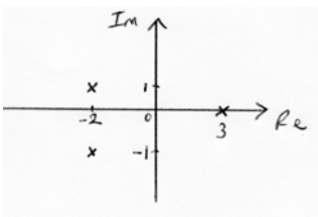
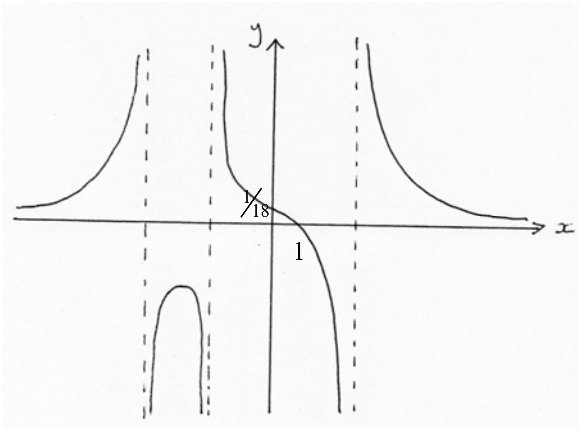
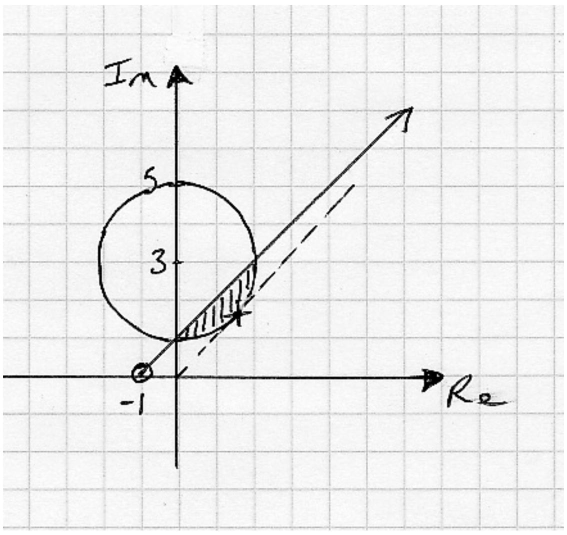
4755 (FP1) Further Concepts for Advanced Mathematics

Qu	Answer	Mark	Comment
Section A			
1(i)	$\mathbf{BA}=\begin{pmatrix}3&1\\-2&4\end{pmatrix}\begin{pmatrix}2&-1\\0&3\end{pmatrix}=\begin{pmatrix}6&0\\-4&14\end{pmatrix}$	M1 A1 **[2]**	Attempt to multiply c.a.o.
1(ii)	$\det\mathbf{BA}=(6\times14)-(-4\times0)=84$ $3\times84=252$ square units	M1 A1 A1(ft) **[3]**	Attempt to calculate any determinant c.a.o. Correct area
2(i)	$\alpha^2=(-3+4\text{j})(-3+4\text{j})=(-7-24\text{j})$	M1 A1 **[2]**	Attempt to multiply with use of $\text{j}^2=-1$ c.a.o.
2(ii)	$\lvert\alpha\rvert=5$ $\arg\alpha=\pi-\arctan\frac{4}{3}$ = 2.21 (2d.p.) (or 126.87°) $\alpha=5(\cos2.21+\text{j}\sin2.21)$	B1 B1 B1(ft) **[3]**	Accept 2.2 or 127° Accept degrees and $(r,\ \theta)$ form s.c. lose 1 mark only if α^2 used throughout (ii)
3(i)	$3^3+3^2-7\times3-15=0$ $z^3+z^2-7z-15=(z-3)(z^2+4z+5)$ $z=\dfrac{-4\pm\sqrt{16-20}}{2}=-2\pm\text{j}$ So other roots are $-2+\text{j}$ and $-2-\text{j}$	B1 M1 A1 M1 A1 **[5]**	Showing 3 satisfies the equation (may be implied) Valid attempt to factorise Correct quadratic factor Use of quadratic formula, or other valid method One mark for both c.a.o.
3(ii)		B2 **[2]**	Minus 1 for each error ft provided conjugate imaginary roots

4	$\sum_{r=1}^{n}\left[(r+1)(r-2)\right]=\sum_{r=1}^{n}r^2-\sum_{r=1}^{n}r-2n$	M1	Attempt to split sum up
	$=\frac{1}{6}n(n+1)(2n+1)-\frac{1}{2}n(n+1)-2n$	A2	Minus one each error
	$=\frac{1}{6}n\left[(n+1)(2n+1)-3(n+1)-12\right]$	M1	Attempt to factorise
	$=\frac{1}{6}n\left(2n^2+3n+1-3n-3-12\right)$	M1	Collecting terms
	$=\frac{1}{6}n\left(2n^2-14\right)$		
	$=\frac{1}{3}n\left(n^2-7\right)$	A1 **[6]**	All correct
5(i)	$p=-3,\ r=7$	B2 **[2]**	One mark for each s.c. B1 if *b* and *d* used instead of *p* and *r*
5(ii)	$q=\alpha\beta+\alpha\gamma+\beta\gamma$	B1	
	$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)$ $=(\alpha+\beta+\gamma)^2-2q$ $\Rightarrow 13=3^2-2q$	M1	Attempt to find *q* using $\alpha^2+\beta^2+\gamma^2$ and $\alpha+\beta+\gamma$, but not $\alpha\beta\gamma$
	$\Rightarrow q=-2$	A1 **[3]**	c.a.o.
6(i)	$a_2=7\times 7-3=46$	M1	Use of inductive definition
	$a_3=7\times 46-3=319$	A1 **[2]**	c.a.o.
6(ii)	When *n* = 1, $\frac{13\times 7^0+1}{2}=7$, so true for *n* = 1	B1	Correct use of part (i) (may be implied)
	Assume true for *n* = *k* $a_k=\frac{13\times 7^{k-1}+1}{2}$	E1	Assuming true for *k*
	$\Rightarrow a_{k+1}=7\times\frac{13\times 7^{k-1}+1}{2}-3$	M1	Attempt to use $a_{k+1}=7a_k-3$
	$=\frac{13\times 7^k+7}{2}-3$ $=\frac{13\times 7^k+7-6}{2}$ $=\frac{13\times 7^k+1}{2}$	A1	Correct simplification
	But this is the given result with *k* + 1 replacing *k*. Therefore if it is true for *k* it is true for *k* + 1. Since it is true for *k* = 1, it is true for *k* = 1, 2, 3 and so true for all positive integers.	E1 E1 **[6]**	Dependent on A1 and previous E1 Dependent on B1 and previous E1
			Section A Total: 36

Section B

7(i)	$(1,\ 0)$ and $\left(0,\ \frac{1}{18}\right)$	B1 B1 **[2]**	
7(ii)	x = 2, $x=-3$, $x=\frac{-3}{2}$, $y=0$	B4 **[4]**	Minus 1 for each error
7(iii)		B1 B1 **[2]**	Correct approaches to vertical asymptotes Through clearly marked $(1,\ 0)$ and $\left(0,\ \frac{1}{18}\right)$
7(iv)	$x<-3$, $x>2$ $\frac{-3}{2}<x\le 1$	B1 B2 **[3]**	 B1 for $\frac{-3}{2}<x<1$, or $\frac{-3}{2}\le x\le 1$
8(i) **8(ii)**		B3 B3 **[6]** B2(ft) **[2]**	Circle, B1; radius 2, B1; centre 3j, B1 Half line, B1; from -1, B1; $\frac{\pi}{4}$ to *x*-axis, B1 Correct region between their circle and half line indicated s.c. B1 for interior of circle
	Sketch should clearly show the radius and centre of the circle and the starting point and angle of the half-line.	M1 A1(ft)	Tangent from origin to circle Correct point placed by eye where tangent from origin meets circle
8(iii)	$\arg z=\dfrac{\pi}{2}-\arcsin\frac{2}{3}$ = 0.84 (2d.p.)	M1 A1 **[4]**	Attempt to use right angled triangle c.a.o. Accept 48.20° (2d.p.)

9(i)	$(-3,\ -3)$	B1 **[1]**	
9(ii)	$(x,\ x)$	B1 B1 **[2]**	
9(iii)	$\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$	B3 **[3]**	Minus 1 each error to min of 0
9(iv)	Rotation through $\frac{\pi}{2}$ anticlockwise about the origin	B1 B1 **[2]**	Rotation and angle (accept 90°) Centre and sense
9(v)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}$	M1 A1 **[2]**	Attempt to multiply using their **T** in correct order c.a.o.
9(vi)	$\begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ x \end{pmatrix}$ So (-*x*, *x*) Line *y* = -*x*	M1 A1(ft) A1 **[3]**	May be implied c.a.o. from correct matrix