4755 (FP1) Further Concepts for Advanced Mathematics

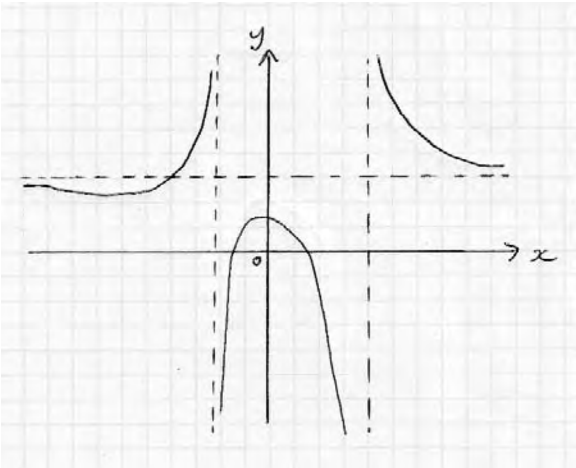
Section A

1(i)	$z=\dfrac{6\pm\sqrt{36-40}}{2}$ $\Rightarrow z=3+\text{j}$ or $z=3-\text{j}$	M1 A1 **[2]**	Use of quadratic formula/completing the square For both roots
1(ii)	$\lvert 3+\text{j}\rvert=\sqrt{10}=3.16$ (3s.f.) $\arg(3+\text{j})=\arctan\left(\frac{1}{3}\right)=0.322$ (3s.f.) $\Rightarrow$ roots are $\sqrt{10}(\cos 0.322+\text{j}\sin 0.322)$ and $\sqrt{10}(\cos 0.322-\text{j}\sin 0.322)$ or √10(cos(-0.322)+jsin(-0.322))	M1 M1 A1 **[3]**	Method for modulus Method for argument (both methods must be seen following A0) One mark for both roots in modulus-argument form – accept surd and decimal equivalents and (r, θ) form. Allow $\pm 18.4^{\circ}$ for θ.
2	$2x^2-13x+25=A(x-3)^2-B(x-2)+C$ $\Rightarrow 2x^2-13x+25$ $=Ax^2-(6A+B)x+(2B+C)+9A$ A = 2 B = 1 C = 5	B1 M1 A1 A1 **[4]**	For A=2 Attempt to compare coefficients of x^1 or x^0, or other valid method. For B and C, cao.
3(i)	$\begin{pmatrix}2 & 1\\0 & 2\end{pmatrix}$	B1 **[1]**	
3(ii)	$\begin{pmatrix}2 & -1\\0 & 3\end{pmatrix}\begin{pmatrix}0 & 2 & 3 & 1\\0 & 0 & 2 & 2\end{pmatrix}=\begin{pmatrix}0 & 4 & 4 & 0\\0 & 0 & 6 & 6\end{pmatrix}$ $\Rightarrow$ A″=(4, 0), B″=(4, 6), C″=(0, 6)	M1 A1 **[2]**	Applying matrix to column vectors, with a result. All correct
3(iii)	Stretch factor 4 in x-direction. Stretch factor 6 in y-direction	B1 B1 **[2]**	Both factor and direction for each mark. SC1 for “enlargement”, not stretch.

4	$\arg\left(z-(2-2\text{j})\right)=\dfrac{\pi}{4}$	B1 B1 B1 **[3]**	Equation involving arg(complex variable). Argument (complex expression) = $\dfrac{\pi}{4}$ All correct
5	Sum of roots = $\alpha+(-3\alpha)+\alpha+3=3-\alpha=5$ $\Rightarrow \alpha=-2$ Product of roots $= -2\times 6\times 1=-12$ Product of roots in pairs $=-2\times 6+(-2)\times 1+6\times 1=-8$ $\Rightarrow p=-8$ and $q=12$ Alternative solution $(x-\alpha)(x+3\alpha)(x-\alpha-3)$ $=x^3+(\alpha-3)x^2+(-5\alpha^2-6\alpha)x+3\alpha^3+9\alpha^2$ => $\alpha = -2$, $p = -8$ and $q = 12$	M1 A1 M1 M1 A1 A1 **[6]** M1 M1A1 M1 A1A1 **[6]**	Use of sum of roots Attempt to use product of roots Attempt to use sum of products of roots in pairs One mark for each, ft if α incorrect Attempt to multiply factors Matching coefficient of x^2 ,cao. Matching other coefficients One mark for each, ft incorrect α.
6	$\sum_{r=1}^{n}\left[r\left(r^2-3\right)\right]=\sum_{r=1}^{n}r^3-3\sum_{r=1}^{n}r$ $=\frac{1}{4}n^2(n+1)^2-\frac{3}{2}n(n+1)$ $=\frac{1}{4}n(n+1)\left(n(n+1)-6\right)$ $=\frac{1}{4}n(n+1)\left(n^2+n-6\right)=\frac{1}{4}n(n+1)(n+3)(n-2)$	M1 M1 A2 M1 A1 **[6]**	Separate into separate sums. (may be implied) Substitution of standard result in terms of n. For two correct terms (indivisible) Attempt to factorise with $n(n+1)$. Correctly factorised to give fully factorised form

7	When $n = 1$, $6\left(3^n - 1\right) = 12$, so true for $n = 1$ Assume true for $n = k$ $12 + 36 + 108 + ... + (4 \times 3^k) = 6\left(3^k - 1\right)$ $\Rightarrow 12 + 36 + 108 + ... + (4 \times 3^{k+1})$ $= 6\left(3^k - 1\right) + (4 \times 3^{k+1})$ $= 6\left[\left(3^k - 1\right) + \frac{2}{3} \times 3^{k+1}\right]$ $= 6\left[3^k - 1 + 2 \times 3^k\right]$ $= 6\left(3^{k+1} - 1\right)$ But this is the given result with $k + 1$ replacing k. Therefore if it is true for $n = k$, it is true for $n = k + 1$. Since it is true for $n = 1$, it is true for $n = 1, 2, 3...$ and so true for all positive integers.	B1 E1 M1 M1 A1 E1 E1 **[7]**	 Assume true for k Add correct next term to both sides Attempt to factorise with a factor 6 c.a.o. with correct simplification Dependent on A1 and first E1 Dependent on B1 and second E1
			Section A Total: 36

Section B

8(i)	$(\sqrt{3},\ 0),(-\sqrt{3},\ 0)\ \left(0,\ \frac{3}{8}\right)$	B1 B1 **[2]**	Intercepts with x axis (both) Intercept with y axis SC1 if seen on graph or if $x = \underline{+}\sqrt{3}$, $y = 3/8$ seen without $y = 0$, $x = 0$ specified.
8(ii)	$x = 4,\ x = -2,\ y = 1$	B3 **[3]**	Minus 1 for each error. Accept equations written on the graph.
8(iii)		B1 B1B1 B1 **[4]**	Correct approaches to vertical asymptotes, LH and RH branches LH and RH branches approaching horizontal asymptote On LH branch 0<y<1 as x→-∞.
8(iv)	$-2 < x \le -\sqrt{3}$ and $4 > x \ge \sqrt{3}$	B1 B2 **[3]**	LH interval and RH interval correct (Award this mark even if errors in inclusive/exclusive inequality signs) All inequality signs correct, minus 1 each error

9(i)	$\alpha + \beta = 3$ $\alpha\alpha^* = (1+\text{j})(1-\text{j}) = 2$ $\frac{\alpha+\beta}{\alpha} = \frac{3}{1+\text{j}} = \frac{3(1-\text{j})}{(1+\text{j})(1-\text{j})} = \frac{3}{2} - \frac{3}{2}\text{j}$	B1 M1 A1 M1 A1 **[5]**	 Attempt to multiply $(1+\text{j})(1-\text{j})$ Multiply top and bottom by $1-\text{j}$
9(ii)	$(z-(1+\text{j}))(z-(1-\text{j}))$ $= z^2 - 2z + 2$	M1 A1 **[2]**	Or alternative valid methods (Condone no "=0" here)
9(iii)	$1-\text{j}$ and $2+\text{j}$ Either $(z-(2-\text{j}))(z-(2+\text{j}))$ $= z^2 - 4z + 5$ $(z^2-2z+2)(z^2-4z+5)$ $= z^4 - 6z^3 + 15z^2 - 18z + 10$ So equation is $z^4 - 6z^3 + 15z^2 - 18z + 10 = 0$	B1 M1 M1 A2 **[5]**	For both For attempt to obtain an equation using the product of linear factors involving complex conjugates Using the correct four factors All correct, -1 each error (including omission of "=0") to min of 0
	Or alternative solution Use of $\sum\alpha = 6$, $\sum\alpha\beta = 15$, $\sum\alpha\beta\gamma = 18$ and $\alpha\beta\gamma\delta = 10$ to obtain the above equation.	M1 A3 **[5]**	Use of relationships between roots and coefficients. All correct, -1 each error, to min of 0

10(i)	$\alpha = 3\times -5 + 4\times 11 + -1\times 29 = 0$ $\beta = -2\times -7 + 7\times(5+k) + -3\times 7 = 28+7k$	B1 M1 A1 **[3]**	 Attempt at row 3 x column 3
10(ii)	$\mathbf{AB} = \begin{pmatrix} 42 & 0 & 0 \\ 0 & 42 & 0 \\ 0 & 0 & 42 \end{pmatrix}$	B2 **[2]**	Minus 1 each error to min of 0
10(iii)	$\mathbf{A}^{-1} = \frac{1}{42}\begin{pmatrix} 11 & -5 & -7 \\ 1 & 11 & 7 \\ -5 & 29 & 7 \end{pmatrix}$	M1 B1 A1 **[3]**	Use of **B** $\frac{1}{42}$ Correct inverse, allow decimals to 3 sf
10(iv)	$\frac{1}{42}\begin{pmatrix} 11 & -5 & -7 \\ 1 & 11 & 7 \\ -5 & 29 & 7 \end{pmatrix}\begin{pmatrix} 1 \\ -9 \\ 26 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $= \frac{1}{42}\begin{pmatrix} -126 \\ 84 \\ -84 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ -2 \end{pmatrix}$ $x=-3,\ y=2,\ z=-2$	M1 A3 **[4]**	Attempt to pre-multiply by $\mathbf{A}^{-1}$ SC B2 for Gaussian elimination with 3 correct solutions, -1 each error to min of 0 Minus 1 each error
			Section B Total: 36
			Total: 72