GCE

Mathematics (MEI)

Advanced Subsidiary GCE **4755**

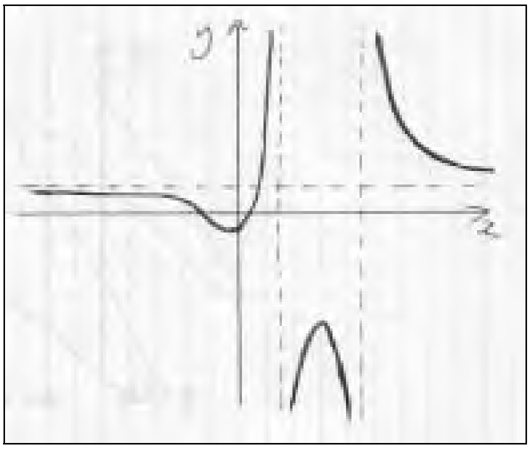
Further Concepts for Advanced Mathematics (FP1)

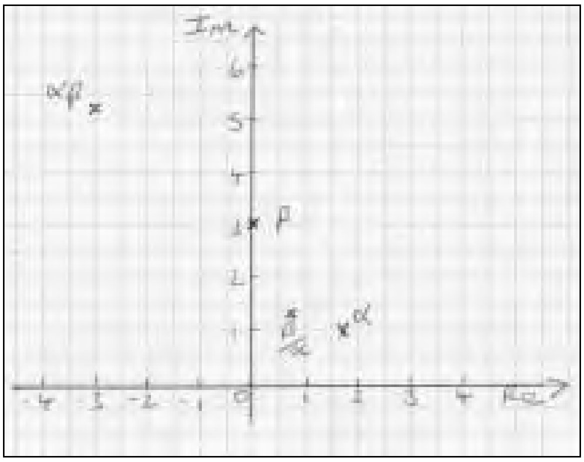
Mark Scheme for June 2010

Qu	Answer	Mark	Comment
Section A			
1	$4x^2-16x+C\equiv A\left(x^2+2Bx+B^2\right)+2$	B1	$A = 4$
	$\Leftrightarrow 4x^2-16x+C\equiv Ax^2+2ABx+AB^2+2$	M1	Attempt to expand RHS or other valid method (may be implied)
	$\Leftrightarrow A=4,\ B=-2,\ C=18$	A2, 1 **[4]**	1 mark each for B and C, c.a.o.
2(i)	$2x-5y=9$	B1	
	$3x+7y=-1$	B1 **[2]**	
2(ii)	$\mathbf{M}^{-1}=\frac{1}{29}\begin{pmatrix}7 & 5\\-3 & 2\end{pmatrix}$	M1 A1 **[2]**	Divide by determinant c.a.o.
	$\frac{1}{29}\begin{pmatrix}7 & 5\\-3 & 2\end{pmatrix}\begin{pmatrix}9\\-1\end{pmatrix}=\frac{1}{29}\begin{pmatrix}58\\-29\end{pmatrix}$ $\Rightarrow x=2,\ y=-1$	M1 A1(ft) **[2]**	Pre-multiply by their inverse For both
3	$z=1-2\text{j}$	B1	
	$1+2\text{j}+1-2\text{j}+\alpha=\frac{1}{2}$	M1	Valid attempt to use sum of roots, or other valid method
	$\Rightarrow \alpha=-\frac{3}{2}$	A1	c.a.o.
	$\frac{-k}{2}=-\frac{3}{2}(1-2\text{j})(1+2\text{j})=-\frac{15}{2}$	M1 A1(ft)	Valid attempt to use product of roots, or other valid method Correct equation – can be implied
	$k=15$	A1 **[6]**	c.a.o.
	OR		
	$\left(z-(1+2\text{j})\right)\left(z-(1-2\text{j})\right)=z^2-2z+5$	M1 A1	Multiplying correct factors Correct quadratic, c.a.o.
	$2z^3-z^2+4z+k=\left(z^2-2z+5\right)(2z+3)$	M1	Attempt to find linear factor
	$\alpha=\frac{-3}{2}$	A1(ft)	
	k = 15	A1 **[6]**	c.a.o.

4	$w = x+1 \Rightarrow x = w-1$	B1	Substitution. For $x = w+1$ give B0 but then follow for a maximum of 3 marks
	$x^3 - 2x^2 - 8x + 11 = 0,\ w = x-1$ $\Rightarrow (w-1)^3 - 2(w-1)^2 - 8(w-1) + 11 = 0$ $\Rightarrow w^3 - 5w^2 - w + 16 = 0$	M1 M1 A3 **[6]**	Attempt to substitute into cubic Attempt to expand -1 for each error (including omission of = 0)
	OR		
	$\alpha + \beta + \gamma = 2$ $\alpha\beta + \alpha\gamma + \beta\gamma = -8$ $\alpha\beta\gamma = -11$	B1	All 3 correct
	Let the new roots be *k*, *l* and *m* then		
	$k + l + m = \alpha + \beta + \gamma + 3 = 2 + 3 = 5$	M1	Valid attempt to use their sum of roots in original equation to find sum of roots in new equation
	$kl + km + lm = (\alpha\beta + \alpha\gamma + \beta\gamma) + 2(\alpha + \beta + \gamma) + 3$ $= -8 + 4 + 3 = -1$ $klm = \alpha\beta\gamma + (\alpha\beta + \alpha\gamma + \beta\gamma) + (\alpha + \beta + \gamma) + 1$ $= -11 - 8 + 2 + 1 = -16$	M1	Valid attempt to use their product of roots in original equation to find one of $\sum \alpha\beta$ or $\alpha\beta\gamma$
	$\Rightarrow w^3 - 5w^2 - w + 16 = 0$	A3 **[6]**	-1 each error (including omission of = 0)
5	$\sum_{r=1}^{n} \frac{1}{(5r-1)(5r+4)} = \frac{1}{5}\sum_{r=1}^{n}\left(\frac{1}{5r-1} - \frac{1}{5r+4}\right)$	M1	Attempt to use identity – may be implied
	$= \frac{1}{5}\left(\left(\frac{1}{4} - \frac{1}{9}\right) + \left(\frac{1}{9} - \frac{1}{14}\right) + ... + \left(\frac{1}{5n-1} - \frac{1}{5n+4}\right)\right)$	A1	Terms in full (at least first and last)
	$= \frac{1}{5}\left(\frac{1}{4} - \frac{1}{5n+4}\right) = \frac{1}{5}\left(\frac{5n+4-4}{4(5n+4)}\right) = \frac{n}{4(5n+4)}$	M1	Attempt at cancelling
		A1	$\left(\frac{1}{4} - \frac{1}{5n+4}\right)$
		A1	factor of $\frac{1}{5}$
		A1 **[6]**	Correct answer as a single algebraic fraction

6(i)	$u_2 = \frac{2}{1+2} = \frac{2}{3},\ u_3 = \frac{\frac{2}{3}}{1+\frac{2}{3}} = \frac{2}{5}$	M1 A1 **[2]**	Use of inductive definition c.a.o.
6(ii)	When n = 1, $\frac{2}{2\times1-1} = 2$, so true for n = 1	B1	Showing use of $u_n = \frac{2}{2n-1}$
	Assume $u_k = \frac{2}{2k-1}$	E1	Assuming true for k
	$\Rightarrow u_{k+1} = \frac{\frac{2}{2k-1}}{1+\frac{2}{2k-1}}$	M1	u_{k+1}
	$= \frac{\frac{2}{2k-1}}{\frac{2k-1+2}{2k-1}} = \frac{2}{2k+1}$	A1	Correct simplification
	$= \frac{2}{2(k+1)-1}$		
	But this is the given result with k + 1 replacing k. Therefore if it is true for k it is also true for k + 1.	E1	Dependent on A1 and previous E1
	Since it is true for k = 1, it is true for all positive integers.	E1 **[6]**	Dependent on B1 and previous E1
			Section A Total: 36

Section B			
7(i)	$\left(0,\ -\frac{1}{2}\right)$	B1	
	$(-3,\ 0),\ \left(\frac{1}{2},\ 0\right)$	B1 **[2]**	For both
7(ii)	$x = 3$, $x = 2$ and $y = 2$	B1 B1 B1 **[3]**	
7(iii)	Large positive x, $y \to 2^+$ (e.g. substitute $x = 100$ to give 2.15…, or convincing algebraic argument)	M1 A1	Must show evidence of method A0 if no valid method
		B1 **[3]**	Correct RH branch
7(iv)	$\frac{(2x-1)(x+3)}{(x-3)(x-2)} = 2$ $\Rightarrow (2x-1)(x+3) = 2(x-3)(x-2)$ $\Rightarrow x = 1$	M1 A1	Or other valid method to find intersection with horizontal asymptote
	From graph $x < 1$ or $2 < x < 3$	B1 B1 **[4]**	For $x < 1$ For $2 < x < 3$

8(i)	$\arg\alpha = \frac{\pi}{6}, \ \lvert\alpha\rvert = 2$ $\arg\beta = \frac{\pi}{2}, \ \lvert\beta\rvert = 3$	B1 B1 B1 **[3]**	Modulus of α Argument of α (allow 30°) Both modulus and argument of β (allow 90°)
8(ii)	$\alpha\beta = \left(\sqrt{3}+\mathrm{j}\right)3\mathrm{j} = -3+3\sqrt{3}\mathrm{j}$ $\frac{\beta}{\alpha} = \frac{3\mathrm{j}}{\sqrt{3}+\mathrm{j}} = \frac{3\mathrm{j}\left(\sqrt{3}-\mathrm{j}\right)}{\left(\sqrt{3}+\mathrm{j}\right)\left(\sqrt{3}-\mathrm{j}\right)}$ $= \frac{3+3\sqrt{3}\mathrm{j}}{4} = \frac{3}{4}+\frac{3\sqrt{3}\mathrm{j}}{4}$	M1 A1 M1 A1 A1 **[5]**	Use of $\mathrm{j}^2 = -1$ Correct . Correct use of conjugate of denominator Denominator = 4 All correct
8(iii)		M1 A1(ft) **[2]**	Argand diagram with at least one correct point Correct relative positions with appropriate labelling

Qu	Answer	Mark	Comment
Section B (continued)			
9(i)	P is a rotation through 90 degrees about the origin in a clockwise direction. Q is a stretch factor 2 parallel to the *x*-axis	B1 B1 B1 B1 **[4]**	Rotation about origin 90 degrees clockwise, or equivalent Stretch factor 2 Parallel to the *x*-axis
9(ii)	$\mathbf{QP} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix}$	M1 A1 **[2]**	Correct order c.a.o.
9(iii)	$\begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 4 & 2 \\ -2 & -1 & -3 \end{pmatrix}$ $A' = (0,\ -2),\ B' = (4,\ -1),\ C' = (2,\ -3)$	M1 A1(ft) **[2]**	Pre-multiply by their **QP** - may be implied For all three points
9(iv)	$\mathbf{R} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1 B1 **[2]**	One for each correct column
9(v)	$\mathbf{RQP} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$ $(\mathbf{RQP})^{-1} = \frac{-1}{2}\begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$	M1 A1(ft) M1 A1 **[4]**	Multiplication of their matrices in correct order Attempt to calculate inverse of their **RQP** c.a.o.
			Section B Total: 36
			Total: 72