

**Friday 20 January 2012 – Afternoon**

**AS GCE MATHEMATICS (MEI)**

**4755** Further Concepts for Advanced Mathematics (FP1)

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4755
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

**Section A** (36 marks)

- 1 You are given that  $\mathbf{A} = \begin{pmatrix} 2 & -1 & 1 \\ 0 & p & -4 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} 0 & q \\ 2 & -2 \\ 1 & -3 \end{pmatrix}$ .
- (i) Find  $\mathbf{AB}$ . [3]
- (ii) Hence prove that matrix multiplication is not commutative. [2]
- 2 Find the values of  $A$ ,  $B$ ,  $C$  and  $D$  in the identity  $2x^3 - 3 \equiv (x + 3)(Ax^2 + Bx + C) + D$ . [5]
- 3 Given that  $z = 6$  is a root of the cubic equation  $z^3 - 10z^2 + 37z + p = 0$ , find the value of  $p$  and the other roots. [6]
- 4 Using the standard summation formulae, find  $\sum_{r=1}^n r^2(r-1)$ . Give your answer in a fully factorised form. [6]
- 5 The equation  $z^3 - 5z^2 + 3z - 4 = 0$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ . Find the cubic equation whose roots are  $\frac{\alpha}{2} + 1$ ,  $\frac{\beta}{2} + 1$ ,  $\frac{\gamma}{2} + 1$ , expressing your answer in a form with integer coefficients. [6]
- 6 Prove by induction that  $\sum_{r=1}^n r3^{r-1} = \frac{1}{4}[3^n(2n-1) + 1]$ . [8]

**Section B** (36 marks)

- 7 A curve has equation  $y = \frac{(x+1)(2x-1)}{x^2-3}$ .
- (i) Find the coordinates of the points where the curve crosses the axes. [2]
- (ii) Write down the equations of the three asymptotes. [3]
- (iii) Determine whether the curve approaches the horizontal asymptote from above or from below for
- (A) large positive values of  $x$ ,
- (B) large negative values of  $x$ . [3]
- (iv) Sketch the curve. [3]
- (v) Solve the inequality  $\frac{(x+1)(2x-1)}{x^2-3} < 2$ . [3]

- 8 (i) Sketch on an Argand diagram the locus,  $C$ , of points for which  $|z - 4| = 3$ . [3]
- (ii) By drawing appropriate lines through the origin, indicate on your Argand diagram the point A on the locus  $C$  where  $\arg z$  has its maximum value. Indicate also the point B on the locus  $C$  where  $\arg z$  has its minimum value. [2]
- (iii) Given that  $\arg z = \alpha$  at A and  $\arg z = \beta$  at B, indicate on your Argand diagram the set of points for which  $\beta \leq \arg z \leq \alpha$  and  $|z - 4| \geq 3$ . [2]
- (iv) Calculate the value of  $\alpha$  and the value of  $\beta$ . [3]
- 9 The matrix  $\mathbf{R}$  is  $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ .
- (i) Explain in terms of transformations why  $\mathbf{R}^4 = \mathbf{I}$ . [3]
- (ii) Describe the transformation represented by  $\mathbf{R}^{-1}$  and write down the matrix  $\mathbf{R}^{-1}$ . [2]
- (iii)  $\mathbf{S}$  is the matrix representing rotation through  $60^\circ$  anticlockwise about the origin. Find  $\mathbf{S}$ . [2]
- (iv) Write down the smallest positive integers  $m$  and  $n$  such that  $\mathbf{S}^m = \mathbf{R}^n$ , explaining your answer in terms of transformations. [2]
- (v) Find  $\mathbf{RS}$  and explain in terms of transformations why  $\mathbf{RS} = \mathbf{SR}$ . [3]

**THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE**



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