{1}{6}n(n+1)(2n-5)$	A1	Or $n(n+1)(2n-5)/6$ only, ie 1/6 must be a factor
				[5]	
2	(i)		$\begin{pmatrix} -3 & -2 \\ -2 & 1 \end{pmatrix}$	B1,B1 **[2]**	1 mark for each column. Must be a 2×2 matrix Condone lack of brackets throughout
2	(ii)		$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	B1 **[1]**	
2	(iii)		$\begin{pmatrix} -3 & -2 \\ 2 & -1 \end{pmatrix}$	B1,B1 **[2]**	1 mark for each column (no ft). Must be a 2×2 matrix

Question			Answer	Marks	Guidance
3			$z = 2 - 3\mathrm{j}$ is also a root	B1	
			Either		
			$\left(z-(2+3\mathrm{j})\right)\left(z-(2-3\mathrm{j})\right)=\left((z-2)+3\mathrm{j}\right)\left((z-2)-3\mathrm{j}\right)$	M1	Condone (z+ 2+ 3j)(z+ 2–3j)
			$= z^2 - 4z + 13$	A1	Correct quadratic
			$z^4 - 5z^3 + 15z^2 - 5z - 26 = \left(z^2 - 4z + 13\right)\left(z^2 - z - 2\right)$	M1 A1	Valid method to find the other quadratic factor. Correct quadratic
			$\left(z^2 - z - 2\right) = (z-2)(z+1)$		
			So the other roots are 2 and –1	A1,A1 **[7]**	1 mark for each root, cao
			Or		
			$2+3j+2-3j+\gamma+\delta=5$ oe	B1	Sum of roots with substitution of roots $2\pm3j$ for α and β
			$(2+3j)(2-3j)\gamma\delta=-26$ $\gamma\delta=-2$	M1	Attempt to obtain equation in $\gamma\delta$ using a root relation and $2\pm3j$
			$\Rightarrow 4+\gamma+\delta=5 \Rightarrow \gamma=1-\delta$		
			and $13\gamma\delta=-26 \Rightarrow \gamma\delta=-2$	M1	Eliminating γ or δ leading to a quadratic equation
			$\Rightarrow \delta(1-\delta)=-2 \Rightarrow \delta^2-\delta-2=0$	A1	Correct equation obtained
			$\Rightarrow (\delta+1)(\delta-2)=0$		
			So the other roots are – 1 and 2.	A1,A1 **[7]**	1 mark for each, cao If 2, –1 guessed from $\gamma+\delta=1$ and $\gamma\delta=-2$ give A1 A1 for these equations and A1A1 for the roots. **SC** factor theorem used. M1 for substitution of $z=-1$ (or 2) or division by (z + 1) (or by z – 2), A1 if zero obtained, B1 for the root stated to be – 1(or 2). For the other root, similarly but M1A1A1 Max [**7/7**] Answers only get M0M0, max [**1/7**]

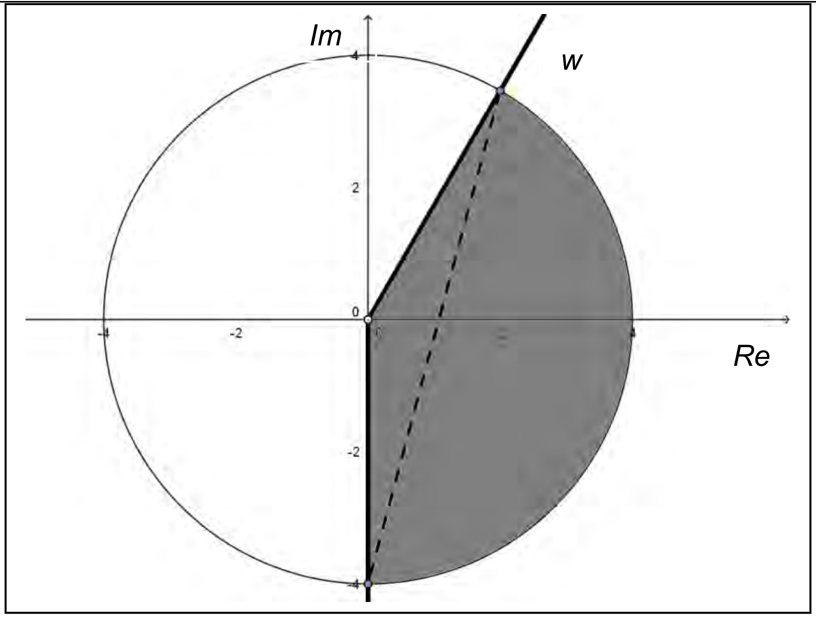
Question			Answer	Marks	Guidance
4			$\sum_{r=1}^{n}\frac{1}{(2r+3)(2r+5)}=\frac{1}{2}\sum_{r=1}^{n}\left[\frac{1}{2r+3}-\frac{1}{2r+5}\right]$	M1	Split to partial fractions. Allow missing $\frac{1}{2}$
			$=\frac{1}{2}\left[\left(\frac{1}{5}-\frac{1}{7}\right)+\left(\frac{1}{7}...\right)+.....+\left(...-\frac{1}{2n+5}\right)\right]$	M1 A1	Expand to show pattern of cancelling, at least 4 fractions All correct, allow missing $\frac{1}{2}$, condone r
			$=\frac{1}{2}\left[\frac{1}{5}-\frac{1}{2n+5}\right]=\frac{n}{5(2n+5)}$	M1 A1	Cancel to first minus last term must be in terms of n. oe single fraction
				[5]	

Question			Answer	Marks	Guidance
5			**Either**		
			$y = 3x - 1 \Rightarrow x = \frac{y+1}{3}$	M1*	Change of variable, condone $\frac{y-1}{3}, \frac{y}{3} \pm 1$.
			$\Rightarrow 3\left(\frac{y+1}{3}\right)^3 - 9\left(\frac{y+1}{3}\right)^2 + \left(\frac{y+1}{3}\right) - 1 = 0$	M1dep* A1	Substitute into cubic expression Correct
			Correct coefficients in cubic expression (may be fractions)	A3ft	ft their substitution (-1 each error)
			$\Rightarrow y^3 - 6y^2 - 12y - 14 = 0$	A1	cao. Must be an equation with integer coefficients
				[7]	
			Or		
			$\alpha + \beta + \gamma = \frac{9}{3} = 3$	M1	All three root relations, condone incorrect signs
			$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{1}{3}$		
			$\alpha\beta\gamma = \frac{1}{3}$	A1	All correct
			Let new roots be k, l, m then $k + l + m = 3(\alpha + \beta + \gamma) - 3 = 6$ $kl + km + lm = 9(\alpha\beta + \alpha\gamma + \beta\gamma) - 6(\alpha + \beta + \gamma) + 3 = -12$	M1	Using (3α-1) etc in $\sum k, \sum kl, klm$, at least two attempted, and using $\sum \alpha, \sum \alpha\beta, \alpha\beta\gamma$
			$klm = 27\alpha\beta\gamma - 9(\alpha\beta + \beta\gamma + \beta\gamma) + 3(\alpha + \beta + \gamma) - 1 = 14$	A3ft	One each for 6, – 12, 14, ft their $3, \frac{1}{3}, \frac{1}{3}$.
			$\Rightarrow y^3 - 6y^2 - 12y - 14 = 0$	A1	cao. Must be an equation with integer coefficients
				[7]	

Question			Answer	Marks	Guidance
6			When $n = 1$, $\frac{1}{1\times 3}=\frac{1}{3}$ and $\frac{n}{2n+1}=\frac{1}{3}$, so true for $n = 1$	B1	Condone eg "$\frac{1}{3}=\frac{1}{3}$ "
			Assume true for $n = k$	E1	Assuming true for k, (some work to follow) If in doubt look for unambiguous "if…then" at next E1 Statement of assumed result not essential but further work should be seen
			Sum of $k + 1$ terms		NB "last term = sum of terms" seen anywhere earns final E0
			$=\frac{k}{2k+1}+\frac{1}{(2k+1)(2k+3)}$	M1	Adding correct $(k + 1)$th term to sum for k terms
			$=\frac{k(2k+3)+1}{(2k+1)(2k+3)}$	M1	Combining their fractions
			$=\frac{2k^2+3k+1}{(2k+1)(2k+3)}$		
			$=\frac{(k+1)(2k+1)}{(2k+1)(2k+3)}=\frac{k+1}{2k+3}$	A1	Complete accurate work
			which is $\frac{n}{2n+1}$ with $n=k+1$		May be shown earlier
			Therefore if true for $n = k$ it is also true for $n = k+1$.	E1	Dependent on A1 and previous E1.
			Since it is true for $n = 1$, it is true for all positive integers, n.	E1	Dependent on B1 and previous E1 E0 if "last term"= "sum of terms " seen above
				[7]	

Question			Answer	Marks	Guidance
7	(i)		$\left(0, -\frac{5}{6}\right)$	B1	Allow for both $x=0$ and $y=-\frac{5}{6}$ seen
			$\left(\sqrt{5},\ 0\right), \left(-\sqrt{5},\ 0\right)$	B1	(both) Allow $(\pm\sqrt{5},0)$ or for both $y=0$ and $x=\pm\sqrt{5}$ seen
				[2]	
7	(ii)		$a=2$	B1	
			$y=0$	B1	
			$x=-3, x=2$	B1	Must be two equations
				[3]	
7	(iii)			B1	Two outer branches correctly placed Inner branches correctly placed Correct asymptotes and intercepts labelled
				B1	For good drawing. Dep all 3 marks above
				B1	Look for a clear maximum point on the right-hand branch, (not really shown here).
				B1	Condone turning points in $-\sqrt{5} < x < \frac{1}{2}, y < 0$
				[4]	
	(iv)		$-3 < x < -\sqrt{5},\ \frac{1}{2} < x < 2,\ x > \sqrt{5}$	B3	One mark for each. Strict inequalities. Allow 2.24 for $\sqrt{5}$ (if B3 then – 1 if more than 3 inequalities)
				[3]	

Question			Answer	Marks	Guidance
8	(i)		$\lvert w\rvert = \sqrt{\left(2^2 + \left(2\sqrt{3}\right)^2\right)} = 4$	B1	
			$\arg w = \arctan\dfrac{2\sqrt{3}}{2} = \dfrac{\pi}{3}$	M1	
			$w = 4\left(\cos\dfrac{\pi}{3} + \mathrm{j}\sin\dfrac{\pi}{3}\right)$	A1	Accept $\left(4,\ \dfrac{\pi}{3}\right)$, 1.05 rad , 60° in place of $\dfrac{\pi}{3}$, or $4e^{j\frac{\pi}{3}}$
				[3]	

Question			Answer	Marks	Guidance
8	(ii)			B1	Circle, or arc of circle, centre the origin
				B1	Radius 4
				B1	Half line from origin $\frac{\pi}{4} < \text{angle} < \frac{\pi}{2}$ with positive real axis or acute angle labelled as π/3
				B1	Use of negative Im axis clearly indicated
				B1	Correct region indicated. Dependent on first 4 B marks Ignore placing of *w*.
				B1	*w* at intersection of $\frac{\pi}{3}$ line and circle (dep 1st 3 B marks)
			Maximum $\lvert z-w\rvert = \sqrt{\left(2^2+\left(4+2\sqrt{3}\right)^2\right)}$ = 7.73 (3 s.f.)	B1	Maximum $\lvert z-w\rvert$ indicated by chord on diagram oe or sight of $-4j-(2+2\sqrt{3}j)$ oe
			Or 2x 4cos15□ = $2\sqrt{6}+2\sqrt{2}$	M1	Valid attempt to calculate maximum $\lvert z-w\rvert$
				A1	allow $\sqrt{32+16\sqrt{3}}$ oe (accept 2 s.f. or better)
				[9]	

Question			Answer	Marks	Guidance
9	(i)		$\beta = (-1)(3\alpha-1)+5\alpha+(-1)(2\alpha+1)$	M1	multiply second row of **A** with first column of **B**
			$= -3\alpha+1+5\alpha-2\alpha-1=0$	A1	Correct
				[2]	
9	(ii)		$\gamma = (1)(3\alpha-1)+15+(-1)(2\alpha+1)$	M1	Attempt to multiply relevant row of **A** with relevant column of **B.** Condone use of **BA** instead
			$=\alpha+13$	A1	Correct
				[2]	
9	(iii)		When $\alpha=2,\ \gamma=15$	M1	Multiplication of **B** by $\dfrac{1}{\text{their } \gamma}$,($\gamma \neq 1$) using $\alpha=2$ in both
			$\mathbf{A}^{-1}=\frac{1}{15}\begin{pmatrix}5&-8&-1\\5&1&2\\5&-5&5\end{pmatrix}$	A1	Correct elements in matrix and correct γ.
			$\mathbf{A}^{-1}$ does not exist when $\alpha=-13$	B1ft	ft their $\gamma=0$. Condone "$\alpha\neq-13$"
				[3]	
9	(iv)		$\frac{1}{15}\begin{pmatrix}5&-8&-1\\5&1&2\\5&-5&5\end{pmatrix}\begin{pmatrix}25\\11\\-23\end{pmatrix}=\begin{pmatrix}x\\y\\z\end{pmatrix}$	M1	Set-up of pre-multiplication by their 3x3 $\mathbf{A}^{-1}$, or by **B** (using $\alpha=2$)
			$=\frac{1}{15}\begin{pmatrix}60\\90\\-45\end{pmatrix}=\begin{pmatrix}4\\6\\-3\end{pmatrix}$	B1	$(60\ \ 90\ \ -45)'$ soi need not be fully evaluated
			$\Rightarrow x=4,\ y=6,\ z=-3$	A3	cao A1 for each explicit identification of x, y, z in a vector or a list. (-1 unidentified) Answers only or solution by other method, M0A0
				[5]	