4755 Mark scheme (FP1) Jan 05

Qu	Answer	Mark	Comment
Section A			
1	Det **M** = 8 $\mathbf{M}^{-1} = \frac{1}{8}\begin{pmatrix} 1 & -3 \\ 2 & 2 \end{pmatrix}$ Area = 16 square units	B1 B1 B1 **[3]**	
2(i)	$\frac{1}{r+1} - \frac{1}{r+2} \equiv \frac{(r+2)-(r+1)}{(r+1)(r+2)} \equiv \frac{1}{(r+1)(r+2)}$	M1 A1 **[2]**	
2(ii)	$\sum_{r=1}^{n} \frac{1}{(r+1)(r+2)} = \sum_{r=1}^{n}\left[\frac{1}{(r+1)} - \frac{1}{(r+2)}\right]$	M1	
	$= \left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\ldots\ldots$	M1	First two terms in full.
	$+\left(\frac{1}{n}-\frac{1}{n+1}\right)+\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$	M1	Last two terms in full.
	$= \frac{1}{2} - \frac{1}{n+2}$	A1 **[4]**	Give B4 for correct without working.
3(i)	$3x^2 + 10x + 7 = 0$	M1	Quadratic from equation
	$x = \frac{-7}{3}$ or $x = -1$	A1, A1 **[3]**	1 mark for each solution.
3(ii)	Graph sketch or valid algebraic method.	G2 or M2	Sketch with $y = \frac{1}{x+2}$ and $y = 3x+4$ or algebra to derive $0 \le \frac{(3x+7)(x+1)}{(x+2)}$ with consideration of behaviour near critical values of x.
	$-2 > x \ge -\frac{7}{3}$	A1	
	or $x \ge -1$	A1 **[4]**	Lose one mark if incorrect inequality symbol used in $-2 > x \ge -\frac{7}{3}$ <u>Alternative</u> For sensible attempt but vertical asymptote at x = -2 not considered, give M1 only.

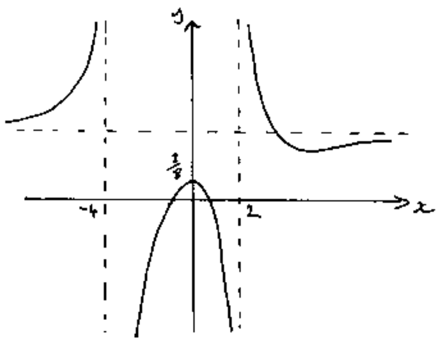
4755 Mark scheme (FP1) Jan 05

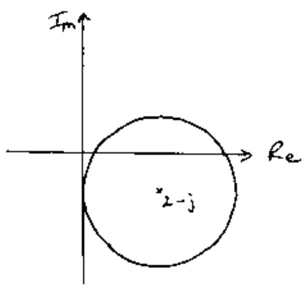
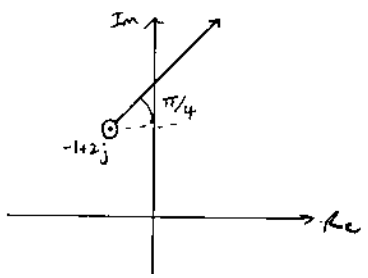
Qu	Answer	Mark	Comment
4	$\sum_{r=1}^{n} r^2(r+2) = \sum_{r=1}^{n} r^3 + 2\sum_{r=1}^{n} r^2$	M1,A1	Separate sums
	$= \frac{1}{4}n^2(n+1)^2 + \frac{1}{3}n(n+1)(2n+1)$	M1,A1	Use of formulae. Follow through from incorrect expansion in line 1.
	$= \frac{1}{12}n(n+1)[3n(n+1)+4(2n+1)]$	M1	Factorising
	$= \frac{1}{12}n(n+1)(3n^2+11n+4)$ i.s.w.	A1 **[6]**	
5	$w = x+1 \Rightarrow x = w-1$	B1	Substitution. For substitution $w = x-1$ give B0 but then follow through.
	$\Rightarrow (w-1)^3 + 2(w-1)^2 + (w-1) - 3 = 0$	M1	Substitute into cubic
	$\Rightarrow w^3 - 3w^2 + 3w - 1 + 2w^2 - 4w + 2 + w - 1 - 3 = 0$	A1, A1,A1	Expansion
	$\Rightarrow w^3 - w^2 - 3 = 0$	A1 **[6]**	Simplifying

5	Alternative		
	$\alpha + \beta + \gamma = -2$ $\alpha\beta + \beta\gamma + \alpha\gamma = 1$ $\alpha\beta\gamma = 3$	M1 A1	Attempt to calculate these All correct
	Coefficients: $w^2 = -1$ $w = 0$ constant $= -3$	B1 B1 B1	
	Correct final cubic expression $w^3 - w^2 - 3 = 0$	B1 **[6]**	

Qu	Answer	Mark	Comment
6	For $k = 1$, $1 \times 2^{1-1}$ and $1 + (1-1)2^1 = 1$, so true for $k = 1$	B1	
	Assume true for $n = k$	E1	Explicit statement: 'assume true for $n = k$' Ignore irrelevant work
	Next term is $(k+1)2^{k+1-1} = (k+1)2^k$	M1 A1	Attempt to find (k +1)th term Correct
	Add to both sides RHS = $1 + (k-1)2^k + (k+1)2^k$	M1	Add to both sides
		A1	Correct simplification of RHS

	$=1+(k-1)2^k+(k+1)2^k$ $=1+2^k(k-1+k+1)$ $=1+2^k\times 2k$ $=1+2^{k+1}k$ $=1+((k+1)-1)2^{k+1}$ But this is the given result with $(k+1)$ replacing *k*. Therefore if it is true for *k* it is true for $(k+1)$. Since it is true for $k=1$ it is true for *k* = 1, 2, 3, … .	E1 E1 **[8]**	Statement 'if true for *k*, true for *k* + 1', or equivalent (only give if simplification valid) Relating to *k* = 1, accept 'by induction'
			Section A Total: 36

Section B			
7(i)	$x = \frac{3}{2}$ and -1	B1 **[1]**	Both.
7(ii)	$x = 2,\ x = -4$ and $y = 2$	B1, B1,B1 **[3]**	
7(iii)	Large positive *x*, $y \to 2^-$ (e.g. consider $x = 100$) Large negative *x*, $y \to 2^+$ (e.g. consider $x = -100$)	B1 B1 B1 **[3]**	Evidence of method needed for this mark.
7(iv)	Curve 3 branches Asymptotes marked Correctly located and no extra intercepts  	B1 B1 B1 **[3]**	Consistent with their (iii)
7(v)	$y = 2 \Rightarrow 2 = \frac{(2x-3)(x+1)}{(x+4)(x-2)}$ $\Rightarrow 2x^2 + 4x - 16 = 2x^2 - x - 3$ $\Rightarrow x = \frac{13}{5}$ From sketch, $y \le 2$ for $x \ge \frac{13}{5}$ or $2 > x > -4$	M1 A1 A1, B1 **[4]**	Some attempt at rearrangement May be given retrospectively B1 for $2 > x > -4$

8(i)	$a + b = 1 + \mathrm{j}$	B1	
	$ab = (2-\mathrm{j})(-1+2\mathrm{j})$ $= -2 + 4\mathrm{j} + \mathrm{j} - 2\mathrm{j}^2$ $= 5\mathrm{j}$	M1 A1	.
	$\frac{a}{b} = \frac{(2-\mathrm{j})(-1-2\mathrm{j})}{(-1+2\mathrm{j})(-1-2\mathrm{j})} = \frac{-2-4\mathrm{j}+\mathrm{j}+2\mathrm{j}^2}{5}$ $= \frac{-4}{5} - \frac{3}{5}\mathrm{j}$ or $\frac{-4-3\mathrm{j}}{5}$	M1, A1 A1 **[6]**	Use of conjugate of denominator
8(ii)	$r = \lvert a \rvert = \sqrt{5}$ $q = \arg a = -0.464$	B1 B1 **[2]**	Accept degree equivalent (-26.6°)
8(iii)	Circle, centre $2 - \mathrm{j}$, radius 2  	B1 B1 **[2]**	Argand diagram with circle. 1 mark for centre, one mark for radius.
8(iv)	Half line from $-1 + 2\mathrm{j}$, making an angle of $\frac{\pi}{4}$ to the positive real axis.  	B1 B1 **[2]**	Argand diagram with half line. One mark for angle.

Qu	Answer	Mark	Comment
Section B (continued)			
9(i)	$\mathbf{M}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I}$	B1 **[1]**	.
9(ii)	$\mathbf{M}^2$ gives the identity because a reflection, followed by a second reflection in the same mirror line will get you back where you started OR reflection matrices are self-inverse.	E1 **[1]**	
9(iii)	$\begin{pmatrix} 0.8 & 0.6 \\ 0.8 & -0.6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ $\Rightarrow 0.8x + 0.6y = x$ and $0.6x - 0.8y = y$ Both of these lead to $y = \frac{1}{3}x$ as the equation of the mirror line.	M1 A1 A1 **[3]**	Give both marks for either equation or for a correct geometrical argument
9(iv)	Rotation, centre origin, 36.9° anticlockwise.	B1, B1 **[2]**	One for rotation and centre, one for angle and sense. Accept 323.1° clockwise or radian equivalents (0.644 or 5.64).
9(v)	$\mathbf{MP} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	M1, A1 **[2]**	
9(vi)	$y = 0$	B1 **[1]**	
			Section B Total: 36
			Total: 72