OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4755

Further Concepts For Advanced Mathematics (FP1)

Tuesday **7 JUNE 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.

This question paper consists of 3 printed pages and 1 blank page.

Section A (36 marks)

1 **(i)** Find the inverse of the matrix $\mathbf{A} = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$. [2]

(ii) Use this inverse to solve the simultaneous equations

$$\begin{aligned} 4x + 3y &= 5, \\ x + 2y &= -4, \end{aligned}$$

showing your working clearly. [3]

2 Find the roots of the quadratic equation $x^2 - 8x + 17 = 0$ in the form $a + b\mathrm{j}$.

Express these roots in modulus-argument form. [5]

3 Find the equation of the line of invariant points under the transformation given by the matrix $\mathbf{M} = \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}$. [3]

4 The quadratic equation $x^2 - 2x + 4 = 0$ has roots α and β.

(i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [1]

(ii) Hence find the value of $\alpha^2 + \beta^2$. [2]

(iii) Find a quadratic equation which has roots 2α and 2β. [2]

5 **(i)** Sketch the locus $|z - (3 + 4\mathrm{j})| = 2$ on an Argand diagram. [2]

(ii) On the same diagram, sketch the locus $\arg(z - 4) = \frac{1}{2}\pi$. [2]

(iii) Indicate clearly on your sketch the points which satisfy **both**

$$|z - (3 + 4\mathrm{j})| = 2 \quad \textbf{and} \quad \arg(z - 4) = \tfrac{1}{2}\pi.$$ [1]

6 Prove by induction that $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$. [7]

7 Find $\sum_{r=1}^{n} 3r(r-1)$, expressing your answer in a fully factorised form. [6]

Section B (36 marks)

8 A curve has equation $y = \dfrac{x^2 - 4}{(3x-2)^2}$.

(i) Find the equations of the asymptotes. [2]

(ii) Describe the behaviour of the curve for large positive and large negative values of x, justifying your description. [3]

(iii) Sketch the curve. [5]

(iv) Solve the inequality $\dfrac{x^2 - 4}{(3x-2)^2} \geqslant -1$. [4]

9 The quartic equation $x^4 + Ax^3 + Bx^2 + Cx + D = 0$, where A, B, C and D are real numbers, has roots $2 + \mathrm{j}$ and $-2\mathrm{j}$.

(i) Write down the other roots of the equation. [2]

(ii) Find the values of A, B, C and D. [8]

10 **(i)** You are given that

$$\frac{2}{r(r+1)(r+2)} = \frac{1}{r} - \frac{2}{r+1} + \frac{1}{r+2}.$$

Use the method of differences to show that

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)} = \frac{1}{2} - \frac{1}{(n+1)(n+2)}.$$ [9]

(ii) Hence find the sum of the infinite series

$$\frac{1}{1 \times 2 \times 3} + \frac{1}{2 \times 3 \times 4} + \frac{1}{3 \times 4 \times 5} + \dots .$$ [3]