OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4755

Further Concepts For Advanced Mathematics (FP1)

Wednesday **18 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages.

Section A (36 marks)

1 You are given that $\mathbf{A} = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 1 & -1 \\ 0 & 2 \\ 0 & 1 \end{pmatrix}$.

(i) Calculate, where possible, $2\mathbf{B}$, $\mathbf{A} + \mathbf{C}$, $\mathbf{CA}$ and $\mathbf{A} - \mathbf{B}$. [5]

(ii) Show that matrix multiplication is not commutative. [2]

2 **(i)** Given that $z = a + b\mathrm{j}$, express $|z|$ and z^* in terms of a and b. [2]

(ii) Prove that $zz^* - |z|^2 = 0$. [3]

3 Find $\sum_{r=1}^{n} (r+1)(r-1)$, expressing your answer in a fully factorised form. [6]

4 The matrix equation $\begin{pmatrix} 6 & -2 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$ represents two simultaneous linear equations in x and y.

(i) Write down the two equations. [2]

(ii) Evaluate the determinant of $\begin{pmatrix} 6 & -2 \\ -3 & 1 \end{pmatrix}$.

What does this value tell you about the solution of the equations in part **(i)**? [3]

5 The cubic equation $x^3 + 3x^2 - 7x + 1 = 0$ has roots α, β and γ.

(i) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. [2]

(ii) Find the cubic equation with roots 2α, 2β and 2γ, simplifying your answer as far as possible. [4]

6 Prove by induction that $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$. [7]

Section B (36 marks)

7 A curve has equation $y = \dfrac{3 + x^2}{4 - x^2}$.

(i) Show that y can never be zero. [1]

(ii) Write down the equations of the two vertical asymptotes and the one horizontal asymptote. [3]

(iii) Describe the behaviour of the curve for large positive and large negative values of x, justifying your description. [2]

(iv) Sketch the curve. [3]

(v) Solve the inequality $\dfrac{3 + x^2}{4 - x^2} \leqslant -2$. [4]

8 You are given that the complex number $\alpha = 1 + \mathrm{j}$ satisfies the equation $z^3 + 3z^2 + pz + q = 0$, where p and q are real constants.

(i) Find α^2 and α^3 in the form $a + b\mathrm{j}$. Hence show that $p = -8$ and $q = 10$. [6]

(ii) Find the other two roots of the equation. [3]

(iii) Represent the three roots on an Argand diagram. [2]

9 A transformation T acts on all points in the plane. The image of a general point P is denoted by P′.

P′ always lies on the line $y = 2x$ and has the same y-coordinate as P. This is illustrated in Fig. 9.

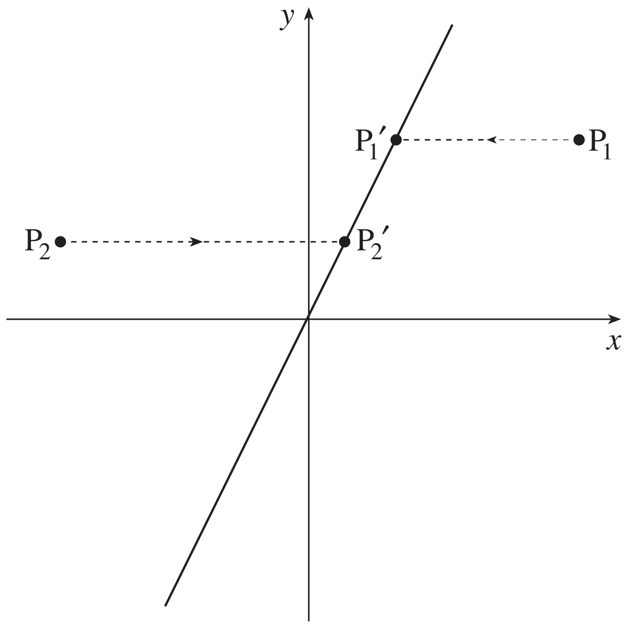

Fig. 9

(i) Write down the image of the point $(10, 50)$ under transformation T. [1]

(ii) P has coordinates (x, y). State the coordinates of P′. [2]

(iii) All points on a particular line l are mapped onto the point $(3, 6)$. Write down the equation of the line l. [1]

(iv) In part **(iii)**, the whole of the line l was mapped by T onto a single point. There are an infinite number of lines which have this property under T. Describe these lines. [1]

(v) For a different set of lines, the transformation T has the same effect as translation parallel to the x-axis. Describe this set of lines. [1]

(vi) Find the 2×2 matrix which represents the transformation. [3]

(vii) Show that this matrix is singular. Relate this result to the transformation. [3]