OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4755

Further Concepts for Advanced Mathematics (FP1)

Thursday **8 JUNE 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 3 printed pages and 1 blank page.

Section A (36 marks)

1 **(i)** State the transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. [1]

(ii) Write down the 2×2 matrix for rotation through 90° anticlockwise about the origin. [1]

(iii) Find the 2×2 matrix for rotation through 90° anticlockwise about the origin, followed by reflection in the x-axis. [2]

2 Find the values of A, B, C and D in the identity

$$2x^3 - 3x^2 + x - 2 \equiv (x+2)(Ax^2 + Bx + C) + D.$$ [5]

3 The cubic equation $z^3 + 4z^2 - 3z + 1 = 0$ has roots α, β and γ.

(i) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. [3]

(ii) Show that $\alpha^2 + \beta^2 + \gamma^2 = 22$. [3]

4 Indicate, on separate Argand diagrams,

(i) the set of points z for which $|z - (3 - \mathrm{j})| \leqslant 3$, [3]

(ii) the set of points z for which $1 < |z - (3 - \mathrm{j})| \leqslant 3$, [2]

(iii) the set of points z for which $\arg(z - (3 - \mathrm{j})) = \frac{1}{4}\pi$. [3]

5 **(i)** The matrix $\mathbf{S} = \begin{pmatrix} -1 & 2 \\ -3 & 4 \end{pmatrix}$ represents a transformation.

(*A*) Show that the point $(1, 1)$ is invariant under this transformation. [1]

(*B*) Calculate $\mathbf{S}^{-1}$. [2]

(*C*) Verify that $(1, 1)$ is also invariant under the transformation represented by $\mathbf{S}^{-1}$. [1]

(ii) Part **(i)** may be generalised as follows.

If (x, y) is an invariant point under a transformation represented by the non-singular matrix $\mathbf{T}$, it is also invariant under the transformation represented by $\mathbf{T}^{-1}$.

Starting with $\mathbf{T}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$, or otherwise, prove this result. [2]

6 Prove by induction that $3 + 6 + 12 + \ldots + 3 \times 2^{n-1} = 3(2^n - 1)$ for all positive integers n. [7]

Section B (36 marks)

7 A curve has equation $y = \dfrac{x^2}{(x-2)(x+1)}$.

(i) Write down the equations of the three asymptotes. [3]

(ii) Determine whether the curve approaches the horizontal asymptote from above or from below for

(A) large positive values of x,

(B) large negative values of x. [3]

(iii) Sketch the curve. [4]

(iv) Solve the inequality $\dfrac{x^2}{(x-2)(x+1)} > 0$. [3]

8 **(i)** Verify that $2 + \mathrm{j}$ is a root of the equation $2x^3 - 11x^2 + 22x - 15 = 0$. [5]

(ii) Write down the other complex root. [1]

(iii) Find the third root of the equation. [4]

9 **(i)** Show that $r(r+1)(r+2) - (r-1)r(r+1) \equiv 3r(r+1)$. [2]

(ii) Hence use the method of differences to find an expression for $\sum_{r=1}^{n} r(r+1)$. [6]

(iii) Show that you can obtain the same expression for $\sum_{r=1}^{n} r(r+1)$ using the standard formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$. [5]