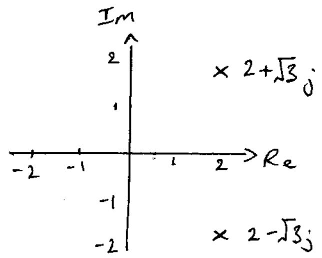
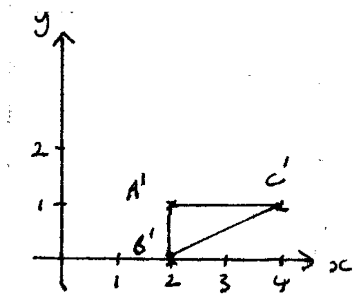
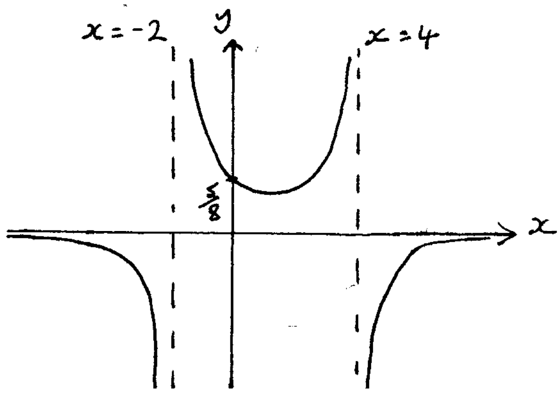
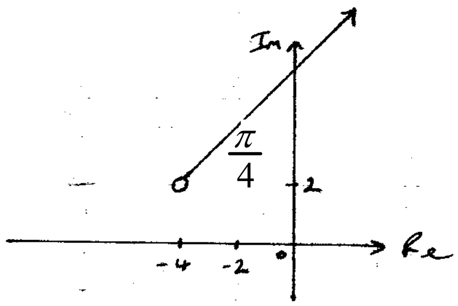
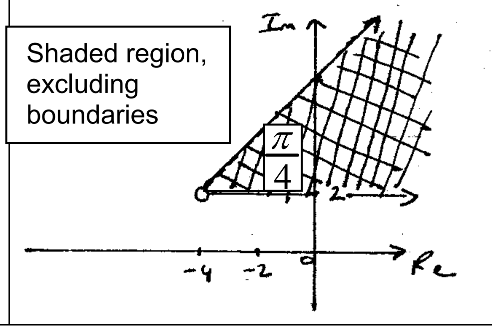
Mark Scheme 4755
January 2007

Qu	Answer	Mark	Comment
Section A			
1	The statement is false. The 'if' part is true, but the 'only if' is false since $x = -2$ also satisfies the equation.	M1 A1 **[2]**	'False', with attempted justification (may be implied) Correct justification
2(i)	$\frac{4 \pm \sqrt{16-28}}{2}$ $= \frac{4 \pm \sqrt{12}}{2}\text{j} = 2 \pm \sqrt{3}\text{j}$	M1 A1 A1 A1 **[4]**	Attempt to use quadratic formula or other valid method Correct Unsimplified form. Fully simplified form.
2(ii)		B1(ft) B1(ft) **[2]**	One correct, with correct labelling Other in correct relative position s.c. give B1 if both points consistent with (i) but no/incorrect labelling
3(i)	 $\begin{pmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 \\ 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 4 \\ 1 & 0 & 1 \end{pmatrix}$	B3 B1 ***ELSE*** M1 A1 **[4]**	Points correctly plotted Points correctly labelled Applying matrix to points Minus 1 each error
3(ii)	Stretch, factor 2 in *x*-direction, stretch factor half in *y*-direction.	B1 B1 B1 **[3]**	1 mark for stretch (withhold if rotation, reflection or translation mentioned incorrectly) 1 mark for each factor and direction

4	$\sum_{r=1}^{n} r(r^2+1) = \sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r$ $= \frac{1}{4}n^2(n+1)^2 + \frac{1}{2}n(n+1)$ $= \frac{1}{4}n(n+1)[n(n+1)+2]$ $= \frac{1}{4}n(n+1)(n^2+n+2)$	M1 M1 A1 M1 A1 A1 **[6]**	Separate into two sums (may be implied by later working) Use of standard results Correct Attempt to factorise (dependent on previous M marks) Factor of *n*(*n* + 1) c.a.o.
5	$\omega = 2x+1 \Rightarrow x = \frac{\omega-1}{2}$ $2\left(\frac{\omega-1}{2}\right)^3 - 3\left(\frac{\omega-1}{2}\right)^2 + \left(\frac{\omega-1}{2}\right) - 4 = 0$ $\Rightarrow \frac{1}{4}(\omega^3 - 3\omega^2 + 3\omega - 1) - \frac{3}{4}(\omega^2 - 2\omega + 1)$ $+\frac{1}{2}(\omega - 1) - 4 = 0$ $\Rightarrow \omega^3 - 6\omega^2 + 11\omega - 22 = 0$	M1 A1 M1 A1(ft) A1(ft) A2 **[7]**	Attempt to give substitution Correct Substitute into cubic Cubic term Quadratic term Minus 1 each error (missing '= 0' is an error)
5	OR $\alpha + \beta + \gamma = \frac{3}{2}$ $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{1}{2}$ $\alpha\beta\gamma = 2$ Let new roots be *k*, *l*, *m* then $k + l + m = 2(\alpha + \beta + \gamma) + 3 = 6 = \frac{-B}{A}$ $kl + km + lm = 4(\alpha\beta + \alpha\gamma + \beta\gamma) +$ $4(\alpha + \beta + \gamma) + 3 = 11 = \frac{C}{A}$ $klm = 8\alpha\beta\gamma + 4(\alpha\beta + \beta\gamma + \beta\gamma)$ $+2(\alpha + \beta + \gamma) + 1 = 22 = \frac{-D}{A}$ $\Rightarrow \omega^3 - 6\omega^2 + 11\omega - 22 = 0$	M1 A1 M1 M1 M1 A2 **[7]**	Attempt to find sums and products of roots All correct Use of sum of roots Use of sum of product of roots in pairs Use of product of roots Minus 1 each error (missing '= 0' is an error)

6	$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ n = 1, LHS = RHS = 1 Assume true for $n = k$ Next term is $(k+1)^2$ Add to both sides $\text{RHS} = \frac{1}{6}k(k+1)(2k+1)+(k+1)^2$ $= \frac{1}{6}(k+1)[k(2k+1)+6(k+1)]$ $= \frac{1}{6}(k+1)[2k^2+7k+6]$ $= \frac{1}{6}(k+1)(k+2)(2k+3)$ $= \frac{1}{6}(k+1)((k+1)+1)(2(k+1)+1)$ But this is the given result with k + 1 replacing k. Therefore if it is true for k it is true for k + 1. Since it is true for k = 1, it is true for k = 1, 2, 3 and so true for all positive integers.	B1 M1 B1 M1 M1 A1 E1 E1 **[8]**	 Assuming true for k. (k + 1)th term. Add to both sides Attempt to factorise Correct brackets required – also allow correct unfactorised form Showing this is the expression with $n = k + 1$ Only if both previous E marks awarded
			Section A Total: 36

Section B			
7(i)	$y=\frac{5}{8}$	B1 **[1]**	
7(ii)	$x=-2,\ x=4,\ y=0$	B1, B1 B1 **[3]**	
7(iii)	3 correct branches Correct, labelled asymptotes *y*-intercept labelled 	B1 B1 B1 **[3]**	Ft from (ii) Ft from (i)
7(iv)	$\frac{5}{(x+2)(4-x)}=1$ $\Rightarrow 5=(x+2)(4-x)$ $\Rightarrow 5=-x^2+2x+8$ $\Rightarrow x^2-2x-3=0$ $\Rightarrow (x-3)(x+1)=0$ $\Rightarrow x=3$ or $x=-1$ From graph: $x<-2$ or $-1<x<3$ or $x>4$	M1 A1 B1 B1 B1 **[5]**	Or evidence of other valid method Both values Ft from previous A1 Penalise inclusive inequalities only once

8(i)	$\frac{1}{m}=\frac{1}{-4+2\text{j}}=\frac{-4-2\text{j}}{(-4+2\text{j})(-4-2\text{j})}$	M1	Attempt to multiply top and bottom by conjugate
	$=\frac{-1}{5}-\frac{1}{10}\text{j}$	A1 **[2]**	Or equivalent
8(ii)	$\lvert m\rvert=\sqrt{(-4)^2+2^2}=\sqrt{20}$	B1	
	$\arg m=\pi-\arctan\left(\frac{1}{2}\right)=2.68$	M1 A1	Attempt to calculate angle Accept any correct expression for angle, including 153.4 degrees, –206 degrees and –3.61 (must be at least 3s.f.)
	So $m=\sqrt{20}\left(\cos 2.68+\text{j}\sin 2.68\right)$	A1(ft) **[4]**	Also accept $(r,\ \theta)$ form
8(iii) (*A*)		B1 B1 **[2]**	Correct initial point Half-line at correct angle
8(iii) (*B*)		B1(ft) B1(ft) B1(ft) **[3]**	Correct horizontal half-line from starting point Correct region indicated Boundaries excluded (accept dotted lines)

Qu	Answer	Mark	Comment
Section B (continued)			
9(i)	$\mathbf{M}^{-1} = \frac{1}{3}\begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix}$ $\mathbf{N}^{-1} = \frac{1}{7}\begin{pmatrix} 4 & 3 \\ -1 & 1 \end{pmatrix}$	M1 A1 A1 **[3]**	Dividing by determinant One for each inverse c.a.o.
9(ii)	$\mathbf{MN} = \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -3 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 5 & -1 \\ 1 & 4 \end{pmatrix}$	M1 A1	Must multiply in correct order
	$(\mathbf{MN})^{-1} = \frac{1}{21}\begin{pmatrix} 4 & 1 \\ -1 & 5 \end{pmatrix}$	A1	Ft from **MN**
	$\mathbf{N}^{-1}\mathbf{M}^{-1} = \frac{1}{7}\begin{pmatrix} 4 & 3 \\ -1 & 1 \end{pmatrix} \times \frac{1}{3}\begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix}$ $= \frac{1}{21}\begin{pmatrix} 4 & 1 \\ -1 & 5 \end{pmatrix}$ $= (\mathbf{MN})^{-1}$	M1 A1 A1 **[6]**	Multiplication in correct order Ft from (i) Statement of equivalence to $(\mathbf{MN})^{-1}$
9(iii)	$\Rightarrow (\mathbf{PQ})^{-1}\mathbf{PQQ}^{-1} = \mathbf{IQ}^{-1}$ $\Rightarrow (\mathbf{PQ})^{-1}\mathbf{PI} = \mathbf{Q}^{-1}$ $\Rightarrow (\mathbf{PQ})^{-1}\mathbf{P} = \mathbf{Q}^{-1}$ $\Rightarrow (\mathbf{PQ})^{-1}\mathbf{PP}^{-1} = \mathbf{Q}^{-1}\mathbf{P}^{-1}$ $\Rightarrow (\mathbf{PQ})^{-1}\mathbf{I} = \mathbf{Q}^{-1}\mathbf{P}^{-1}$ $\Rightarrow (\mathbf{PQ})^{-1} = \mathbf{Q}^{-1}\mathbf{P}^{-1}$	M1 M1 M1 A1 **[4]**	$\mathbf{QQ}^{-1} = \mathbf{I}$ Correctly eliminate **I** from LHS Post-multiply both sides by $\mathbf{P}^{-1}$ at an appropriate point Correct and complete argument
			Section B Total: 36
			Total: 72