OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4755

Further Concepts For Advanced Mathematics (FP1)

Friday **21 JANUARY 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages.

[Turn over

Section A (36 marks)

1 You are given the matrix $\mathbf{M} = \begin{pmatrix} 2 & 3 \\ -2 & 1 \end{pmatrix}$.

Find the inverse of **M**.

The transformation associated with **M** is applied to a figure of area 2 square units. What is the area of the transformed figure? [3]

2 **(i)** Show that $\dfrac{1}{r+1} - \dfrac{1}{r+2} = \dfrac{1}{(r+1)(r+2)}$. [2]

(ii) Hence use the method of differences to find the sum of the series

$$\sum_{r=1}^{n} \frac{1}{(r+1)(r+2)}.$$ [4]

3 **(i)** Solve the equation $\dfrac{1}{x+2} = 3x+4$. [3]

(ii) Solve the inequality $\dfrac{1}{x+2} \leqslant 3x+4$. [4]

4 Find $\sum_{r=1}^{n} r^2(r+2)$, giving your answer in a factorised form. [6]

5 The roots of the cubic equation $x^3 + 2x^2 + x - 3 = 0$ are α, β and γ.

Find the cubic equation whose roots are $\alpha + 1$, $\beta + 1$ and $\gamma + 1$, simplifying your answer as far as you can. [6]

6 Prove by induction that $\sum_{r=1}^{n} r2^{r-1} = 1 + (n-1)2^n$. [8]

Section B (36 marks)

7 A curve has equation $y = \dfrac{(2x-3)(x+1)}{(x+4)(x-2)}$.

(i) Write down the values of x for which $y = 0$. [1]

(ii) Write down the equations of the three asymptotes. [3]

(iii) Determine whether the curve approaches the horizontal asymptote from above or from below for

(A) large positive values of x,

(B) large negative values of x. [3]

(iv) Sketch the curve. [3]

(v) Solve the inequality $\dfrac{(2x-3)(x+1)}{(x+4)(x-2)} \leqslant 2$. [4]

8 Two complex numbers are given by $\alpha = 2 - \mathrm{j}$ and $\beta = -1 + 2\mathrm{j}$.

(i) Find $\alpha + \beta$, $\alpha\beta$ and $\dfrac{\alpha}{\beta}$ in the form $a + b\mathrm{j}$, showing your working. [6]

(ii) Find the modulus of α, leaving your answer in surd form. Find also the argument of α. [2]

(iii) Sketch the locus $|z - \alpha| = 2$ on an Argand diagram. [2]

(iv) On a separate Argand diagram, sketch the locus $\arg(z - \beta) = \frac{1}{4}\pi$. [2]

9 You are given the matrix $\mathbf{M} = \begin{pmatrix} 0.8 & 0.6 \\ 0.6 & -0.8 \end{pmatrix}$.

(i) Calculate $\mathbf{M}^2$. [1]

You are now given that the matrix **M** represents a reflection in a line through the origin.

(ii) Explain how your answer to part **(i)** relates to this information. [1]

(iii) By investigating the invariant points of the reflection, find the equation of the mirror line. [3]

(iv) Describe fully the transformation represented by the matrix $\mathbf{P} = \begin{pmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{pmatrix}$. [2]

(v) A composite transformation is formed by the transformation represented by **P** followed by the transformation represented by **M**. Find the single matrix that represents this composite transformation. [2]

(vi) The composite transformation described in part **(v)** is equivalent to a single reflection. What is the equation of the mirror line of this reflection? [1]