

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI)

Further Concepts for Advanced Mathematics (FP1)

4755

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

None

Wednesday 20 January 2010
Afternoon

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

- 1 Two complex numbers are given by $\alpha = -3 + j$ and $\beta = 5 - 2j$.

Find $\alpha\beta$ and $\frac{\alpha}{\beta}$, giving your answers in the form $a + bj$, showing your working. [5]

- 2 You are given that $\mathbf{A} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 5 & 1 \\ 2 & -3 \end{pmatrix}$, $\mathbf{C} = (5 \quad 1 \quad 8)$ and $\mathbf{D} = \begin{pmatrix} -2 & 0 \\ 4 & 1 \end{pmatrix}$.

(i) Calculate, where they exist, \mathbf{AB} , \mathbf{CA} , $\mathbf{B} + \mathbf{D}$ and \mathbf{AC} and indicate any that do not exist. [5]

(ii) Matrices \mathbf{B} and \mathbf{D} represent transformations B and D respectively. Find the single matrix that represents transformation B followed by transformation D. [2]

- 3 The roots of the cubic equation $4x^3 - 12x^2 + kx - 3 = 0$ may be written $a - d$, a and $a + d$. Find the roots and the value of k . [6]

- 4 You are given that if $\mathbf{M} = \begin{pmatrix} 4 & 0 & 1 \\ -6 & 1 & 1 \\ 5 & 2 & 5 \end{pmatrix}$ then $\mathbf{M}^{-1} = \frac{1}{k} \begin{pmatrix} -3 & -2 & 1 \\ -35 & -15 & 10 \\ 17 & 8 & -4 \end{pmatrix}$.

Find the value of k . Hence solve the following simultaneous equations. [6]

$$\begin{aligned} 4x + z &= 9 \\ -6x + y + z &= 32 \\ 5x + 2y + 5z &= 81 \end{aligned}$$

- 5 Use standard series formulae to show that $\sum_{r=1}^n (r+2)(r-3) = \frac{1}{3}n(n^2 - 19)$. [6]

- 6 Prove by induction that $1 \times 2 + 2 \times 3 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$ for all positive integers n . [6]

Section B (36 marks)

7 A curve has equation $y = \frac{5x - 9}{(2x - 3)(2x + 7)}$.

- (i) Write down the equations of the two vertical asymptotes and the one horizontal asymptote. [3]
- (ii) Describe the behaviour of the curve for large positive and large negative values of x , justifying your answers. [3]
- (iii) Sketch the curve. [3]
- (iv) Solve the inequality $\frac{5x - 9}{(2x - 3)(2x + 7)} \leq 0$. [3]

- 8 (a) Fig. 8 shows an Argand diagram.

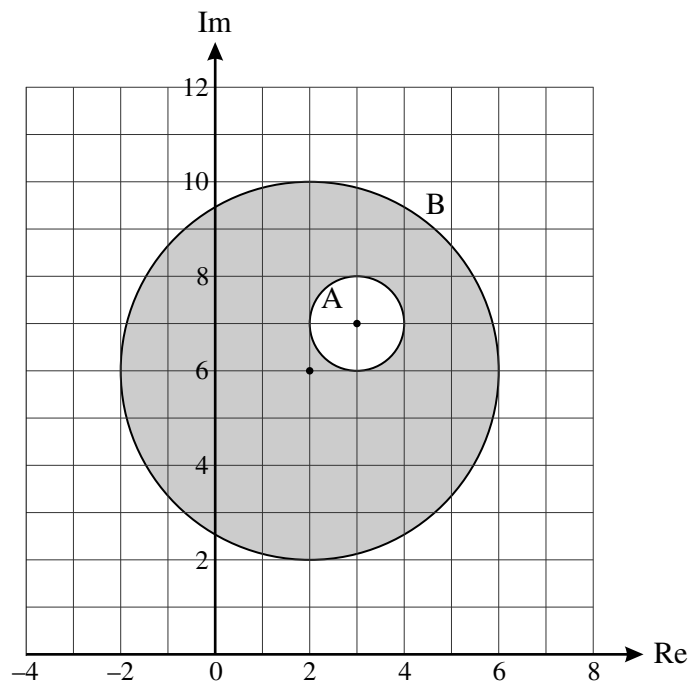


Fig. 8

- (i) Write down the equation of the locus represented by the circumference of circle B. [3]
- (ii) Write down the two inequalities that define the shaded region between, but not including, circles A and B. [3]
- (b) (i) Draw an Argand diagram to show the region where
- $$\frac{\pi}{4} < \arg(z - (2 + j)) < \frac{3\pi}{4}. \quad [3]$$
- (ii) Determine whether the point $43 + 47j$ lies within this region. [3]

9 (i) Verify that $\frac{4+r}{r(r+1)(r+2)} = \frac{2}{r} - \frac{3}{r+1} + \frac{1}{r+2}$. [2]

(ii) Use the method of differences to show that

$$\sum_{r=1}^n \frac{4+r}{r(r+1)(r+2)} = \frac{3}{2} - \frac{2}{n+1} + \frac{1}{n+2}. \quad [6]$$

(iii) Write down the limit to which $\sum_{r=1}^n \frac{4+r}{r(r+1)(r+2)}$ converges as n tends to infinity. [1]

(iv) Find $\sum_{r=50}^{100} \frac{4+r}{r(r+1)(r+2)}$, giving your answer to 3 significant figures. [3]



Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations, is given to all schools that receive assessment material and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.