Mark Scheme 4755
January 2006

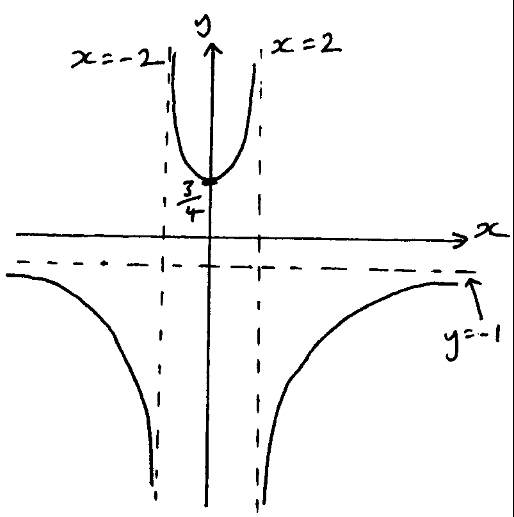
Section A

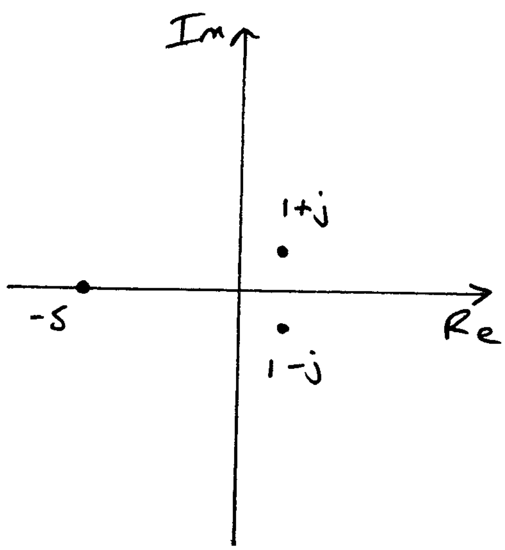
1(i)	$2\mathbf{B}=\begin{pmatrix}4 & -6\\ 2 & 8\end{pmatrix}$, **A + C** is impossible, $\mathbf{CA}=\begin{pmatrix}3 & 1\\ 2 & 4\\ 1 & 2\end{pmatrix}$, $\mathbf{A}-\mathbf{B}=\begin{pmatrix}2 & 6\\ 0 & -2\end{pmatrix}$	B1 B1 M1, A1 B1 **[5]**	**CA** 3×2 matrix M1
1(ii)	$\mathbf{AB}=\begin{pmatrix}4 & 3\\ 1 & 2\end{pmatrix}\begin{pmatrix}2 & -3\\ 1 & 4\end{pmatrix}=\begin{pmatrix}11 & 0\\ 4 & 5\end{pmatrix}$ $\mathbf{BA}=\begin{pmatrix}2 & -3\\ 1 & 4\end{pmatrix}\begin{pmatrix}4 & 3\\ 1 & 2\end{pmatrix}=\begin{pmatrix}5 & 0\\ 8 & 11\end{pmatrix}$ $\mathbf{AB}\neq\mathbf{BA}$	M1 E1 **[2]**	Or **AC** impossible, or student's own correct example. Allow M1 even if slip in multiplication Meaning of commutative
2(i)	$\lvert z\rvert=\sqrt{\left(a^2+b^2\right)}$, $z^*=a-b\mathrm{j}$	B1 B1 **[2]**	
2(ii)	$zz^*=(a+bj)(a-bj)=a^2+b^2$ $\Rightarrow zz^*-\lvert z\rvert^2=a^2+b^2-\left(a^2+b^2\right)=0$	M1 M1 A1 **[3]**	Serious attempt to find zz^*, consistent with their z^* ft their $\lvert z\rvert$ in subtraction All correct
3	$\sum_{r=1}^{n}(r+1)(r-1)=\sum_{r=1}^{n}\left(r^2-1\right)$ $=\frac{1}{6}n(n+1)(2n+1)-n$ $=\frac{1}{6}n\left[(n+1)(2n+1)-6\right]$ $=\frac{1}{6}n\left(2n^2+3n-5\right)$ $=\frac{1}{6}n(2n+5)(n-1)$	M1 M1, A1, A1 M1 A1 **[6]**	Condone missing brackets Attempt to use standard results Each part correct Attempt to collect terms with common denominator c.a.o.

4(i)	$6x-2y=a$ $-3x+y=b$	B1 B1 **[2]**	
4(ii)	Determinant = 0 The equations have no solutions or infinitely many solutions.	B1 E1 E1 **[3]**	No solution or infinitely many solutions Give E2 for 'no unique solution' s.c. 1: Determinant = 12, allow 'unique solution' B0 E1 E0 s.c. 2: Determinant = $\frac{1}{0}$ give maximum of B0 E1
5(i)	$\alpha+\beta+\gamma=-3,\ \alpha\beta+\beta\gamma+\gamma\alpha=-7,\ \alpha\beta\gamma=-1$	B2 **[2]**	Minus 1 each error to minimum of 0
5(ii)	Coefficients *A*, *B* and *C* $2\alpha+2\beta+2\gamma=2\times-3=-6=\frac{-B}{A}$ $2\alpha\times2\beta+2\beta\times2\gamma+2\gamma\times2\alpha=4\times-7=-28=\frac{C}{A}$ $2\alpha\times2\beta\times2\gamma=8\times-1=-8=\frac{-D}{A}$ $\Rightarrow x^3+6x^2-28x+8=0$	M1 A3 **[4]**	Attempt to use sums and products of roots ft their coefficients, minus one each error (including '= 0' missing), to minimum of 0
	OR $\omega=2x\Rightarrow x=\frac{\omega}{2}$ $\left(\frac{\omega}{2}\right)^3+3\left(\frac{\omega}{2}\right)^2-7\left(\frac{\omega}{2}\right)+1=0$ $\Rightarrow\frac{\omega^3}{8}+\frac{3\omega^2}{4}-\frac{7\omega}{2}+1=0$ $\Rightarrow\omega^3+6\omega^2-28\omega+8=0$	M1 A1 A1 A1 **[4]**	Attempt at substitution Correct substitution Substitute into cubic (ft) c.a.o.

6	$\sum_{r=1}^{n}\frac{1}{r(r+1)}=\frac{n}{n+1}$ n = 1, LHS = RHS = $\frac{1}{2}$ Assume true for $n = k$ Next term is $\frac{1}{(k+1)(k+2)}$ Add to both sides $\text{RHS}=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}$ $=\frac{k(k+2)+1}{(k+1)(k+2)}$ $=\frac{k^2+2k+1}{(k+1)(k+2)}$ $=\frac{(k+1)^2}{(k+1)(k+2)}$ $=\frac{k+1}{k+2}$ But this is the given result with k + 1 replacing k. Therefore if it is true for k it is true for k + 1. Since it is true for k = 1, it is true for k = 1, 2, 3	B1 E1 B1 M1 A1 E1 E1 **[7]**	Assuming true for k (must be explicit) $(k + 1)^{th}$ term seen c.a.o. Add to $\frac{k}{k+1}$ (ft) c.a.o. with correct working True for k, therefore true for k + 1 (dependent on $\frac{k+1}{k+2}$ seen) Complee argument

Section A Total: 36

	Section B		
7(i)	$3+x^2 \neq 0$ for any real x.	E1 **[1]**	
7(ii)	$y = -1,\ x = 2,\ x = -2$	B1, B1 B1 **[3]**	
7(iii)	Large positive x, $y \to -1^-$ (e.g. consider $x = 100$) Large negative x, $y \to -1^-$ (e.g. consider $x = -100$)	M1 B1 **[2]**	Evidence of method required From below on each side c.a.o.
7(iv)	Curve 3 branches correct Asymptotes labelled Intercept labelled  	B1 B1 B1 **[3]**	Consistent with (i) and their (ii), (iii) Consistent with (i) and their (ii), (iii) Labels may be on axes Lose 1 mark if graph not symmetrical May be written in script
7(v)	$\frac{3+x^2}{4-x^2} = -2 \Rightarrow 3+x^2 = -8+2x^2$ $\Rightarrow 11 = x^2$ $\Rightarrow x = (\pm)\sqrt{11}$ From graph, $-\sqrt{11} \le x < -2$ or $2 < x \le \sqrt{11}$	M1 A1 B1 A1 **[4]**	Reasonable attempt to solve Accept $\sqrt{11}$ $x < -2$ and $2 < x$ seen c.a.o.

8(i)	$\alpha^2 = (1+\text{j})^2 = 2\text{j}$ $\alpha^3 = (1+\text{j})2\text{j} = -2+2\text{j}$	M1, A1 A1	
	$z^3 + 3z^2 + pz + q = 0$ $\Rightarrow 2\text{j} - 2 + 3\times 2\text{j} + p(1+\text{j}) + q = 0$ $\Rightarrow (8+p)j + p + q - 2 = 0$ $p = -8$ and $p + q - 2 = 0 \Rightarrow q = 10$	M1 M1 A1 **[6]**	Substitute their α^2 and α^3 into cubic Equate real and imaginary parts to 0 Results obtained correctly
8(ii)	$1-\text{j}$ must also be a root. The roots must sum to –3, so the other root is z = -5	B1 M1 A1 **[3]**	Any valid method c.a.o.
8(iii)		B2 **[2]**	Argand diagram with all three roots clearly shown; minus 1 for each error to minimum of 0 ft their real root

Section B (continued)			
9(i)	$(25,50)$	B1 **[1]**	
9(ii)	$\left(\frac{1}{2}y, y\right)$	B1, B1 **[2]**	
9(iii)	$y = 6$	B1 **[1]**	
9(iv)	All such lines are parallel to the *x*-axis.	B1 **[1]**	Or equivalent
9(v)	All such lines are parallel to $y = 2x$.	B1 **[1]**	Or equivalent
9(vi)	$\begin{pmatrix} 0 & \frac{1}{2} \\ 0 & 1 \end{pmatrix}$	B3 **[3]**	Minus 1 each error s.c. Allow 1 for reasonable attempt but incorrect working
9(vii)	$\det\begin{pmatrix} 0 & \frac{1}{2} \\ 0 & 1 \end{pmatrix} = 0\times 1 - 0\times\frac{1}{2} = 0$ Transformation many to one.	M1 E2 **[3]**	Attempt to show determinant = 0 or other valid argument May be awarded without previous M1 Allow E1 for 'transformation has no inverse' or other partial explanation
			Section B Total: 36
			Total: 72