OCR MEI Maths FP1

Mark Scheme Pack

2005-2014

4755 Mark scheme (FP1) Jan 05

Qu	Answer	Mark	Comment
Section A			
1	Det **M** = 8 $\mathbf{M}^{-1} = \frac{1}{8}\begin{pmatrix} 1 & -3 \\ 2 & 2 \end{pmatrix}$ Area = 16 square units	B1 B1 B1 **[3]**	
2(i)	$\frac{1}{r+1} - \frac{1}{r+2} \equiv \frac{(r+2)-(r+1)}{(r+1)(r+2)} \equiv \frac{1}{(r+1)(r+2)}$	M1 A1 **[2]**	
2(ii)	$\sum_{r=1}^{n} \frac{1}{(r+1)(r+2)} = \sum_{r=1}^{n}\left[\frac{1}{(r+1)} - \frac{1}{(r+2)}\right]$	M1	
	$= \left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\ldots\ldots$	M1	First two terms in full.
	$+\left(\frac{1}{n}-\frac{1}{n+1}\right)+\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$	M1	Last two terms in full.
	$= \frac{1}{2} - \frac{1}{n+2}$	A1 **[4]**	Give B4 for correct without working.
3(i)	$3x^2 + 10x + 7 = 0$	M1	Quadratic from equation
	$x = \frac{-7}{3}$ or $x = -1$	A1, A1 **[3]**	1 mark for each solution.
3(ii)	Graph sketch or valid algebraic method.	G2 or M2	Sketch with $y = \frac{1}{x+2}$ and $y = 3x+4$ or algebra to derive $0 \le \frac{(3x+7)(x+1)}{(x+2)}$ with consideration of behaviour near critical values of x.
	$-2 > x \ge -\frac{7}{3}$	A1	
	or $x \ge -1$	A1 **[4]**	Lose one mark if incorrect inequality symbol used in $-2 > x \ge -\frac{7}{3}$ <u>Alternative</u> For sensible attempt but vertical asymptote at x = -2 not considered, give M1 only.

Qu	Answer	Mark	Comment
4	$\sum_{r=1}^{n} r^2(r+2) = \sum_{r=1}^{n} r^3 + 2\sum_{r=1}^{n} r^2$	M1,A1	Separate sums
	$= \frac{1}{4}n^2(n+1)^2 + \frac{1}{3}n(n+1)(2n+1)$	M1,A1	Use of formulae. Follow through from incorrect expansion in line 1.
	$= \frac{1}{12}n(n+1)[3n(n+1)+4(2n+1)]$	M1	Factorising
	$= \frac{1}{12}n(n+1)(3n^2+11n+4)$ i.s.w.	A1 **[6]**	
5	$w = x+1 \Rightarrow x = w-1$	B1	Substitution. For substitution $w = x$-1 give B0 but then follow through.
	$\Rightarrow (w-1)^3 + 2(w-1)^2 + (w-1) - 3 = 0$	M1	Substitute into cubic
	$\Rightarrow w^3 - 3w^2 + 3w - 1 + 2w^2 - 4w + 2 + w - 1 - 3 = 0$	A1, A1,A1	Expansion
	$\Rightarrow w^3 - w^2 - 3 = 0$	A1 **[6]**	Simplifying

5	Alternative $\alpha + \beta + \gamma = -2$ $\alpha\beta + \beta\gamma + \alpha\gamma = 1$ $\alpha\beta\gamma = 3$ Coefficients: $w^2 = -1$ $w = 0$ constant $= -3$ Correct final cubic expression $w^3 - w^2 - 3 = 0$	M1 A1 B1 B1 B1 B1 **[6]**	Attempt to calculate these All correct

Qu	Answer	Mark	Comment
6	For $k = 1, 1 \times 2^{1-1}$ and $1 + (1-1)2^1 = 1$, so true for $k = 1$ Assume true for $n = k$ Next term is $(k+1)2^{k+1-1} = (k+1)2^k$ Add to both sides RHS $= 1 + (k-1)2^k + (k+1)2^k$	B1 E1 M1 A1 M1 A1	 Explicit statement: 'assume true for $n = k$' Ignore irrelevant work Attempt to find $(k+1)$th term Correct Add to both sides Correct simplification of RHS

	$= 1 + (k-1)2^k + (k+1)2^k$ $= 1 + 2^k(k - 1 + k + 1)$ $= 1 + 2^k \times 2k$ $= 1 + 2^{k+1}k$ $= 1 + ((k+1) - 1)2^{k+1}$ But this is the given result with $(k+1)$ replacing k. Therefore if it is true for k it is true for $(k+1)$. Since it is true for $k = 1$ it is true for $k = 1, 2, 3, \ldots$.	E1 E1 **[8]**	Statement 'if true for k, true for $k + 1$', or equivalent (only give if simplification valid) Relating to $k = 1$, accept 'by induction'
			Section A Total: 36

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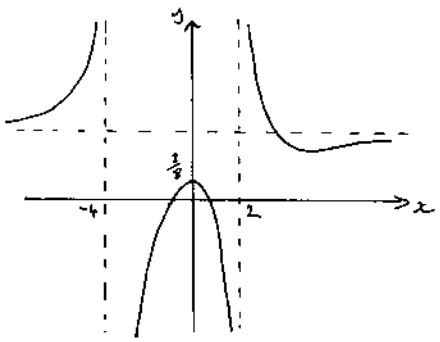
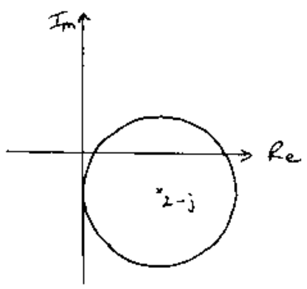
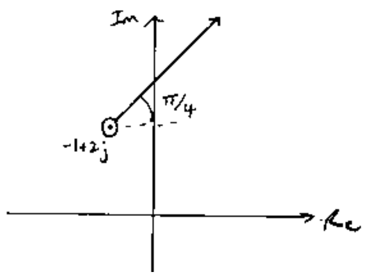
Section B			
7(i)	$x = \frac{3}{2}$ and -1	B1 **[1]**	Both.
7(ii)	$x = 2$, $x = -4$ and $y = 2$	B1, B1,B1 **[3]**	
7(iii)	Large positive x, $y \to 2^-$ (e.g. consider $x = 100$) Large negative x, $y \to 2^+$ (e.g. consider $x = -100$)	B1 B1 B1 **[3]**	Evidence of method needed for this mark.
7(iv)	Curve 3 branches Asymptotes marked Correctly located and no extra intercepts 	B1 B1 B1 **[3]**	Consistent with their (iii)
7(v)	$y = 2 \Rightarrow 2 = \frac{(2x-3)(x+1)}{(x+4)(x-2)}$ $\Rightarrow 2x^2 + 4x - 16 = 2x^2 - x - 3$ $\Rightarrow x = \frac{13}{5}$ From sketch, $y \le 2$ for $x \ge \frac{13}{5}$ or $2 > x > -4$	M1 A1 A1, B1 **[4]**	Some attempt at rearrangement May be given retrospectively B1 for $2 > x > -4$

8(i)	$\alpha+\beta=1+\mathrm{j}$	B1	
	$\alpha\beta=(2-\mathrm{j})(-1+2\mathrm{j})$ $=-2+4\mathrm{j}+\mathrm{j}-2\mathrm{j}^2$ $=5\mathrm{j}$	M1 A1	.
	$\dfrac{\alpha}{\beta}=\dfrac{(2-\mathrm{j})(-1-2\mathrm{j})}{(-1+2\mathrm{j})(-1-2\mathrm{j})}=\dfrac{-2-4\mathrm{j}+\mathrm{j}+2\mathrm{j}^2}{5}$ $=\dfrac{-4}{5}-\dfrac{3}{5}\mathrm{j}$ or $\dfrac{-4-3\mathrm{j}}{5}$	M1, A1 A1 **[6]**	Use of conjugate of denominator
8(ii)	$r=\lvert\alpha\rvert=\sqrt{5}$ $\theta=\arg\alpha=-0.464$	B1 B1 **[2]**	Accept degree equivalent (-26.6°)
8(iii)	Circle, centre $2-\mathrm{j}$, radius 2 	B1 B1 **[2]**	Argand diagram with circle. 1 mark for centre, one mark for radius.
8(iv)	Half line from $-1+2\mathrm{j}$, making an angle of $\dfrac{\pi}{4}$ to the positive real axis. 	B1 B1 **[2]**	Argand diagram with half line. One mark for angle.

Qu	Answer	Mark	Comment
Section B (continued)			
9(i)	$\mathbf{M}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I}$	B1 **[1]**	.
9(ii)	$\mathbf{M}^2$ gives the identity because a reflection, followed by a second reflection in the same mirror line will get you back where you started OR reflection matrices are self-inverse.	E1 **[1]**	
9(iii)	$\begin{pmatrix} 0.8 & 0.6 \\ 0.8 & -0.6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ $\Rightarrow 0.8x + 0.6y = x$ and $0.6x - 0.8y = y$ Both of these lead to $y = \frac{1}{3}x$ as the equation of the mirror line.	M1 A1 A1 **[3]**	Give both marks for either equation or for a correct geometrical argument
9(iv)	Rotation, centre origin, 36.9° anticlockwise.	B1, B1 **[2]**	One for rotation and centre, one for angle and sense. Accept 323.1° clockwise or radian equivalents (0.644 or 5.64).
9(v)	$\mathbf{MP} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	M1, A1 **[2]**	
9(vi)	$y = 0$	B1 **[1]**	
			Section B Total: 36
			Total: 72

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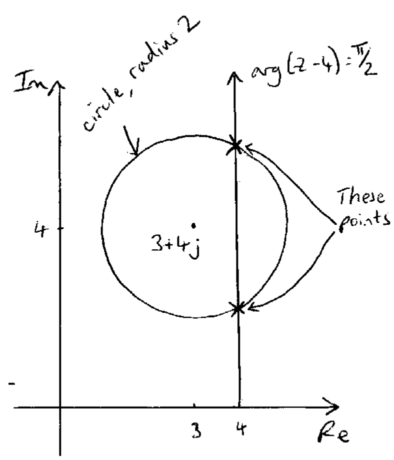
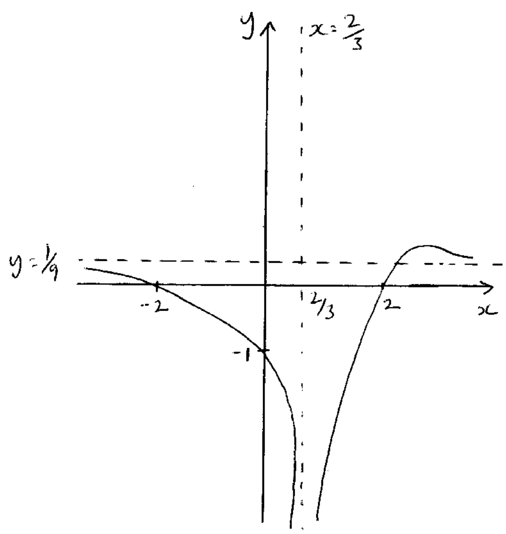
Section A			
1(i)	$\mathbf{A}^{-1} = \frac{1}{5}\begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix}$	M1 A1	Dividing by determinant
1(ii)	$\frac{1}{5}\begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 5 \\ -4 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 22 \\ -21 \end{pmatrix}$ $\Rightarrow x = \frac{22}{5},\ y = \frac{-21}{5}$	M1 A1(ft), A1(ft) **[5]**	Pre-multiplying by their inverse Follow through use of their inverse No marks for solving without using inverse matrix
2	$4 - \mathrm{j},\ 4 + \mathrm{j}$ $\sqrt{17}\left(\cos 0.245 + \mathrm{j}\sin 0.245\right)$ $\sqrt{17}\left(\cos 0.245 - \mathrm{j}\sin 0.245\right)$	M1 A1 **[2]** M1 F1, F1 **[3]**	Use of quadratic formula Both roots correct Attempt to find modulus and argument One mark for each root Accept $(r,\ \theta)$ form Allow any correct arguments in radians or degrees, including negatives: 6.04, 14.0°, 346°. Accuracy at least 2s.f. S.C. F1 for consistent use of their incorrect modulus or argument (not both, F0)
3	$\begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow x = 3x - y,\ y = 2x$ $\Rightarrow y = 2x$	M1 A1 A1 **[3]**	M1 for $\begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ (allow if implied) $\begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} k \\ mk \end{pmatrix} = \begin{pmatrix} K \\ mK \end{pmatrix}$ can lead to full marks if correctly used. Lose second A1 if answer includes two lines
4(i) 4(ii) 4(iii)	$\alpha + \beta = 2,\ \alpha\beta = 4$ $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4 - 8 = -4$ Sum of roots = $2\alpha + 2\beta = 2(\alpha + \beta) = 4$	B1 M1A1 (ft) M1	Both Accept method involving calculation of roots Or substitution method, or method

	Product of roots = $2\alpha \times 2\beta = 4\alpha\beta = 16$ $x^2 - 4x + 16 = 0$	A1(ft) **[5]**	involving calculation of roots The = 0, or equivalent, is necessary for final A1

5(i)	Sketch of Argand diagram with: Point $3+4\mathrm{j}$. Circle, radius 2.	B1 B1 **[2]**	Circle must not touch either axis. B1 max if no labelling or scales. Award even if centre incorrect.
5(ii)	Half-line: Starting from (4, 0) Vertically upwards	B1 B1 **[2]**	
5(iii)	Points where line crosses circle clearly indicated. 	B1 **[1]**	Identifying 2 points where their line cuts the circle

Qu	Answer	Mark	Comment
Section A (continued)			
6	For $k=1$, $1^3=1$ and $\frac{1}{4}1^2(1+1)^2=1$, so true for $k=1$ Assume true for $n=k$ Next term is $(k+1)^3$ Add to both sides RHS = $\frac{1}{4}k^2(k+1)^2+(k+1)^3$ $=\frac{1}{4}(k+1)^2\left[k^2+4(k+1)\right]$ $=\frac{1}{4}(k+1)^2(k+2)^2$ $=\frac{1}{4}(k+1)^2((k+1)+1)^2$ But this is the given result with $(k+1)$ replacing *k*. Therefore if it is true for *k* it is true for $(k+1)$. Since it is true for $k=1$ it is true for k = 1, 2, 3, … .	B1 B1 B1 M1 M1 A1 E1 **[7]**	Assuming true for *k*, $(k+1)^{th}$ term - for alternative statement, give this mark if whole argument logically correct Add to both sides Factor of $(k+1)^2$ Allow alternative correct methods For fully convincing algebra leading to true for $k \Rightarrow$ true for k + 1 Accept 'Therefore true by induction' only if previous A1 awarded S.C. Give E1 if convincing explanation of induction following acknowledgement of earlier error
7	$3\sum r^2-3\sum r$ $=3\times\frac{1}{6}n(n+1)(2n+1)-3\times\frac{1}{2}n(n+1)$ $=\frac{1}{2}n(n+1)[(2n+1)-3]$ $=\frac{1}{2}n(n+1)(2n-2)$ $=n(n+1)(n-1)$	M1,A1 M1,A1 M1 A1 c.a.o. **[6]**	Separate sums Use of formulae Attempt to factorise, only if earlier M marks awarded Must be fully factorised
			Section A Total: 36

8(i)	$x=\frac{2}{3}$ and $y=\frac{1}{9}$	B1, B1 **[2]**	-1 if any others given. Accept min of 2s.f. accuracy
8(ii)	Large positive x, $y\to\frac{1}{9}^{+}$ (e.g. consider $x=100$) Large negative x, $y\to\frac{1}{9}^{-}$ (e.g. consider $x=-100$)	M1 A1 E1 **[3]**	Approaches horizontal asymptote, not inconsistent with their (i) Correct approaches Reasonable attempt to justify approaches
8(iii)	Curve $x=\frac{2}{3}$ shown with correct approaches $y=\frac{1}{9}$ shown with correct approaches (from below on left, above on right). (2, 0), (-2, 0) and (0, -1) shown 	B1(ft) B1(ft) B1(ft) B1 B1 **[5]**	 1 for each branch, consistent with horizontal asymptote in (i) or (ii) Both x intercepts y intercept (give these marks if coordinates shown in workings, even if not shown on graph)
8(iv)	$-1=\frac{x^2-4}{(3x-2)^2}\Rightarrow -9x^2+12x-4=x^2-4$ $\Rightarrow 10x^2-12x=0$ $\Rightarrow 2x(5x-6)=0$ $\Rightarrow x=0$ or $x=\frac{6}{5}$ From sketch, $y\geq -1$ for $x\leq 0$ and $x\geq\frac{6}{5}$	M1 A1 B1 F1 **[4]**	Reasonable attempt at solving inequality Both values – give for seeing 0 and $\frac{6}{5}$, even if inequalities are wrong For $x\leq 0$ Lose only one mark if any strict inequalities given

9(i)	2 - j 2j	B1 B1 **[2]**	
9(iii)	$(x-2-\text{j})(x-2+\text{j})(x+2\text{j})(x-2\text{j})$ $=(x^2-4x+5)(x^2+4)$ $=x^4-4x^3+9x^2-16x+20$ So A = -4, B = 9, C = -16 and D = 20	M1, M1 A1,A1 A4 **[8]**	M1 for each attempted factor pair A1 for each quadratic - follow through sign errors Minus 1 each error – follow through sign errors only
OR	$-\text{A}=\sum\alpha=4\Rightarrow\text{A}=-4$ $\text{B}=\sum\alpha\beta=9\Rightarrow\text{B}=9$ $-\text{C}=\sum\alpha\beta\gamma=16\Rightarrow\text{C}=-16$ $\text{D}=\sum\alpha\beta\gamma\delta=20\Rightarrow\text{D}=20$	M1, A1 M1, A1 M1, A1 M1, A1 **[8]**	M1s for reasonable attempt to find sums S.C. If one sign incorrect, give total of A3 for A, B, C, D values If more than one sign incorrect, give total of A2 for A, B, C, D values
OR	Attempt to substitute two correct roots into $x^4+Ax^3+Bx^2+Cx+D=0$ Produce 2 correct equations in two unknowns A = -4, B = 9, C = -16, D = 20	M1 M1 A2 A4	One for each root One for each equation One mark for each correct. S.C. If one sign incorrect, give total of A3 for A, B, C, D values If more than one sign incorrect, give total of A2 for A, B, C, D values

10(i)	$\sum_{r=1}^{n}\frac{2}{r(r+1)(r+2)}=\sum_{r=1}^{n}\left[\frac{1}{r}-\frac{2}{(r+1)}+\frac{1}{(r+2)}\right]$	M1	Give if implied by later working
	$=\left(\frac{1}{1}-\frac{2}{2}+\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{2}{3}+\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{2}{4}+\frac{1}{5}\right)+$ $\ldots\ldots+\left(\frac{1}{n-1}-\frac{2}{n}+\frac{1}{n+1}\right)+\left(\frac{1}{n}-\frac{2}{n+1}+\frac{1}{n+2}\right)$	M1 A2	Writing out terms in full, at least three terms All terms correct. A1 for at least two correct
	$=\frac{1}{1}-\frac{2}{2}+\frac{1}{2}+\frac{1}{n+1}-\frac{2}{n+1}+\frac{1}{n+2}$ $=\frac{1}{2}-\frac{1}{n+1}+\frac{1}{n+2}$ $=\frac{1}{2}-\frac{1}{(n+1)(n+2)}$	M1 A3 M1 [9]	Attempt at cancelling terms Correct terms retained (minus 1 each error) Attempt at single fraction leading to given answer.
10(ii)	$\frac{1}{1\times2\times3}+\frac{1}{2\times3\times4}+\frac{1}{3\times4\times5}+\ldots$ $=\frac{1}{2}\sum_{r=1}^{n}\frac{2}{r(r+1)(r+2)}=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{(n+1)(n+2)}\right)$	M1 M1	M1 relating to previous sum, M1 for recognising that $\frac{1}{(n+1)(n+2)}\to 0$ as $n\to\infty$ (could be implied)
	$\Rightarrow\frac{1}{1\times2\times3}+\frac{1}{2\times3\times4}+\frac{1}{3\times4\times5}+\ldots=\frac{1}{4}$	A1 [3]	

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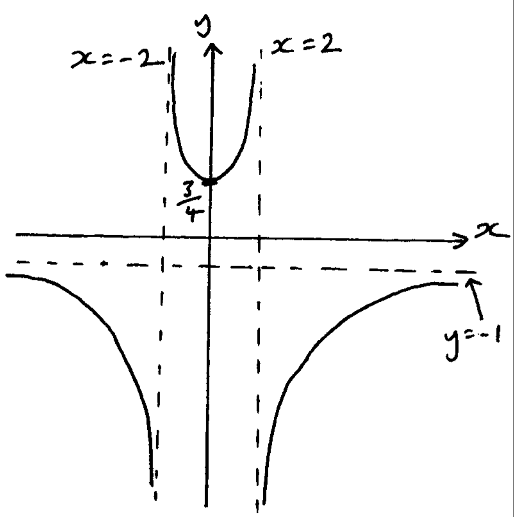
Section A

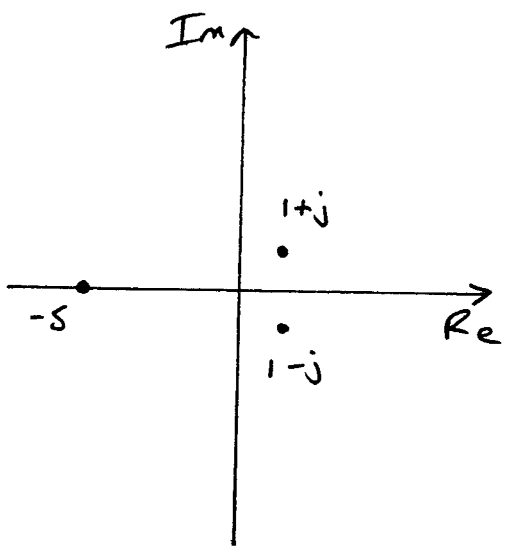
1(i)	$2\mathbf{B}=\begin{pmatrix}4&-6\\2&8\end{pmatrix}$, **A + C** is impossible, $\mathbf{CA}=\begin{pmatrix}3&1\\2&4\\1&2\end{pmatrix}$, $\mathbf{A}-\mathbf{B}=\begin{pmatrix}2&6\\0&-2\end{pmatrix}$	B1 B1 M1, A1 B1 **[5]**	**CA** 3×2 matrix M1
1(ii)	$\mathbf{AB}=\begin{pmatrix}4&3\\1&2\end{pmatrix}\begin{pmatrix}2&-3\\1&4\end{pmatrix}=\begin{pmatrix}11&0\\4&5\end{pmatrix}$ $\mathbf{BA}=\begin{pmatrix}2&-3\\1&4\end{pmatrix}\begin{pmatrix}4&3\\1&2\end{pmatrix}=\begin{pmatrix}5&0\\8&11\end{pmatrix}$ $\mathbf{AB}\neq\mathbf{BA}$	M1 E1 **[2]**	Or **AC** impossible, or student's own correct example. Allow M1 even if slip in multiplication Meaning of commutative
2(i)	$\lvert z\rvert=\sqrt{(a^2+b^2)}$, $z^*=a-b\mathrm{j}$	B1 B1 **[2]**	
2(ii)	$zz^*=(a+bj)(a-bj)=a^2+b^2$ $\Rightarrow zz^*-\lvert z\rvert^2=a^2+b^2-(a^2+b^2)=0$	M1 M1 A1 **[3]**	Serious attempt to find zz^*, consistent with their z^* ft their $\lvert z\rvert$ in subtraction All correct
3	$\sum_{r=1}^{n}(r+1)(r-1)=\sum_{r=1}^{n}(r^2-1)$ $=\frac{1}{6}n(n+1)(2n+1)-n$ $=\frac{1}{6}n[(n+1)(2n+1)-6]$ $=\frac{1}{6}n(2n^2+3n-5)$ $=\frac{1}{6}n(2n+5)(n-1)$	M1 M1, A1, A1 M1 A1 **[6]**	Condone missing brackets Attempt to use standard results Each part correct Attempt to collect terms with common denominator c.a.o.

4(i)	$6x - 2y = a$ $-3x + y = b$	B1 B1 **[2]**	
4(ii)	Determinant = 0 The equations have no solutions or infinitely many solutions.	B1 E1 E1 **[3]**	No solution or infinitely many solutions Give E2 for 'no unique solution' s.c. 1: Determinant = 12, allow 'unique solution' B0 E1 E0 s.c. 2: Determinant = $\frac{1}{0}$ give maximum of B0 E1
5(i)	$\alpha + \beta + \gamma = -3$, $\alpha\beta + \beta\gamma + \gamma\alpha = -7$, $\alpha\beta\gamma = -1$	B2 **[2]**	Minus 1 each error to minimum of 0
5(ii)	Coefficients A, B and C $2\alpha + 2\beta + 2\gamma = 2 \times -3 = -6 = \frac{-B}{A}$ $2\alpha \times 2\beta + 2\beta \times 2\gamma + 2\gamma \times 2\alpha = 4 \times -7 = -28 = \frac{C}{A}$ $2\alpha \times 2\beta \times 2\gamma = 8 \times -1 = -8 = \frac{-D}{A}$ $\Rightarrow x^3 + 6x^2 - 28x + 8 = 0$ **OR** $\omega = 2x \Rightarrow x = \frac{\omega}{2}$ $\left(\frac{\omega}{2}\right)^3 + 3\left(\frac{\omega}{2}\right)^2 - 7\left(\frac{\omega}{2}\right) + 1 = 0$ $\Rightarrow \frac{\omega^3}{8} + \frac{3\omega^2}{4} - \frac{7\omega}{2} + 1 = 0$ $\Rightarrow \omega^3 + 6\omega^2 - 28\omega + 8 = 0$	M1 A3 **[4]** M1 A1 A1 A1 **[4]**	Attempt to use sums and products of roots ft their coefficients, minus one each error (including '= 0' missing), to minimum of 0 Attempt at substitution Correct substitution Substitute into cubic (ft) c.a.o.

6	$\sum_{r=1}^{n}\frac{1}{r(r+1)}=\frac{n}{n+1}$ $n = 1$, LHS = RHS = $\frac{1}{2}$ Assume true for $n = k$ Next term is $\frac{1}{(k+1)(k+2)}$ Add to both sides $\text{RHS}=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}$ $=\frac{k(k+2)+1}{(k+1)(k+2)}$ $=\frac{k^2+2k+1}{(k+1)(k+2)}$ $=\frac{(k+1)^2}{(k+1)(k+2)}$ $=\frac{k+1}{k+2}$ But this is the given result with $k + 1$ replacing k. Therefore if it is true for k it is true for $k + 1$. Since it is true for $k = 1$, it is true for $k = 1, 2, 3$	B1 E1 B1 M1 A1 E1 E1 **[7]**	Assuming true for k (must be explicit) $(k + 1)^{th}$ term seen c.a.o. Add to $\frac{k}{k+1}$ (ft) c.a.o. with correct working True for k, therefore true for $k + 1$ (dependent on $\frac{k+1}{k+2}$ seen) Complee argument

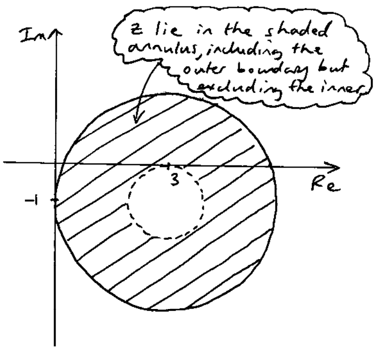
Section A Total: 36

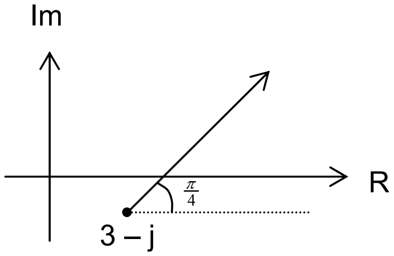
	Section B		
7(i)	$3+x^2 \neq 0$ for any real x.	E1 **[1]**	
7(ii)	$y = -1,\ x = 2,\ x = -2$	B1, B1 B1 **[3]**	
7(iii)	Large positive x, $y \to -1^-$ (e.g. consider $x = 100$) Large negative x, $y \to -1^-$ (e.g. consider $x = -100$)	M1 B1 **[2]**	Evidence of method required From below on each side c.a.o.
7(iv)	Curve 3 branches correct Asymptotes labelled Intercept labelled 	B1 B1 B1 **[3]**	Consistent with (i) and their (ii), (iii) Consistent with (i) and their (ii), (iii) Labels may be on axes Lose 1 mark if graph not symmetrical May be written in script
7(v)	$\frac{3+x^2}{4-x^2} = -2 \Rightarrow 3+x^2 = -8+2x^2$ $\Rightarrow 11 = x^2$ $\Rightarrow x = (\pm)\sqrt{11}$ From graph, $-\sqrt{11} \le x < -2$ or $2 < x \le \sqrt{11}$	M1 A1 B1 A1 **[4]**	Reasonable attempt to solve Accept $\sqrt{11}$ $x < -2$ and $2 < x$ seen c.a.o.

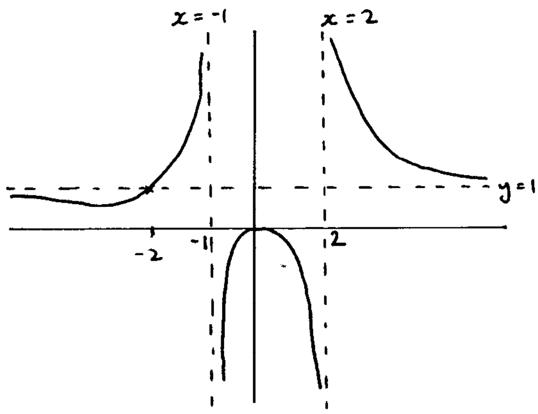
8(i)	$\alpha^2 = (1+j)^2 = 2j$ $\alpha^3 = (1+j)2j = -2+2j$ $z^3 + 3z^2 + pz + q = 0$ $\Rightarrow 2j - 2 + 3\times 2j + p(1+j) + q = 0$ $\Rightarrow (8+p)j + p + q - 2 = 0$ $p = -8$ and $p + q - 2 = 0 \Rightarrow q = 10$	M1, A1 A1 M1 M1 A1 **[6]**	Substitute their α^2 and α^3 into cubic Equate real and imaginary parts to 0 Results obtained correctly
8(ii)	$1-j$ must also be a root. The roots must sum to –3, so the other root is $z = -5$	B1 M1 A1 **[3]**	Any valid method c.a.o.
8(iii)		B2 **[2]**	Argand diagram with all three roots clearly shown; minus 1 for each error to minimum of 0 ft their real root

Section B (continued)			
9(i)	$(25, 50)$	B1 **[1]**	
9(ii)	$\left(\frac{1}{2}y, y\right)$	B1, B1 **[2]**	
9(iii)	$y = 6$	B1 **[1]**	
9(iv)	All such lines are parallel to the x-axis.	B1 **[1]**	Or equivalent
9(v)	All such lines are parallel to $y = 2x$.	B1 **[1]**	Or equivalent
9(vi)	$\begin{pmatrix} 0 & \frac{1}{2} \\ 0 & 1 \end{pmatrix}$	B3 **[3]**	Minus 1 each error s.c. Allow 1 for reasonable attempt but incorrect working
9(vii)	$\det\begin{pmatrix} 0 & \frac{1}{2} \\ 0 & 1 \end{pmatrix} = 0\times1 - 0\times\frac{1}{2} = 0$ Transformation many to one.	M1 E2 **[3]**	Attempt to show determinant = 0 or other valid argument May be awarded without previous M1 Allow E1 for 'transformation has no inverse' or other partial explanation
			Section B Total: 36
			Total: 72

Mark Scheme 4755
June 2006

Qu	Answer	Mark	Comment
Section A			
1 (i)	Reflection in the x-axis.	B1 **[1]**	
1(ii)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	B1 **[1]**	
1(iii)	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	M1 A1 c.a.o. **[2]**	Multiplication of their matrices in the correct order or B2 for correct matrix without working
2	$(x+2)(Ax^2+Bx+C)+D$ $= Ax^3+Bx^2+Cx+2Ax^2+2Bx+2C+D$ $= Ax^3+(2A+B)x^2+(2B+C)x+2C+D$ $\Rightarrow A=2,\ B=-7,\ C=15,\ D=-32$	M1 B1 B1 F1 F1 **OR** B5 **[5]**	Valid method to find all coefficients For A = 2 For D = -32 F1 for each of B and C For all correct
3(i)	$\alpha+\beta+\gamma=-4$ $\alpha\beta+\beta\gamma+\alpha\gamma=-3$ $\alpha\beta\gamma=-1$	B1 B1 B1 **[3]**	
3(ii)	$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\alpha\gamma)$ $=16+6=22$	M1 A1 E1 **[3]**	Attempt to use $(\alpha+\beta+\gamma)^2$ Correct Result shown
4 (i)	Argand diagram with solid circle, centre 3 – j, radius 3, with values of z on and within the circle clearly indicated as satisfying the inequality.	B1 B1 B1 **[3]**	Circle, radius 3, shown on diagram Circle centred on 3 - j Solution set indicated (solid circle with region inside)
4(ii)		B1 B1 **[2]**	 Hole, radius 1, shown on diagram Boundaries dealt with correctly

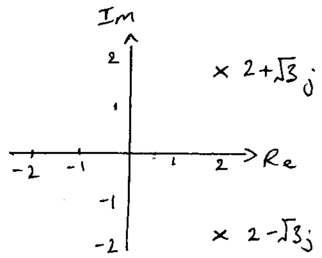
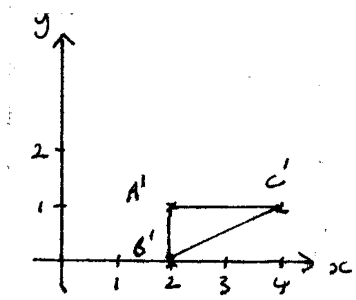
Qu	Answer	Mark	Comment
Section A (continued)			
4(iii)		B1 B1 B1 [3]	Line through their 3 – j Half line $\frac{\pi}{4}$ to real axis
5(i)	$\begin{pmatrix}-1 & 2\\-3 & 4\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}$	B1	
	$\mathbf{S}^{-1}=\frac{1}{2}\begin{pmatrix}4 & -2\\3 & -1\end{pmatrix}$	M1, A1	Attempt to divide by determinant and manipulate contents Correct
	$\frac{1}{2}\begin{pmatrix}4 & -2\\3 & -1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}$	E1 [4]	
5(ii)	$\mathbf{T}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$		
	$\Rightarrow \mathbf{T}^{-1}\mathbf{T}\begin{pmatrix}x\\y\end{pmatrix}=\mathbf{T}^{-1}\begin{pmatrix}x\\y\end{pmatrix}$	M1	Pre-multiply by $\mathbf{T}^{-1}$
	$\Rightarrow \begin{pmatrix}x\\y\end{pmatrix}=\mathbf{T}^{-1}\begin{pmatrix}x\\y\end{pmatrix}$	A1 [2]	Invariance shown
6	$3+6+12+........+3\times 2^{n-1}=3\left(2^n-1\right)$ n = 1, LHS = 3, RHS = 3	B1	
	Assume true for $n = k$ Next term is $3\times 2^{k+1-1}=3\times 2^k$ Add to both sides $\text{RHS}=3\left(2^k-1\right)+3\times 2^k$	E1 B1 M1	Assuming true for k $(k + 1)^{\text{th}}$ term. Add to both sides
	$=3\left(2^k-1+2^k\right)$ $=3\left(2\times 2^k-1\right)$ $=3\left(2^{k+1}-1\right)$	A1	Working must be valid
	But this is the given result with k + 1 replacing k. Therefore if it is true for k it is true for k + 1. Since it is true for k = 1, it is true for all positive integers n.	E1 E1 [7]	Dependent on previous A1and E1 Dependent on B1 and previous E1
			Section A Total: 36

Section B			
7(i)	$x = 2$, $x = -1$ and $y = 1$	B1 B1B1 [3]	One mark for each
7(ii) (*A*) (*B*)	Large positive *x*, $y \to 1^+$ (from above) (e.g. consider $x = 100$) Large negative *x*, $y \to 1^-$ (from below) (e.g. consider $x = -100$)	M1 B1 B1 [3]	Evidence of method needed for M1
7(iii)	Curve 3 branches Correct approaches to horizontal asymptote Asymptotes marked Through origin 	B1 B1 B1 B1 [4]	With correct approaches to vertical asymptotes Consistent with their (i) and (ii) Equations or values at axes clear
7(iv)	$x < -1$, $x > 2$	B1B1, B1, [3]	s.c. 1 for inclusive inequalities Final B1 for all correct with no other solutions

8(i)	$(2+j)^2 = 3+4j$ $(2+j)^3 = 2+11j$ Substituting into $2x^3 - 11x^2 + 22x - 15$: $2(2+11j) - 11(3+4j) + 22(2+j) - 15$ $= 4 + 22j - 33 - 44j + 44 + 22j - 15$ $= 0$ So 2 + j is a root.	B1 B1 M1 A1 A1 **[5]**	Attempt at substitution Correctly substituted Correctly cancelled (Or other valid methods)
8(ii)	2 - j	B1 **[1]**	
8(iii)	$(x-(2+j))(x-(2-j))$ $= (x-2-j)(x-2+j)$ $= x^2 - 2x + jx - 2x + 4 - 2j - jx + 2j + 1$ $= x^2 - 4x + 5$ $(x^2 - 4x + 5)(ax + b) = 2x^3 - 11x^2 + 22x - 15$ $(x^2 - 4x + 5)(2x - 3) = 2x^3 - 11x^2 + 22x - 15$ $(2x-3) = 0 \Rightarrow x = \frac{3}{2}$ **OR** Sum of roots = $\frac{11}{2}$ **or** product of roots = $\frac{15}{2}$ leading to $\alpha + 2 + j + 2 - j = \frac{11}{2}$ $\Rightarrow \alpha = \frac{3}{2}$ **or** $\alpha(2+j)(2-j) = \frac{15}{2}$ $\Rightarrow 5\alpha = \frac{15}{2} \Rightarrow \alpha = \frac{3}{2}$	M1 A1 M1 A1 **[4]** M1 A1 M1 A1 **[4]** M1 A1 M1 A1 **[4]**	Use of factor theorem Comparing coefficients or long division Correct third root (Or other valid methods)

9(i)	$r(r+1)(r+2)-(r-1)r(r+1)$ $\equiv (r^2+r)(r+2)-r^3-r$ $\equiv r^3+2r^2+r^2+2r-r^3+r$ $\equiv 3r^2+3r \equiv 3r(r+1)$	M1 E1 **[2]**	Accept '=' in place of '≡' throughout working Clearly shown
9(ii)	$\sum_{r=1}^{n} r(r+1)$ $=\frac{1}{3}\sum_{r=1}^{n}[r(r+1)(r+2)-(r-1)r(r+1)]$ $=\frac{1}{3}[(1\times2\times3-0\times1\times2)+(2\times3\times4-1\times2\times3)+$ $(3\times4\times5-2\times3\times4)+......$ $+(n(n+1)(n+2)-(n-1)n(n+1))]$ $=\frac{1}{3}n(n+1)(n+2)$ or equivalent	M1 M1 A2 M1 A1 **[6]**	Using identity from (i) Writing out terms in full At least 3 terms correct (minus 1 each error to minimum of 0) Attempt at eliminating terms (telescoping) Correct result
9(iii)	$\sum_{r=1}^{n} r(r+1)=\sum_{r=1}^{n} r^2+\sum_{r=1}^{n} r$ $=\frac{1}{6}n(n+1)(2n+1)+\frac{1}{2}n(n+1)$ $=\frac{1}{6}n(n+1)[(2n+1)+3]$ $=\frac{1}{6}n(n+1)(2n+4)$ $=\frac{1}{3}n(n+1)(n+2)$ or equivalent	B1 B1 M1 A1 E1 **[5]**	Use of standard sums (1 mark each) Attempt to combine Correctly simplified to match result from (ii)
			Section B Total: 36
			Total: 72

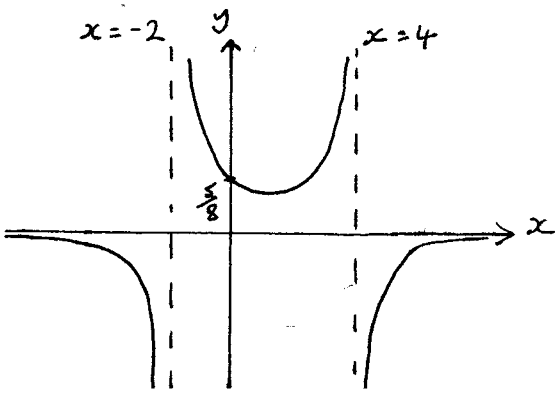
Mark Scheme 4755
January 2007

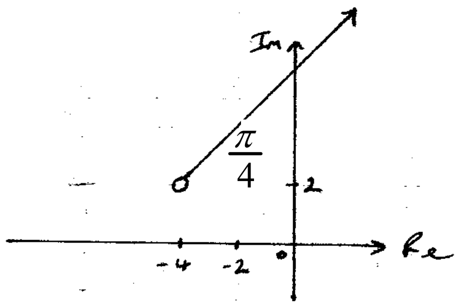
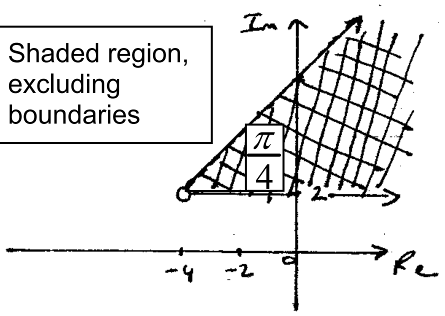
Qu	Answer	Mark	Comment
Section A			
1	The statement is false. The 'if' part is true, but the 'only if' is false since $x=-2$ also satisfies the equation.	M1 A1 **[2]**	'False', with attempted justification (may be implied) Correct justification
2(i)	$\frac{4\pm\sqrt{16-28}}{2}$ $=\frac{4\pm\sqrt{12}}{2}\text{j}=2\pm\sqrt{3}\text{j}$	M1 A1 A1 A1 **[4]**	Attempt to use quadratic formula or other valid method Correct Unsimplified form. Fully simplified form.
2(ii)		B1(ft) B1(ft) **[2]**	One correct, with correct labelling Other in correct relative position s.c. give B1 if both points consistent with (i) but no/incorrect labelling
3(i)	 $\begin{pmatrix}2&0\\0&\frac{1}{2}\end{pmatrix}\begin{pmatrix}1&1&2\\2&0&2\end{pmatrix}=\begin{pmatrix}2&2&4\\1&0&1\end{pmatrix}$	B3 B1 ***ELSE*** M1 A1 **[4]**	Points correctly plotted Points correctly labelled Applying matrix to points Minus 1 each error
3(ii)	Stretch, factor 2 in *x*-direction, stretch factor half in *y*-direction.	B1 B1 B1 **[3]**	1 mark for stretch (withhold if rotation, reflection or translation mentioned incorrectly) 1 mark for each factor and direction

4	$\sum_{r=1}^{n} r\left(r^2+1\right)=\sum_{r=1}^{n} r^3+\sum_{r=1}^{n} r$ $=\frac{1}{4}n^2\left(n+1\right)^2+\frac{1}{2}n\left(n+1\right)$ $=\frac{1}{4}n\left(n+1\right)[n\left(n+1\right)+2]$ $=\frac{1}{4}n\left(n+1\right)\left(n^2+n+2\right)$	M1 M1 A1 M1 A1 A1 **[6]**	Separate into two sums (may be implied by later working) Use of standard results Correct Attempt to factorise (dependent on previous M marks) Factor of $n(n + 1)$ c.a.o.
5	$\omega=2x+1\Rightarrow x=\frac{\omega-1}{2}$ $2\left(\frac{\omega-1}{2}\right)^3-3\left(\frac{\omega-1}{2}\right)^2+\left(\frac{\omega-1}{2}\right)-4=0$ $\Rightarrow\frac{1}{4}\left(\omega^3-3\omega^2+3\omega-1\right)-\frac{3}{4}\left(\omega^2-2\omega+1\right)$ $+\frac{1}{2}(\omega-1)-4=0$ $\Rightarrow\omega^3-6\omega^2+11\omega-22=0$	M1 A1 M1 A1(ft) A1(ft) A2 **[7]**	Attempt to give substitution Correct Substitute into cubic Cubic term Quadratic term Minus 1 each error (missing '= 0' is an error)
5	OR $\alpha+\beta+\gamma=\frac{3}{2}$ $\alpha\beta+\alpha\gamma+\beta\gamma=\frac{1}{2}$ $\alpha\beta\gamma=2$ Let new roots be *k*, *l*, *m* then $k+l+m=2\left(\alpha+\beta+\gamma\right)+3=6=\frac{-B}{A}$ $kl+km+lm=4\left(\alpha\beta+\alpha\gamma+\beta\gamma\right)+$ $4\left(\alpha+\beta+\gamma\right)+3=11=\frac{C}{A}$ $klm=8\alpha\beta\gamma+4\left(\alpha\beta+\beta\gamma+\beta\gamma\right)$ $+2\left(\alpha+\beta+\gamma\right)+1=22=\frac{-D}{A}$ $\Rightarrow\omega^3-6\omega^2+11\omega-22=0$	 M1 A1 M1 M1 M1 A2 **[7]**	 Attempt to find sums and products of roots All correct Use of sum of roots Use of sum of product of roots in pairs Use of product of roots Minus 1 each error (missing '= 0' is an error)

6	$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ n = 1, LHS = RHS = 1 Assume true for $n = k$ Next term is $(k+1)^2$ Add to both sides $\text{RHS} = \frac{1}{6}k(k+1)(2k+1)+(k+1)^2$ $= \frac{1}{6}(k+1)[k(2k+1)+6(k+1)]$ $= \frac{1}{6}(k+1)[2k^2+7k+6]$ $= \frac{1}{6}(k+1)(k+2)(2k+3)$ $= \frac{1}{6}(k+1)((k+1)+1)(2(k+1)+1)$ But this is the given result with k + 1 replacing k. Therefore if it is true for k it is true for k + 1. Since it is true for k = 1, it is true for k = 1, 2, 3 and so true for all positive integers.	B1 M1 B1 M1 M1 A1 E1 E1 **[8]**	 Assuming true for k. (k + 1)th term. Add to both sides Attempt to factorise Correct brackets required – also allow correct unfactorised form Showing this is the expression with $n = k + 1$ Only if both previous E marks awarded
			Section A Total: 36

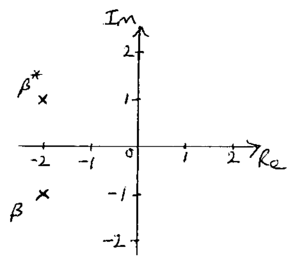
Section B

7(i)	$y = \frac{5}{8}$	B1 **[1]**	
7(ii)	$x = -2,\ x = 4,\ y = 0$	B1, B1 B1 **[3]**	
7(iii)	3 correct branches Correct, labelled asymptotes *y*-intercept labelled  	B1 B1 B1 **[3]**	Ft from (ii) Ft from (i)
7(iv)	$\frac{5}{(x+2)(4-x)} = 1$ $\Rightarrow 5 = (x+2)(4-x)$ $\Rightarrow 5 = -x^2 + 2x + 8$ $\Rightarrow x^2 - 2x - 3 = 0$ $\Rightarrow (x-3)(x+1) = 0$ $\Rightarrow x = 3 \text{ or } x = -1$ From graph: $x < -2$ or $-1 < x < 3$ or $x > 4$	M1 A1 B1 B1 B1 **[5]**	Or evidence of other valid method Both values Ft from previous A1 Penalise inclusive inequalities only once

8(i)	$\frac{1}{m}=\frac{1}{-4+2\mathrm{j}}=\frac{-4-2\mathrm{j}}{(-4+2\mathrm{j})(-4-2\mathrm{j})}$	M1	Attempt to multiply top and bottom by conjugate
	$=\frac{-1}{5}-\frac{1}{10}\mathrm{j}$	A1 **[2]**	Or equivalent
8(ii)	$\lvert m\rvert=\sqrt{(-4)^2+2^2}=\sqrt{20}$	B1	
	$\arg m=\pi-\arctan\left(\frac{1}{2}\right)=2.68$	M1 A1	Attempt to calculate angle Accept any correct expression for angle, including 153.4 degrees, –206 degrees and –3.61 (must be at least 3s.f.)
	So $m=\sqrt{20}\left(\cos 2.68+\mathrm{j}\sin 2.68\right)$	A1(ft) **[4]**	Also accept $(r,\ \theta)$ form
8(iii) (*A*)		B1 B1 **[2]**	Correct initial point Half-line at correct angle
8(iii) (*B*)	Shaded region, excluding boundaries 	B1(ft) B1(ft) B1(ft) **[3]**	Correct horizontal half-line from starting point Correct region indicated Boundaries excluded (accept dotted lines)

Qu	Answer	Mark	Comment
Section B (continued)			
9(i)	$\mathbf{M}^{-1}=\frac{1}{3}\begin{pmatrix}1 & -2\\ 0 & 3\end{pmatrix}$ $\mathbf{N}^{-1}=\frac{1}{7}\begin{pmatrix}4 & 3\\ -1 & 1\end{pmatrix}$	M1 A1 A1 **[3]**	Dividing by determinant One for each inverse c.a.o.
9(ii)	$\mathbf{MN}=\begin{pmatrix}3 & 2\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & -3\\ 1 & 4\end{pmatrix}=\begin{pmatrix}5 & -1\\ 1 & 4\end{pmatrix}$	M1 A1	Must multiply in correct order
	$(\mathbf{MN})^{-1}=\frac{1}{21}\begin{pmatrix}4 & 1\\ -1 & 5\end{pmatrix}$	A1	Ft from **MN**
	$\mathbf{N}^{-1}\mathbf{M}^{-1}=\frac{1}{7}\begin{pmatrix}4 & 3\\ -1 & 1\end{pmatrix}\times\frac{1}{3}\begin{pmatrix}1 & -2\\ 0 & 3\end{pmatrix}$ $=\frac{1}{21}\begin{pmatrix}4 & 1\\ -1 & 5\end{pmatrix}$ $=(\mathbf{MN})^{-1}$	M1 A1 A1 **[6]**	Multiplication in correct order Ft from (i) Statement of equivalence to $(\mathbf{MN})^{-1}$
9(iii)	$\Rightarrow(\mathbf{PQ})^{-1}\mathbf{PQQ}^{-1}=\mathbf{IQ}^{-1}$ $\Rightarrow(\mathbf{PQ})^{-1}\mathbf{PI}=\mathbf{Q}^{-1}$ $\Rightarrow(\mathbf{PQ})^{-1}\mathbf{P}=\mathbf{Q}^{-1}$ $\Rightarrow(\mathbf{PQ})^{-1}\mathbf{PP}^{-1}=\mathbf{Q}^{-1}\mathbf{P}^{-1}$ $\Rightarrow(\mathbf{PQ})^{-1}\mathbf{I}=\mathbf{Q}^{-1}\mathbf{P}^{-1}$ $\Rightarrow(\mathbf{PQ})^{-1}=\mathbf{Q}^{-1}\mathbf{P}^{-1}$	M1 M1 M1 A1 **[4]**	$\mathbf{QQ}^{-1}=\mathbf{I}$ Correctly eliminate **I** from LHS Post-multiply both sides by $\mathbf{P}^{-1}$ at an appropriate point Correct and complete argument
			Section B Total: 36
			Total: 72

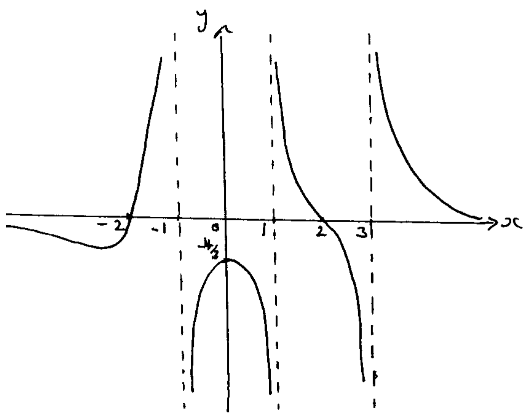
Mark Scheme 4755
June 2007

Section A			
1(i)	$\mathbf{M}^{-1}=\frac{1}{10}\begin{pmatrix}3 & 1\\ -4 & 2\end{pmatrix}$	M1 A1 **[2]**	Attempt to find determinant
1(ii)	20 square units	B1 **[1]**	$2\times$ their determinant
2	$\lvert z-(3-2\mathrm{j})\rvert=2$	B1 B1 B1 **[3]**	$z\pm(3-2\mathrm{j})$ seen radius = 2 seen Correct use of modulus
3	$x^3-4=(x-1)(Ax^2+Bx+C)+D$ $\Rightarrow x^3-4=Ax^3+(B-A)x^2+(C-B)x-C+D$ $\Rightarrow A=1, B=1, C=1, D=-3$	M1 B1 B1 B1 B1 **[5]**	Attempt at equating coefficients or long division (may be implied) For A = 1 B1 for each of B, C and D
4(i)		B1 B1 **[2]**	One for each correctly shown. s.c. B1 if not labelled correctly but position correct
4(ii)	$\alpha\beta=(1-2\mathrm{j})(-2-\mathrm{j})=-4+3\mathrm{j}$	M1 A1 **[2]**	Attempt to multiply
4(iii)	$\frac{\alpha+\beta}{\beta}=\frac{(\alpha+\beta)\beta^*}{\beta\beta^*}=\frac{\alpha\beta^*+\beta\beta^*}{\beta\beta^*}=\frac{5\mathrm{j}+5}{5}=\mathrm{j}+1$	M1 A1 A1 **[3]**	Appropriate attempt to use conjugate, or other valid method 5 in denominator or correct working consistent with their method All correct

5	**Scheme A** $w = 3x \Rightarrow x = \dfrac{w}{3}$ $\Rightarrow \left(\dfrac{w}{3}\right)^3 + 3\left(\dfrac{w}{3}\right)^2 - 7\left(\dfrac{w}{3}\right) + 1 = 0$ $\Rightarrow w^3 + 9w^2 - 63w + 27 = 0$ **OR**	B1 M1 A3 A1 **[6]**	Substitution. For substitution $x = 3w$ give B0 but then follow through for a maximum of 3 marks Substitute into cubic Correct coefficients consistent with x^3 coefficient, minus 1 each error Correct cubic equation c.a.o.
	Scheme B $\alpha + \beta + \gamma = -3$ $\alpha\beta + \alpha\gamma + \beta\gamma = -7$ $\alpha\beta\gamma = -1$ Let new roots be *k*, *l*, *m* then $k + l + m = 3(\alpha + \beta + \gamma) = -9 = \dfrac{-B}{A}$ $kl + km + lm = 9(\alpha\beta + \alpha\gamma + \beta\gamma) = -63 = \dfrac{C}{A}$ $klm = 27\alpha\beta\gamma = -27 = \dfrac{-D}{A}$ $\Rightarrow \omega^3 + 9\omega^2 - 63\omega + 27 = 0$	M1 M1 A3 A1 **[6]**	Attempt to find sums and products of roots (at least two of three) Attempt to use sums and products of roots of original equation to find sums and products of roots in related equation Correct coefficients consistent with x^3 coefficient, minus 1 each error Correct cubic equation c.a.o.
6(i) 6(ii)	$\dfrac{1}{r+2} - \dfrac{1}{r+3} = \dfrac{r+3-(r+2)}{(r+2)(r+3)} = \dfrac{1}{(r+2)(r+3)}$ $\sum_{r=1}^{50} \dfrac{1}{(r+2)(r+3)} = \sum_{r=1}^{50} \left[\dfrac{1}{r+2} - \dfrac{1}{r+3}\right]$ $= \left(\dfrac{1}{3} - \dfrac{1}{4}\right) + \left(\dfrac{1}{4} - \dfrac{1}{5}\right) + \left(\dfrac{1}{5} - \dfrac{1}{6}\right) + ...$ $+ \left(\dfrac{1}{51} - \dfrac{1}{52}\right) + \left(\dfrac{1}{52} - \dfrac{1}{53}\right)$ $= \dfrac{1}{3} - \dfrac{1}{53} = \dfrac{50}{159}$	M1 A1 **[2]** M1 M1, M1 A1 **[4]**	Attempt at common denominator Correct use of part (i) (may be implied) First two terms in full Last two terms in full (allow in terms of *n*) Give B4 for correct without working Allow 0.314 (3s.f.)

7	$\sum_{r=1}^{n} 3^{r-1} = \frac{3^n - 1}{2}$ $n = 1$, LHS = RHS = 1 Assume true for $n = k$ Next term is 3^k Add to both sides $\text{RHS} = \frac{3^k - 1}{2} + 3^k$ $= \frac{3^k - 1 + 2 \times 3^k}{2}$ $= \frac{3 \times 3^k - 1}{2}$ $= \frac{3^{k+1} - 1}{2}$ But this is the given result with $k + 1$ replacing k. Therefore if it is true for k it is true for $k + 1$. Since it is true for $k = 1$, it is true for $k = 1, 2, 3$ and so true for all positive integers.	B1 E1 M1 A1 E1 E1 **[6]**	 Assuming true for k Attempt to add 3^k to RHS c.a.o. with correct simplification Dependent on previous E1 and immediately previous A1 Dependent on B1 and both previous E marks
			Section A Total: 36

Section B

8(i)	$(2,\ 0)$, $(-2,\ 0)$, $\left(0,\ \dfrac{-4}{3}\right)$	B1 B1 B1 **[3]**	1 mark for each s.c. B2 for $2,\ -2,\ \dfrac{-4}{3}$
8(ii)	$x=3,\ x=-1,\ x=1, y=0$	B4 **[4]**	Minus 1 for each error
8(iii)	Large positive *x*, $y \to 0^+$, approach from above (e.g. consider $x=100$) Large negative *x*, $y \to 0^-$, approach from below (e.g. consider $x=-100$)	B1 B1 M1 **[3]**	Direction of approach must be clear for each B mark Evidence of method required
8(iv)	Curve 4 branches correct Asymptotes correct and labelled Intercepts labelled 	B2 B1 B1 **[4]**	Minus 1 each error, min 0

9(i)	$x = 1 - 2\text{j}$	B1 **[1]**	
9(ii)	Complex roots occur in conjugate pairs. A cubic has three roots, so one must be real. Or, valid argument involving graph of a cubic or behaviour for large positive and large negative x.	E1 **[1]**	
9(iii)	**Scheme A**		
	$(x-1-2\text{j})(x-1+2\text{j}) = x^2 - 2x + 5$ $(x-\alpha)(x^2-2x+5) = x^3 + Ax^2 + Bx + 15$	M1 A1 A1(ft)	Attempt to use factor theorem Correct factors Correct quadratic(using their factors)
	comparing constant term: $-5\alpha = 15 \Rightarrow \alpha = -3$	M1 M1	Use of factor involving real root Comparing constant term
	So real root is $x = -3$	A1(ft)	From their quadratic
	$(x+3)(x^2-2x+5) = x^3 + Ax^2 + Bx + 15$ $\Rightarrow x^3 + x^2 - x + 15 = x^3 + Ax^2 + Bx + 15$ $\Rightarrow A = 1,\ B = -1$	M1 M1 A1 **[9]**	Expand LHS Compare coefficients 1 mark for both values
	OR		
	Scheme B		
	Product of roots $= -15$ $(1+2\text{j})(1-2\text{j}) = 5$	M1 A1 M1 A1	Attempt to use product of roots Product is –15 Multiplying complex roots
	$\Rightarrow 5\alpha = -15$	A1	
	$\Rightarrow \alpha = -3$	A1	c.a.o.
	Sum of roots = -A $\Rightarrow -A = 1 + 2j + 1 - 2j - 3 = -1 \Rightarrow A = 1$	M1	Attempt to use sum of roots
	Substitute root $x =$ –3 into cubic $(-3)^3 + (-3)^2 - 3B + 15 = 0 \Rightarrow B = -1$	M1	Attempt to substitute, or to use sum
	A = 1 and B = -1	A1 **[9]**	c.a.o.
	OR		
	Scheme C		
	$\alpha = -3$	6	As scheme A, or other valid method
	$(1+2\text{j})^3 + A(1+2\text{j})^2 + B(1+2\text{j}) + 15 = 0$	M1	Attempt to substitute root
	$\Rightarrow A(-3+4\text{j}) + B(1+2\text{j}) + 4 - 2\text{j} = 0$ $\Rightarrow -3A + B + 4 = 0$ and $4A + 2B - 2 = 0$	M1	Attempt to equate real and imaginary parts, or equivalent.
	$\Rightarrow A = 1$ and $B = -1$	A1 **[9]**	c.a.o.

Section B (continued)			
10(i)	$\mathbf{AB}=\begin{pmatrix}1&-2&k\\2&1&2\\3&2&-1\end{pmatrix}\begin{pmatrix}-5&-2+2k&-4-k\\8&-1-3k&-2+2k\\1&-8&5\end{pmatrix}$ $=\begin{pmatrix}k-21&0&0\\0&k-21&0\\0&0&k-21\end{pmatrix}$ $n=21$	M1 A1 **[2]**	Attempt to multiply matrices (can be implied)
10(ii)	$\mathbf{A}^{-1}=\frac{1}{k-21}\begin{pmatrix}-5&-2+2k&-4-k\\8&-1-3k&-2+2k\\1&-8&5\end{pmatrix}$ $k\neq 21$	M1 M1 A1 A1 **[4]**	Use of **B** Attempt to use their answer to (i) Correct inverse Accept n in place of 21 for full marks
10 (iii)	**Scheme A** $\frac{1}{-20}\begin{pmatrix}-5&0&-5\\8&-4&0\\1&-8&5\end{pmatrix}\begin{pmatrix}1\\12\\3\end{pmatrix}=\frac{1}{-20}\begin{pmatrix}-20\\-40\\-80\end{pmatrix}=\begin{pmatrix}1\\2\\4\end{pmatrix}$ $x=1,\ y=2,\ z=4$ OR **Scheme B** Attempt to eliminate 2 variables Substitute in their value to attempt to find others $x=1,\ y=2,\ z=4$	M1 M1 A3 **[5]** M1 M1 A3 **[5]**	Attempt to use inverse Their inverse with $k=1$ One for each correct (ft) s.c. award 2 marks only for $x=1,\ y=2,\ z=4$ with no working.
			Section B Total: 36
			Total: 72

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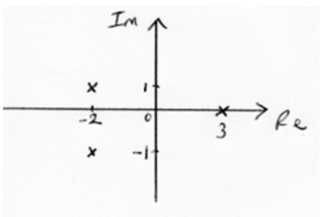
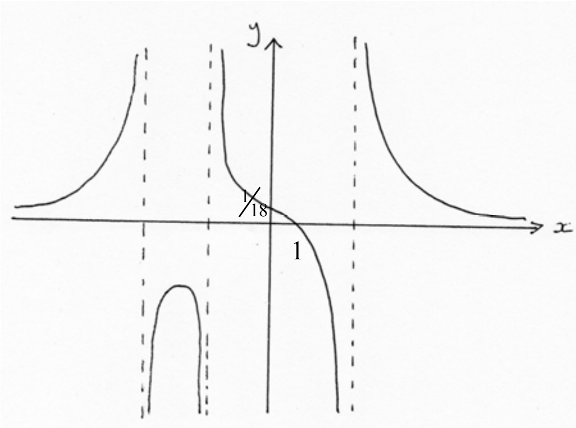
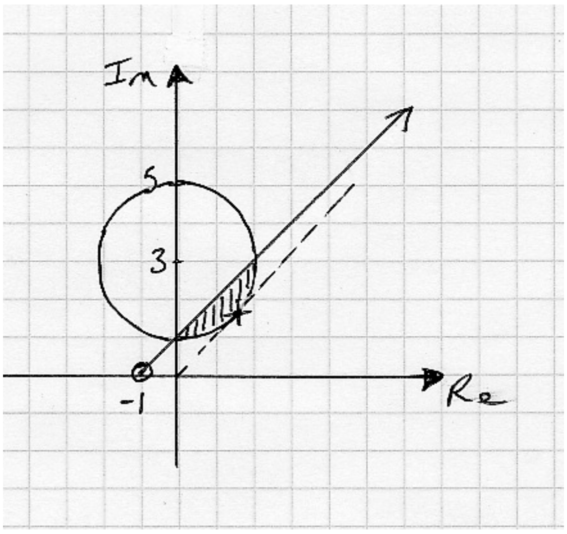
Qu	Answer	Mark	Comment
Section A			
1(i)	$\mathbf{BA} = \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ -4 & 14 \end{pmatrix}$	M1 A1 **[2]**	Attempt to multiply c.a.o.
1(ii)	$\det \mathbf{BA} = (6 \times 14) - (-4 \times 0) = 84$ $3 \times 84 = 252$ square units	M1 A1 A1(ft) **[3]**	Attempt to calculate any determinant c.a.o. Correct area
2(i)	$\alpha^2 = (-3+4\mathrm{j})(-3+4\mathrm{j}) = (-7-24\mathrm{j})$	M1 A1 **[2]**	Attempt to multiply with use of $\mathrm{j}^2 = -1$ c.a.o.
2(ii)	$\lvert\alpha\rvert = 5$ $\arg\alpha = \pi - \arctan\frac{4}{3}$ = 2.21 (2d.p.) (or 126.87°) $\alpha = 5(\cos 2.21 + \mathrm{j}\sin 2.21)$	B1 B1 B1(ft) **[3]**	Accept 2.2 or 127° Accept degrees and (r, θ) form s.c. lose 1 mark only if α^2 used throughout (ii)
3(i)	$3^3 + 3^2 - 7\times 3 - 15 = 0$ $z^3 + z^2 - 7z - 15 = (z-3)(z^2+4z+5)$ $z = \dfrac{-4 \pm \sqrt{16-20}}{2} = -2 \pm \mathrm{j}$ So other roots are $-2+\mathrm{j}$ and $-2-\mathrm{j}$	B1 M1 A1 M1 A1 **[5]**	Showing 3 satisfies the equation (may be implied) Valid attempt to factorise Correct quadratic factor Use of quadratic formula, or other valid method One mark for both c.a.o.
3(ii)		B2 **[2]**	Minus 1 for each error ft provided conjugate imaginary roots

4	$\sum_{r=1}^{n}\left[(r+1)(r-2)\right]=\sum_{r=1}^{n}r^2-\sum_{r=1}^{n}r-2n$	M1	Attempt to split sum up
	$=\frac{1}{6}n(n+1)(2n+1)-\frac{1}{2}n(n+1)-2n$	A2	Minus one each error
	$=\frac{1}{6}n\left[(n+1)(2n+1)-3(n+1)-12\right]$	M1	Attempt to factorise
	$=\frac{1}{6}n\left(2n^2+3n+1-3n-3-12\right)$	M1	Collecting terms
	$=\frac{1}{6}n\left(2n^2-14\right)$		
	$=\frac{1}{3}n\left(n^2-7\right)$	A1 **[6]**	All correct
5(i)	$p=-3,\ r=7$	B2 **[2]**	One mark for each s.c. B1 if *b* and *d* used instead of *p* and *r*
5(ii)	$q=\alpha\beta+\alpha\gamma+\beta\gamma$	B1	
	$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)$ $=(\alpha+\beta+\gamma)^2-2q$ $\Rightarrow 13=3^2-2q$	M1	Attempt to find *q* using $\alpha^2+\beta^2+\gamma^2$ and $\alpha+\beta+\gamma$, but not $\alpha\beta\gamma$
	$\Rightarrow q=-2$	A1 **[3]**	c.a.o.
6(i)	$a_2=7\times7-3=46$	M1	Use of inductive definition
	$a_3=7\times46-3=319$	A1 **[2]**	c.a.o.
6(ii)	When n = 1, $\frac{13\times7^0+1}{2}=7$, so true for n = 1	B1	Correct use of part (i) (may be implied)
	Assume true for $n=k$ $a_k=\frac{13\times7^{k-1}+1}{2}$	E1	Assuming true for *k*
	$\Rightarrow a_{k+1}=7\times\frac{13\times7^{k-1}+1}{2}-3$ $=\frac{13\times7^k+7}{2}-3$ $=\frac{13\times7^k+7-6}{2}$	M1	Attempt to use $a_{k+1}=7a_k-3$
	$=\frac{13\times7^k+1}{2}$	A1	Correct simplification
	But this is the given result with $k+1$ replacing k. Therefore if it is true for k it is true for $k+1$. Since it is true for $k=1$, it is true for $k=1, 2, 3$ and so true for all positive integers.	E1 E1 **[6]**	Dependent on A1 and previous E1 Dependent on B1 and previous E1
			Section A Total: 36

Section B

7(i)	$(1,\ 0)$ and $(0,\ \frac{1}{18})$	B1 B1 **[2]**	
7(ii)	x = 2, $x=-3,\ x=\frac{-3}{2},\ y=0$	B4 **[4]**	Minus 1 for each error
7(iii)		B1 B1 **[2]**	Correct approaches to vertical asymptotes Through clearly marked $(1,\ 0)$ and $(0,\ \frac{1}{18})$
7(iv)	$x<-3$, $x>2$ $\frac{-3}{2}<x\le 1$	B1 B2 **[3]**	 B1 for $\frac{-3}{2}<x<1$, or $\frac{-3}{2}\le x\le 1$
8(i) **8(ii)**	 *Sketch should clearly show the radius and centre of the circle and the starting point and angle of the half-line.*	B3 B3 **[6]** B2(ft) **[2]** M1 A1(ft)	Circle, B1; radius 2, B1; centre 3j, B1 Half line, B1; from -1, B1; $\frac{\pi}{4}$ to *x*-axis, B1 Correct region between their circle and half line indicated s.c. B1 for interior of circle Tangent from origin to circle Correct point placed by eye where tangent from origin meets circle
8(iii)	$\arg z=\frac{\pi}{2}-\arcsin\frac{2}{3}$ = 0.84 (2d.p.)	M1 A1 **[4]**	Attempt to use right angled triangle c.a.o. Accept 48.20° (2d.p.)

9(i)	$(-3,\ -3)$	B1 **[1]**	
9(ii)	$(x,\ x)$	B1 B1 **[2]**	
9(iii)	$\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$	B3 **[3]**	Minus 1 each error to min of 0
9(iv)	Rotation through $\frac{\pi}{2}$ anticlockwise about the origin	B1 B1 **[2]**	Rotation and angle (accept 90°) Centre and sense
9(v)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}$	M1 A1 **[2]**	Attempt to multiply using their **T** in correct order c.a.o.
9(vi)	$\begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ x \end{pmatrix}$ So (-*x*, *x*) Line *y* = -*x*	M1 A1(ft) A1 **[3]**	May be implied c.a.o. from correct matrix

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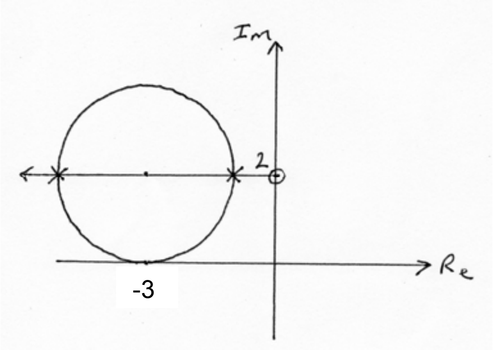
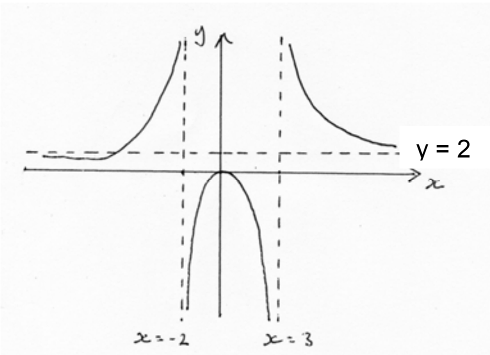
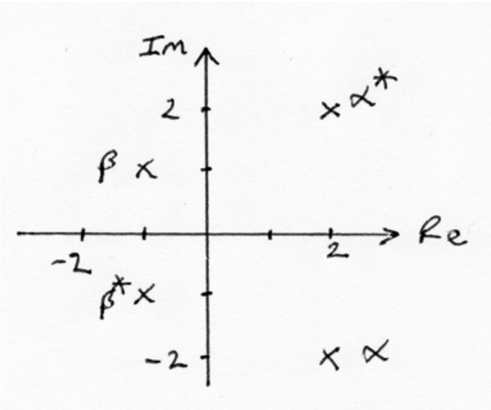
Qu	Answer	Mark	Comment
Section A			
1(i)	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	B1	
1(ii)	$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	B1	
1(iii)	$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}=\begin{pmatrix} -3 & 0 \\ 0 & 3 \end{pmatrix}$	M1 A1 **[4]**	Multiplication, or other valid method (may be implied) c.a.o.
2		B3 B3 B1 **[7]**	Circle, B1; centre $-3+2\text{j}$, B1; radius = 2, B1 Line parallel to real axis, B1; through (0, 2), B1; correct half line, B1 Points $-1+2\text{j}$ and $-5+2\text{j}$ indicated c.a.o.
3	$\begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} x \\ y \end{pmatrix}$ $\Rightarrow -x-y=x,\ 2x+2y=y$ $\Rightarrow y=-2x$	M1 M1 B1 **[3]**	For $\begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} x \\ y \end{pmatrix}$
4	$3x^3-x^2+2\equiv A(x-1)^3+(x^3+Bx^2+Cx+D)$ $\equiv Ax^3-3Ax^2+3Ax-A+x^3+Bx^2+Cx+D$ $\equiv (A+1)x^3+(B-3A)x^2+(3A+C)x+(D-A)$ $\Rightarrow A=2,\ B=5,\ C=-6,\ D=4$	M1 B4 **[5]**	Attempt to compare coefficients One for each correct value

5(i)	$\mathbf{AB} = \begin{pmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix}$	B3 **[3]**	Minus 1 each error to minimum of 0
5(ii)	$\mathbf{A}^{-1} = \frac{1}{7}\begin{pmatrix} -1 & 0 & 2 \\ 14 & -14 & 7 \\ -5 & 7 & -4 \end{pmatrix}$	M1 A1 **[2]**	Use of B c.a.o.
6	$w = 2x \Rightarrow x = \frac{w}{2}$ $\Rightarrow 2\left(\frac{w}{2}\right)^3 + \left(\frac{w}{2}\right)^2 - 3\left(\frac{w}{2}\right) + 1 = 0$ $\Rightarrow w^3 + w^2 - 6w + 4 = 0$	B1 M1 A1 A2 **[5]**	Substitution. For substitution $x = 2w$ give B0 but then follow through for a maximum of 3 marks Substitute into cubic Correct substitution Minus 1 for each error (including '= 0' missing), to a minimum of 0 Give full credit for integer multiple of equation
6	**OR** $\alpha + \beta + \gamma = -\frac{1}{2}$ $\alpha\beta + \alpha\gamma + \beta\gamma = -\frac{3}{2}$ $\alpha\beta\gamma = -\frac{1}{2}$ Let new roots be *k*, *l*, *m* then $k + l + m = 2(\alpha + \beta + \gamma) = -1 = \frac{-B}{A}$ $kl + km + lm = 4(\alpha\beta + \alpha\gamma + \beta\gamma) = -6 = \frac{C}{A}$ $klm = 8\alpha\beta\gamma = -4 = \frac{-D}{A}$ $\Rightarrow \omega^3 + \omega^2 - 6\omega + 4 = 0$	B1 M1 A1 A2 **[5]**	All three Attempt to use sums and products of roots of original equation to find sums and products of roots in related equation Sums and products all correct ft their coefficients; minus one for each error (including '= 0' missing), to minimum of 0 Give full credit for integer multiple of equation

7(i)	$\frac{1}{3r-1}-\frac{1}{3r+2}\equiv\frac{3r+2-(3r-1)}{(3r-1)(3r+2)}$	M1	Attempt at correct method
	$\equiv\frac{3}{(3r-1)(3r+2)}$	A1	Correct, without fudging
		[2]	
7(ii)	$\sum_{r=1}^{n}\frac{1}{(3r-1)(3r+2)}=\frac{1}{3}\sum_{r=1}^{n}\left[\frac{1}{3r-1}-\frac{1}{3r+2}\right]$	M1	Attempt to use identity
	$=\frac{1}{3}\left[\left(\frac{1}{2}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{8}\right)+\ldots\ldots+\left(\frac{1}{3n-1}-\frac{1}{3n+2}\right)\right]$	A1 M1	Terms in full (at least two) Attempt at cancelling
	$=\frac{1}{3}\left[\frac{1}{2}-\frac{1}{3n+2}\right]$	A2	A1 if factor of $\frac{1}{3}$ missing,
		[5]	A1 max if answer not in terms of n
			Section A Total: 36

Section B			
8(i)	$x = 3$, $x = -2$, $y = 2$	B1 B1 B1 **[3]**	
8(ii)	Large positive *x*, $y \to 2^+$ (e.g. consider $x = 100$) Large negative *x*, $y \to 2^-$ (e.g. consider $x = -100$)	M1 B1 B1 **[3]**	Evidence of method required
8(iii)	Curve Central and RH branches correct Asymptotes correct and labelled LH branch correct, with clear minimum  	B1 B1 B1 **[3]**	
8(iv)	$-2 < x < 3$ $x \neq 0$	B2 B1 **[3]**	B2 max if any inclusive inequalities appear B3 for $-2 < x < 0$ and $0 < x < 3$,

9(i)	$2+2\text{j}$ and $-1-\text{j}$	B2 **[2]**	1 mark for each
9(ii)		B2 **[2]**	1 mark for each correct pair
9(iii)	$(x-2-2\text{j})(x-2+2\text{j})(x+1+\text{j})(x+1-\text{j})$ $=(x^2-4x+8)(x^2+2x+2)$ $=x^4+2x^3+2x^2-4x^3-8x^2-8x+8x^2+16x+16$ $=x^4-2x^3+2x^2+8x+16$ $\Rightarrow A=-2,\ B=2,\ C=8,\ D=16$	M1 B2 A1 M1 A2 **[7]**	Attempt to use factor theorem Correct factors, minus 1 each error B1 if only errors are sign errors One correct quadratic with real coefficients (may be implied) Expanding Minus 1 each error, A1 if only errors are sign errors
	OR $\sum\alpha=2$ $\alpha\beta\gamma\delta=16$ $\sum\alpha\beta=\alpha\alpha^*+\alpha\beta+\alpha\beta^*+\beta\beta^*+\beta\alpha^*+\beta^*\alpha^*$ $\sum\alpha\beta\gamma=\alpha\alpha^*\beta+\alpha\alpha^*\beta^*+\alpha\beta\beta^*+\alpha^*\beta\beta^*$ $\sum\alpha\beta=2$, $\sum\alpha\beta\gamma=-8$ $A=-2,\ B=2,\ C=8,\ D=16$	B1 B1 M1 M1 A1 A2 **[7]**	 Both correct Minus 1 each error, A1 if only errors are sign errors
	OR Attempt to substitute in one root Attempt to substitute in a second root Equating real and imaginary parts to 0 Attempt to solve simultaneous equations $A=-2,\ B=2,\ C=8,\ D=16$	M1 M1 A1 M1 M1 A2 **[7]**	 Both correct Minus 1 each error, A1 if only errors are sign errors

Qu	Answer	Mark	Comment
Section B (continued)			
10(i)	$\sum_{r=1}^{n} r^2(r+1) = \sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r^2$	M1	Separation of sums (may be implied)
	$= \frac{1}{4}n^2(n+1)^2 + \frac{1}{6}n(n+1)(2n+1)$	B1	One mark for both parts
	$= \frac{1}{12}n(n+1)[3n(n+1)+2(2n+1)]$	M1	Attempt to factorise (at least two linear algebraic factors)
	$= \frac{1}{12}n(n+1)(3n^2+7n+2)$	A1	Correct
	$= \frac{1}{12}n(n+1)(n+2)(3n+1)$	E1	Complete, convincing argument
		[5]	
10(ii)	$\sum_{r=1}^{n} r^2(r+1) = \frac{1}{12}n(n+1)(n+2)(3n+1)$		
	n = 1, LHS = RHS = 2	B1	2 must be seen
	Assume true for $n = k$	E1	Assuming true for k
	$\sum_{r=1}^{k} r^2(r+1) = \frac{1}{12}k(k+1)(k+2)(3k+1)$		
	$\sum_{r=1}^{k+1} r^2(r+1)$		
	$= \frac{1}{12}k(k+1)(k+2)(3k+1)+(k+1)^2(k+2)$	B1	(k + 1)th term
	$= \frac{1}{12}(k+1)(k+2)[k(3k+1)+12(k+1)]$	M1	Attempt to factorise
	$= \frac{1}{12}(k+1)(k+2)(3k^2+13k+12)$	A1	Correct
	$= \frac{1}{12}(k+1)(k+2)(k+3)(3k+4)$	A1	Complete convincing argument
	$= \frac{1}{12}(k+1)((k+1)+1)((k+1)+2)(3(k+1)+1)$		
	But this is the given result with k + 1 replacing k. Therefore if it is true for k it is true for k + 1.	E1	Dependent on previous A1 and previous E1
	Since it is true for k = 1, it is true for k = 1, 2, 3 and so true for all positive integers.	E1	Dependent on first B1 and previous E1
		[8]	
			Section B Total: 36
			Total: 72

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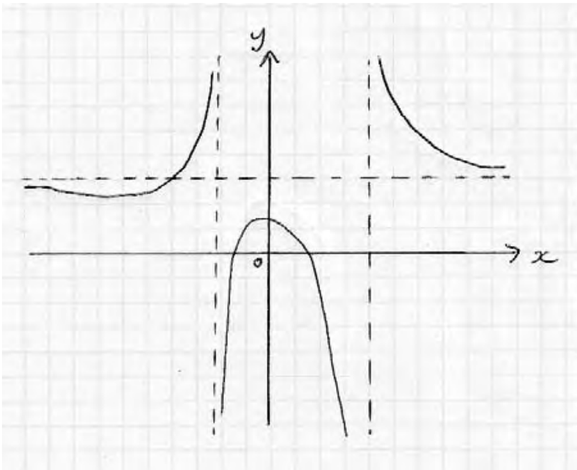
Section A

1(i)	$z = \frac{6 \pm \sqrt{36-40}}{2}$ $\Rightarrow z = 3 + \text{j}$ or $z = 3 - \text{j}$	M1 A1 **[2]**	Use of quadratic formula/completing the square For both roots
1(ii)	$\lvert 3+\text{j} \rvert = \sqrt{10} = 3.16$ (3s.f.) $\arg(3+\text{j}) = \arctan\left(\frac{1}{3}\right) = 0.322$ (3s.f.) $\Rightarrow$ roots are $\sqrt{10}(\cos 0.322 + \text{j}\sin 0.322)$ and $\sqrt{10}(\cos 0.322 - \text{j}\sin 0.322)$ or √10(cos(-0.322)+jsin(-0.322))	M1 M1 A1 **[3]**	Method for modulus Method for argument (both methods must be seen following A0) One mark for both roots in modulus-argument form – accept surd and decimal equivalents and $(r, \ \theta)$ form. Allow $\pm 18.4^{\circ}$ for θ.
2	$2x^2 - 13x + 25 = A(x-3)^2 - B(x-2) + C$ $\Rightarrow 2x^2 - 13x + 25$ $= Ax^2 - (6A+B)x + (2B+C) + 9A$ A = 2 B = 1 C = 5	B1 M1 A1 A1 **[4]**	For A=2 Attempt to compare coefficients of x^1or x^0, or other valid method. For B and C, cao.
3(i)	$\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$	B1 **[1]**	
3(ii)	$\begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 0 & 2 & 3 & 1 \\ 0 & 0 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 4 & 4 & 0 \\ 0 & 0 & 6 & 6 \end{pmatrix}$ $\Rightarrow$ A″$=(4, \ 0)$, B″$=(4, \ 6)$, C″$=(0, \ 6)$	M1 A1 **[2]**	Applying matrix to column vectors, with a result. All correct
3(iii)	Stretch factor 4 in x-direction. Stretch factor 6 in y-direction	B1 B1 **[2]**	Both factor and direction for each mark. SC1 for "enlargement", not stretch.

4	$\arg\left(z-(2-2\text{j})\right)=\frac{\pi}{4}$	B1 B1 B1 **[3]**	Equation involving arg(complex variable). Argument (complex expression) = $\frac{\pi}{4}$ All correct
5	Sum of roots = $\alpha+(-3\alpha)+\alpha+3=3-\alpha=5$ $\Rightarrow \alpha=-2$ Product of roots $=-2\times6\times1=-12$ Product of roots in pairs $=-2\times6+(-2)\times1+6\times1=-8$ $\Rightarrow p=-8$ and $q=12$	M1 A1 M1 M1 A1 A1 **[6]**	Use of sum of roots Attempt to use product of roots Attempt to use sum of products of roots in pairs One mark for each, ft if α incorrect
	Alternative solution $(x-\alpha)(x+3\alpha)(x-\alpha-3)$ $=x^3+(\alpha-3)x^2+(-5\alpha^2-6\alpha)x+3\alpha^3+9\alpha^2$ => $\alpha=-2$, $p=-8$ and $q=12$	M1 M1A1 M1 A1A1 **[6]**	Attempt to multiply factors Matching coefficient of x^2 ,cao. Matching other coefficients One mark for each, ft incorrect α.
6	$\sum_{r=1}^{n}\left[r(r^2-3)\right]=\sum_{r=1}^{n}r^3-3\sum_{r=1}^{n}r$ $=\frac{1}{4}n^2(n+1)^2-\frac{3}{2}n(n+1)$ $=\frac{1}{4}n(n+1)(n(n+1)-6)$ $=\frac{1}{4}n(n+1)(n^2+n-6)=\frac{1}{4}n(n+1)(n+3)(n-2)$	M1 M1 A2 M1 A1 **[6]**	Separate into separate sums. (may be implied) Substitution of standard result in terms of n. For two correct terms (indivisible) Attempt to factorise with $n(n+1)$. Correctly factorised to give fully factorised form

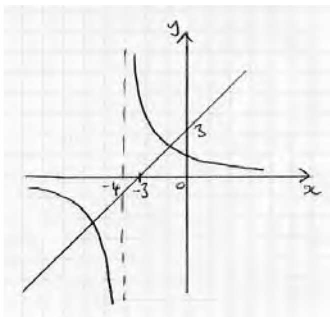
7	When $n = 1,\ 6\left(3^n - 1\right) = 12$, so true for $n = 1$ Assume true for $n = k$ $12 + 36 + 108 + ... + (4 \times 3^k) = 6\left(3^k - 1\right)$ $\Rightarrow 12 + 36 + 108 + ... + (4 \times 3^{k+1})$ $= 6\left(3^k - 1\right) + (4 \times 3^{k+1})$ $= 6\left[\left(3^k - 1\right) + \frac{2}{3} \times 3^{k+1}\right]$ $= 6\left[3^k - 1 + 2 \times 3^k\right]$ $= 6\left(3^{k+1} - 1\right)$ But this is the given result with $k + 1$ replacing k. Therefore if it is true for $n = k$, it is true for $n = k + 1$. Since it is true for $n = 1$, it is true for $n = 1, 2, 3...$ and so true for all positive integers.	B1 E1 M1 M1 A1 E1 E1 **[7]**	 Assume true for k Add correct next term to both sides Attempt to factorise with a factor 6 c.a.o. with correct simplification Dependent on A1 and first E1 Dependent on B1 and second E1
			Section A Total: 36

Section B			
8(i)	$(\sqrt{3},\ 0), (-\sqrt{3},\ 0)\ \left(0,\ \frac{3}{8}\right)$	B1 B1 [2]	Intercepts with x axis (both) Intercept with y axis SC1 if seen on graph or if $x = \pm\sqrt{3}$, $y = 3/8$ seen without $y = 0$, $x = 0$ specified.
8(ii)	$x = 4,\ x = -2,\ y = 1$	B3 [3]	Minus 1 for each error. Accept equations written on the graph.
8(iii)		B1 B1B1 B1 [4]	Correct approaches to vertical asymptotes, LH and RH branches LH and RH branches approaching horizontal asymptote On LH branch 0<y<1 as x→-∞.
8(iv)	$-2 < x \le -\sqrt{3}$ and $4 > x \ge \sqrt{3}$	B1 B2 [3]	LH interval and RH interval correct (Award this mark even if errors in inclusive/exclusive inequality signs) All inequality signs correct, minus 1 each error

9(i)	$\alpha + \beta = 3$ $\alpha\alpha^* = (1+\text{j})(1-\text{j}) = 2$ $\frac{\alpha+\beta}{\alpha} = \frac{3}{1+\text{j}} = \frac{3(1-\text{j})}{(1+\text{j})(1-\text{j})} = \frac{3}{2} - \frac{3}{2}\text{j}$	B1 M1 A1 M1 A1 **[5]**	 Attempt to multiply $(1+\text{j})(1-\text{j})$ Multiply top and bottom by $1-\text{j}$
9(ii)	$(z-(1+\text{j}))(z-(1-\text{j}))$ $= z^2 - 2z + 2$	M1 A1 **[2]**	Or alternative valid methods (Condone no "=0" here)
9(iii)	$1-\text{j}$ and $2+\text{j}$ Either $(z-(2-\text{j}))(z-(2+\text{j}))$ $= z^2 - 4z + 5$ $(z^2-2z+2)(z^2-4z+5)$ $= z^4 - 6z^3 + 15z^2 - 18z + 10$ So equation is $z^4 - 6z^3 + 15z^2 - 18z + 10 = 0$	B1 M1 M1 A2 **[5]**	For both For attempt to obtain an equation using the product of linear factors involving complex conjugates Using the correct four factors All correct, -1 each error (including omission of "=0") to min of 0
	Or alternative solution Use of $\sum\alpha = 6$, $\sum\alpha\beta = 15$, $\sum\alpha\beta\gamma = 18$ and $\alpha\beta\gamma\delta = 10$ to obtain the above equation.	M1 A3 **[5]**	Use of relationships between roots and coefficients. All correct, -1 each error, to min of 0

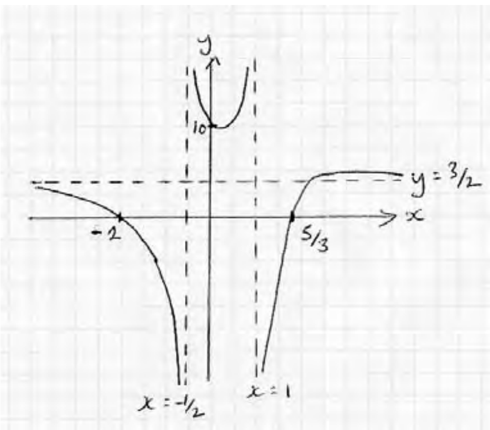
10(i)	$\alpha = 3\times -5 + 4\times 11 + -1\times 29 = 0$ $\beta = -2\times -7 + 7\times(5+k) + -3\times 7 = 28 + 7k$	B1 M1 A1 **[3]**	 Attempt at row 3 x column 3
10(ii)	$\mathbf{AB} = \begin{pmatrix} 42 & 0 & 0 \\ 0 & 42 & 0 \\ 0 & 0 & 42 \end{pmatrix}$	B2 **[2]**	Minus 1 each error to min of 0
10(iii)	$\mathbf{A}^{-1} = \frac{1}{42}\begin{pmatrix} 11 & -5 & -7 \\ 1 & 11 & 7 \\ -5 & 29 & 7 \end{pmatrix}$	M1 B1 A1 **[3]**	Use of **B** $\frac{1}{42}$ Correct inverse, allow decimals to 3 sf
10(iv)	$\frac{1}{42}\begin{pmatrix} 11 & -5 & -7 \\ 1 & 11 & 7 \\ -5 & 29 & 7 \end{pmatrix}\begin{pmatrix} 1 \\ -9 \\ 26 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $= \frac{1}{42}\begin{pmatrix} -126 \\ 84 \\ -84 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ -2 \end{pmatrix}$ $x = -3,\ y = 2,\ z = -2$	M1 A3 **[4]**	Attempt to pre-multiply by $\mathbf{A}^{-1}$ SC B2 for Gaussian elimination with 3 correct solutions, -1 each error to min of 0 Minus 1 each error
			Section B Total: 36
			Total: 72

4755 (FP1) Further Concepts for Advanced Mathematics

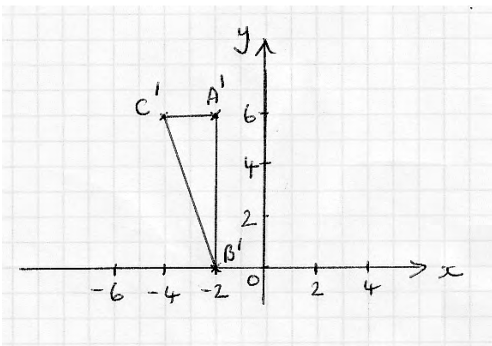
Section A			
1(i)	$\mathbf{M}^{-1} = \frac{1}{11}\begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}$	M1 A1 **[2]**	Dividing by determinant
1(ii)	$\frac{1}{11}\begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 49 \\ 100 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{11}\begin{pmatrix} 198 \\ 253 \end{pmatrix}$ $\Rightarrow x = 18, y = 23$	M1 A1(ft) A1(ft) **[3]**	Pre-multiplying by their inverse
2	$z^3 + z^2 - 7z - 15 = (z-3)(z^2 + 4z + 5)$ $z^2 + 4z + 5 = 0 \Rightarrow z = \frac{-4 \pm \sqrt{16-20}}{2}$ $\Rightarrow z = -2 + \mathrm{j}$ and $z = -2 - \mathrm{j}$	B1 M1 A1 M1 A1 **[5]**	Show z = 3 is a root; may be implied Attempt to find quadratic factor Correct quadratic factor Use of quadratic formula or other valid method Both solutions
3(i)		B1 B1 **[2]**	Asymptote at x = -4 Both branches correct
3(ii)	$\frac{2}{x+4} = x + 3 \Rightarrow x^2 + 7x + 10 = 0$ $\Rightarrow x = -2$ or $x = -5$ $x \geq -2$ or $-4 > x \geq -5$	M1 A1 A1 A2 **[5]**	Attempt to find where graphs cross or valid attempt at solution using inequalities Correct intersections (both), or -2 and -5 identified as critical values $x \geq -2$ $-4 > x \geq -5$ s.c. A1 for $-4 \geq x \geq -5$ or $-4 > x > -5$

4	$2w-6w+3w=\frac{-1}{2}$ $\Rightarrow w=\frac{1}{2}$ $\Rightarrow$ roots are $1, -3, \frac{3}{2}$ $\frac{-q}{2}=\alpha\beta\gamma=\frac{-9}{2}\Rightarrow q=9$ $\frac{p}{2}=\alpha\beta+\alpha\gamma+\beta\gamma=-6\Rightarrow p=-12$	M1 A1 A1 M1 A2(ft) **[6]**	Use of sum of roots – can be implied Correct roots seen Attempt to use relationships between roots s.c. M1 for other valid method One mark each for *p* = -12 and *q* = 9

5(i)	$\frac{1}{5r-2}-\frac{1}{5r+3}\equiv\frac{5r+3-5r+2}{(5r+3)(5r-2)}$ $\equiv\frac{5}{(5r+3)(5r-2)}$	M1 A1 **[2]**	Attempt to form common denominator Correct cancelling
5(ii)	$\sum_{r=1}^{30}\frac{1}{(5r-2)(5r+3)}=\frac{1}{5}\sum_{r=1}^{30}\left[\frac{1}{(5r-2)}-\frac{1}{(5r+3)}\right]$ $=\frac{1}{5}\left[\left(\frac{1}{3}-\frac{1}{8}\right)+\left(\frac{1}{8}-\frac{1}{13}\right)+\left(\frac{1}{13}-\frac{1}{18}\right)+\ldots +\left(\frac{1}{5n-7}-\frac{1}{5n-2}\right)+\left(\frac{1}{5n-2}-\frac{1}{5n+3}\right)\right]$ $=\frac{1}{5}\left[\frac{1}{3}-\frac{1}{5n+3}\right]=\frac{n}{3(5n+3)}$	B1 B1 M1 A1 **[4]**	First two terms in full Last term in full Attempt to cancel terms
6	When *n* = 1, $\frac{1}{2}n(7n-1)=3$, so true for *n* = 1 Assume true for *n* = *k* $3+10+17+.....+(7k-4)=\frac{1}{2}k(7k-1)$ $\Rightarrow 3+10+17+.....+(7(k+1)-4)$ $=\frac{1}{2}k(7k-1)+(7(k+1)-4)$ $=\frac{1}{2}[k(7k-1)+(14(k+1)-8)]$ $=\frac{1}{2}[7k^2+13k+6]$ $=\frac{1}{2}(k+1)(7k+6)$ $=\frac{1}{2}(k+1)(7(k+1)-1)$ But this is the given result with *k* + 1 replacing *k*. Therefore if it is true for *k* it is true for *k* + 1. Since it is true for *n* = 1, it is true for *n* = 1, 2, 3 and so true for all positive integers.	B1 E1 M1 M1 A1 E1 E1 **[7]**	 Assume true for *n* = *k* Add $(k+1)$th term to both sides Valid attempt to factorise c.a.o. with correct simplification Dependent on previous E1 and immediately previous A1 Dependent on B1 and both previous E marks
			Section A Total: 36

Section B			
7(i)	$(0, 10), (-2, 0), \left(\frac{5}{3}, 0\right)$	B1 B1 B1 [3]	
7(ii)	$x = \frac{-1}{2},\ x = 1,\ y = \frac{3}{2}$	B1 B1 B1 [3]	
7(iii)	Large positive *x*, $y \to \frac{3}{2}^{+}$ (e.g. consider $x = 100$) Large negative *x*, $y \to \frac{3}{2}^{-}$ (e.g. consider $x = -100$)	M1 B1 B1 [3]	Clear evidence of method required for full marks
7(iv)	Curve 3 branches of correct shape Asymptotes correct and labelled Intercepts correct and labelled 	B1 B1 B1 [3]	

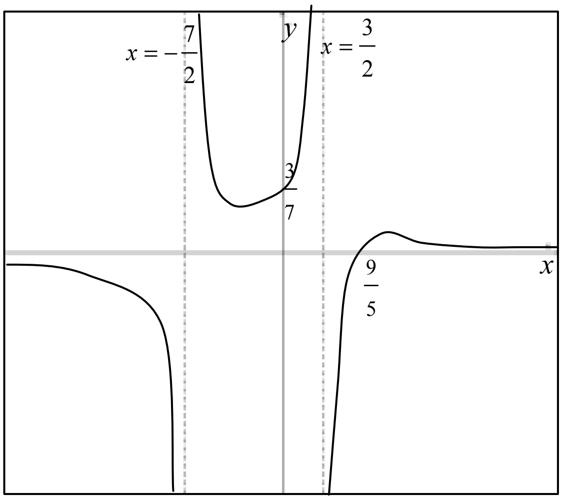
8 (i)	$\left\|z-(4+2j)\right\|=2$	**B1** **B1** **B1** **[3]**	Radius = 2 $z-(4+2j)$ or $z-4-2j$ All correct
8(ii)	$\arg\left(z-(4+2j)\right)=0$	**B1** **B1** **B1** **[3]**	Equation involving the argument of a complex variable Argument = 0 All correct
8(iii)	$a=4-2\cos\frac{\pi}{4}=4-\sqrt{2}$ $b=2+2\sin\frac{\pi}{4}=2+\sqrt{2}$ $P=4-\sqrt{2}+\left(2+\sqrt{2}\right)j$	**M1** **A2** **[3]**	Valid attempt to use trigonometry involving $\frac{\pi}{4}$, or coordinate geometry 1 mark for each of *a* and *b* s.c. A1 only for *a* = 2.59, *b* = 3.41
8(iv)	$\frac{3}{4}\pi>\arg\left(z-(4+2j)\right)>0$ and $\left\|z-(4+2j)\right\|<2$	**B1** **B1** **B1** **[3]**	$\arg\left(z-(4+2j)\right)>0$ $\arg\left(z-(4+2j)\right)<\frac{3}{4}\pi$ $\left\|z-(4+2j)\right\|<2$ Deduct one mark if only error is use of inclusive inequalities

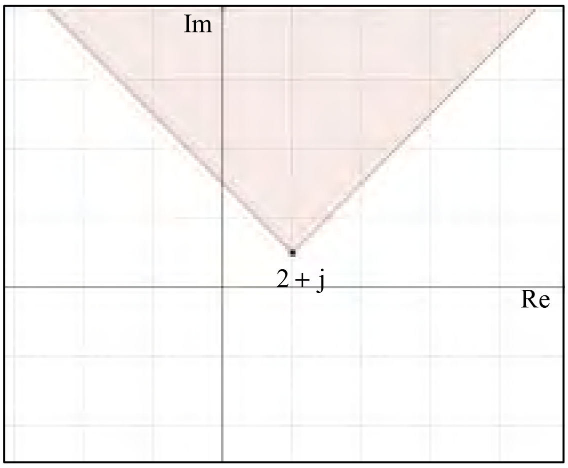
Section B (continued)			
9(i)	Matrix multiplication is associative	B1 [1]	
	$\mathbf{MN}=\begin{pmatrix}3&0\\0&2\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}$	M1	Attempt to find **MN** or **QM**
	$\Rightarrow \mathbf{MN}=\begin{pmatrix}0&3\\2&0\end{pmatrix}$	A1	or $\mathbf{QM}=\begin{pmatrix}0&-2\\3&0\end{pmatrix}$
	$\mathbf{QMN}=\begin{pmatrix}-2&0\\0&3\end{pmatrix}$	A1(ft) [3]	
9(ii)	M is a stretch, factor 3 in the *x* direction, factor 2 in the *y* direction.	B1 B1	Stretch factor 3 in the *x* direction Stretch factor 2 in the *y* direction
	N is a reflection in the line *y* = *x*.	B1	
	Q is an anticlockwise rotation through 90° about the origin.	B1 [4]	
9(iii)	$\begin{pmatrix}-2&0\\0&3\end{pmatrix}\begin{pmatrix}1&1&2\\2&0&2\end{pmatrix}=\begin{pmatrix}-2&-2&-4\\6&0&6\end{pmatrix}$	M1 A1(ft)	Applying their **QMN** to points. Minus 1 each error to a minimum of 0.
		B2 [4]	Correct, labelled image points, minus 1 each error to a minimum of 0. Give B4 for correct diagram with no workings.
			Section B Total: 36
			Total: 72

4755 (FP1) Further Concepts for Advanced Mathematics

1	$\alpha\beta = (-3+j)(5-2j) = -13+11j$	M1 A1 [2]	Use of $j^2 = -1$
	$\dfrac{\alpha}{\beta} = \dfrac{-3+j}{5-2j} = \dfrac{(-3+j)(5+2j)}{29} = \dfrac{-17}{29} - \dfrac{1}{29}j$	M1 A1 A1 [3]	Use of conjugate 29 in denominator All correct
2 (i)	**AB** is impossible $\mathbf{CA} = (50)$ $\mathbf{B}+\mathbf{D} = \begin{pmatrix} 3 & 1 \\ 6 & -2 \end{pmatrix}$ $\mathbf{AC} = \begin{pmatrix} 20 & 4 & 32 \\ -10 & -2 & -16 \\ 20 & 4 & 32 \end{pmatrix}$	B1 B1 B1 B2 [5]	 –1 each error
(ii)	$\mathbf{DB} = \begin{pmatrix} -2 & 0 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 5 & 1 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} -10 & -2 \\ 22 & 1 \end{pmatrix}$	M1 A1 [2]	Attempt to multiply in correct order c.a.o.
3	$\alpha+\beta+\gamma = a-d+a+a+d = \dfrac{12}{4} \Rightarrow a = 1$	M1 A1	Valid attempt to use sum of roots $a = 1$, c.a.o.
	$(a-d)a(a+d) = \dfrac{3}{4} \Rightarrow d = \pm\dfrac{1}{2}$	M1	Valid attempt to use product of roots
	So the roots are $\dfrac{1}{2}$, 1 and $\dfrac{3}{2}$	A1	All three roots
	$\alpha\beta+\alpha\gamma+\beta\gamma = \dfrac{k}{4} = \dfrac{11}{4} \Rightarrow k = 11$	M1 A1 [6]	Valid attempt to use $\alpha\beta+\alpha\gamma+\beta\gamma$, or to multiply out factors, or to substitute a root $k = 11$ c.a.o.

4	$\mathbf{MM}^{-1}=\frac{1}{k}\begin{pmatrix}4&0&1\\-6&1&1\\5&2&5\end{pmatrix}\begin{pmatrix}-3&-2&1\\-35&-15&10\\17&8&-4\end{pmatrix}$	M1	Attempt to consider $\mathbf{MM}^{-1}$ or $\mathbf{M}^{-1}\mathbf{M}$ (may be implied)
	$=\frac{1}{k}\begin{pmatrix}5&0&0\\0&5&0\\0&0&5\end{pmatrix}\Rightarrow k=5$	A1 **[2]**	c.a.o.
	$\begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{1}{5}\begin{pmatrix}-3&-2&1\\-35&-15&10\\17&8&-4\end{pmatrix}\begin{pmatrix}9\\32\\81\end{pmatrix}$	M1	Attempt to pre-multiply by $\mathbf{M}^{-1}$
		M1	Attempt to multiply matrices
	$\frac{1}{5}\begin{pmatrix}-3&-2&1\\-35&-15&10\\17&8&-4\end{pmatrix}\begin{pmatrix}9\\32\\81\end{pmatrix}=\frac{1}{5}\begin{pmatrix}-10\\15\\85\end{pmatrix}$	A1	Correct
	$\Rightarrow x=-2,\ y=3,\ z=17$	A1 **[4]**	All 3 correct s.c. B1 if matrices not used
5	$\sum_{r=1}^{n}(r+2)(r-3)=\sum_{r=1}^{n}(r^2-r-6)$		
	$=\sum_{r=1}^{n}r^2-\sum_{r=1}^{n}r-6n$	M1	Separate into 3 sums
	$=\frac{1}{6}n(n+1)(2n+1)-\frac{1}{2}n(n+1)-6n$	A2	–1 each error
	$=\frac{1}{6}n[(n+1)(2n+1)-3(n+1)-36]$	M1	Valid attempt to factorise (with n as a factor)
	$=\frac{1}{6}n(2n^2-38)=\frac{1}{3}n(n^2-19)$	A1 A1 **[6]**	Correct expression c.a.o. Complete, convincing argument
6	When $n = 1$, $\frac{n(n+1)(n+2)}{3}=2$,	B1	
	so true for $n = 1$ Assume true for $n = k$	E1	Assume true for k
	$2+6+.....+k(k+1)=\frac{k(k+1)(k+2)}{3}$		
	$\Rightarrow 2+6+.....+(k+1)(k+2)$	M1	Add $(k+1)$th term to both sides
	$=\frac{k(k+1)(k+2)}{3}+(k+1)(k+2)$		
	$=\frac{1}{3}(k+1)(k+2)(k+3)$	A1	c.a.o. with correct simplification
	$=\frac{(k+1)((k+1)+1)((k+1)+2)}{3}$		
	But this is the given result with $k + 1$ replacing k. Therefore if it is true for $n = k$ it is true for $n = k + 1$.	E1	Dependent on A1 and previous E1
	Since it is true for $n = 1$, it is true for $n = 1, 2, 3$ and so true for all positive integers.	E1 **[6]**	Dependent on B1 and previous E1

7	(i)	$x = \frac{-7}{2}$, $x = \frac{3}{2}$, $y = 0$	B1 B1 B1 **[3]**	
	(ii)	Large positive x, $y \to 0^+$ (e.g. consider $x = 100$) Large negative x, $y \to 0^-$ (e.g. consider $x = -100$)	B1 B1 M1 **[3]**	Evidence of method
	(iii)		B1 B1 B1 **[3]**	Intercepts correct and labelled LH and central branches correct RH branch correct, with clear maximum
	(iv)	$x < -\frac{7}{2}$ or $\frac{3}{2} < x \le \frac{9}{5}$	B1 B2 **[3]**	 Award B1 if only error relates to inclusive/exclusive inequalities

8(a) (i)	$\lvert z-(2+6\text{j})\rvert=4$	B1 B1 B1 **[3]**	2 + 6j seen (expression in z) = 4 Correct equation
(ii)	$\lvert z-(2+6\text{j})\rvert<4$ and $\lvert z-(3+7\text{j})\rvert>1$	B1 B1 B1 **[3]**	$\lvert z-(2+6\text{j})\rvert<4$ $\lvert z-(3+7\text{j})\rvert>1$ (allow errors in inequality signs) Both inequalities correct
(b)(i)		B1 B1 B1 **[3]**	Any straight line through 2 + j Both correct half lines Region between their two half lines indicated
(ii)	$43+47\text{j}-(2+\text{j})=41+46\text{j}$ $\arg(41+46\text{j})=\arctan\left(\frac{46}{41}\right)=0.843$ $\frac{\pi}{4}<0.843<\frac{3\pi}{4}$ so $43+47\text{j}$ does fall within the region	M1 A1 E1 **[3]**	Attempt to calculate argument, or other valid method such as comparison with $y=x-1$ Correct Justified

9 (i)	$\frac{2}{r}-\frac{3}{r+1}+\frac{1}{r+2}$ $=\frac{2(r+1)(r+2)-3r(r+2)+r(r+1)}{r(r+1)(r+2)}$ $=\frac{2r^2+6r+4-3r^2-6r+r^2+r}{r(r+1)(r+2)}=\frac{4+r}{r(r+1)(r+2)}$	M1 A1 **[2]**	Attempt a common denominator Convincingly shown
(ii)	$\sum_{r=1}^{n}\frac{4+r}{r(r+1)(r+2)}=\sum_{r=1}^{n}\left[\frac{2}{r}-\frac{3}{r+1}+\frac{1}{r+2}\right]$ $=\left(\frac{2}{1}-\frac{3}{2}+\frac{1}{3}\right)+\left(\frac{2}{2}-\frac{3}{3}+\frac{1}{4}\right)+\left(\frac{2}{3}-\frac{3}{4}+\frac{1}{5}\right)+..$ $..+\left(\frac{2}{n-1}-\frac{3}{n}+\frac{1}{n+1}\right)+\left(\frac{2}{n}-\frac{3}{n+1}+\frac{1}{n+2}\right)$ $=\frac{2}{1}-\frac{3}{2}+\frac{2}{2}+\frac{1}{n+1}-\frac{3}{n+1}+\frac{1}{n+2}$ $=\frac{3}{2}-\frac{2}{n+1}+\frac{1}{n+2}$ as required	M1 M1 A2 M1 A1 **[6]**	Use of the given result (may be implied) Terms in full (at least first and one other) At least 3 consecutive terms correct, -1 each error Attempt to cancel, including algebraic terms Convincingly shown
(iii)	$\frac{3}{2}$	B1 **[1]**	
(iv)	$\sum_{r=50}^{100}\frac{4+r}{r(r+1)(r+2)}$ $=\sum_{r=1}^{100}\frac{4+r}{r(r+1)(r+2)}-\sum_{r=1}^{49}\frac{4+r}{r(r+1)(r+2)}$ $=\left(\frac{3}{2}-\frac{2}{101}+\frac{1}{102}\right)-\left(\frac{3}{2}-\frac{2}{50}+\frac{1}{51}\right)$ $=0.0104$ (3s.f.)	M1 M1 A1 **[3]**	Splitting into two parts Use of result from (ii) c.a.o.

RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced Subsidiary GCE **4755**

Further Concepts for Advanced Mathematics (FP1)

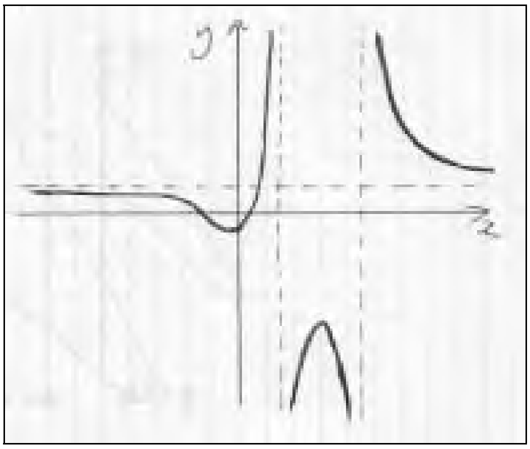
Mark Scheme for June 2010

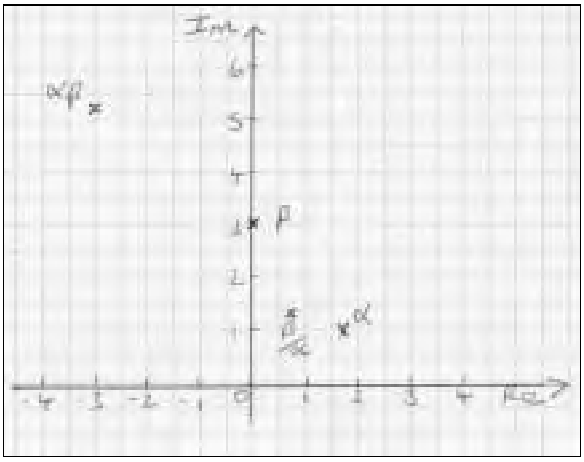
Qu	Answer	Mark	Comment
Section A			
1	$4x^2-16x+C\equiv A\left(x^2+2Bx+B^2\right)+2$	B1	$A = 4$
	$\Leftrightarrow 4x^2-16x+C\equiv Ax^2+2ABx+AB^2+2$	M1	Attempt to expand RHS or other valid method (may be implied)
	$\Leftrightarrow A=4, B=-2, C=18$	A2, 1 **[4]**	1 mark each for B and C, c.a.o.
2(i)	$2x-5y=9$	B1	
	$3x+7y=-1$	B1 **[2]**	
2(ii)	$\mathbf{M}^{-1}=\frac{1}{29}\begin{pmatrix}7&5\\-3&2\end{pmatrix}$	M1 A1 **[2]**	Divide by determinant c.a.o.
	$\frac{1}{29}\begin{pmatrix}7&5\\-3&2\end{pmatrix}\begin{pmatrix}9\\-1\end{pmatrix}=\frac{1}{29}\begin{pmatrix}58\\-29\end{pmatrix}$ $\Rightarrow x=2,\ y=-1$	M1 A1(ft) **[2]**	Pre-multiply by their inverse For both
3	$z=1-2\text{j}$	B1	
	$1+2\text{j}+1-2\text{j}+\alpha=\frac{1}{2}$	M1	Valid attempt to use sum of roots, or other valid method
	$\Rightarrow\alpha=-\frac{3}{2}$	A1	c.a.o.
	$\frac{-k}{2}=-\frac{3}{2}(1-2\text{j})(1+2\text{j})=-\frac{15}{2}$	M1 A1(ft)	Valid attempt to use product of roots, or other valid method Correct equation – can be implied
	$k=15$	A1 **[6]**	c.a.o.
	OR		
	$\left(z-(1+2\text{j})\right)\left(z-(1-2\text{j})\right)=z^2-2z+5$	M1 A1	Multiplying correct factors Correct quadratic, c.a.o.
	$2z^3-z^2+4z+k=\left(z^2-2z+5\right)(2z+3)$	M1	Attempt to find linear factor
	$\alpha=\frac{-3}{2}$	A1(ft)	
	$k = 15$	A1 **[6]**	c.a.o.

4	$w = x+1 \Rightarrow x = w-1$	B1	Substitution. For $x = w+1$ give B0 but then follow for a maximum of 3 marks
	$x^3 - 2x^2 - 8x + 11 = 0,\ w = x-1$ $\Rightarrow (w-1)^3 - 2(w-1)^2 - 8(w-1) + 11 = 0$ $\Rightarrow w^3 - 5w^2 - w + 16 = 0$	M1 M1 A3 **[6]**	Attempt to substitute into cubic Attempt to expand -1 for each error (including omission of = 0)
	OR		
	$\alpha + \beta + \gamma = 2$ $\alpha\beta + \alpha\gamma + \beta\gamma = -8$ $\alpha\beta\gamma = -11$	B1	All 3 correct
	Let the new roots be k, l and m then		
	$k + l + m = \alpha + \beta + \gamma + 3 = 2 + 3 = 5$	M1	Valid attempt to use their sum of roots in original equation to find sum of roots in new equation
	$kl + km + lm = (\alpha\beta + \alpha\gamma + \beta\gamma) + 2(\alpha + \beta + \gamma) + 3$ $= -8 + 4 + 3 = -1$ $klm = \alpha\beta\gamma + (\alpha\beta + \alpha\gamma + \beta\gamma) + (\alpha + \beta + \gamma) + 1$ $= -11 - 8 + 2 + 1 = -16$	M1	Valid attempt to use their product of roots in original equation to find one of $\sum \alpha\beta$ or $\alpha\beta\gamma$
	$\Rightarrow w^3 - 5w^2 - w + 16 = 0$	A3 **[6]**	-1 each error (including omission of = 0)
5	$\sum_{r=1}^{n} \frac{1}{(5r-1)(5r+4)} = \frac{1}{5}\sum_{r=1}^{n}\left(\frac{1}{5r-1} - \frac{1}{5r+4}\right)$	M1	Attempt to use identity – may be implied
	$= \frac{1}{5}\left(\left(\frac{1}{4} - \frac{1}{9}\right) + \left(\frac{1}{9} - \frac{1}{14}\right) + \ldots\ldots + \left(\frac{1}{5n-1} - \frac{1}{5n+4}\right)\right)$	A1	Terms in full (at least first and last)
	$= \frac{1}{5}\left(\frac{1}{4} - \frac{1}{5n+4}\right) = \frac{1}{5}\left(\frac{5n+4-4}{4(5n+4)}\right) = \frac{n}{4(5n+4)}$	M1	Attempt at cancelling
		A1	$\left(\frac{1}{4} - \frac{1}{5n+4}\right)$
		A1	factor of $\frac{1}{5}$
		A1 **[6]**	Correct answer as a single algebraic fraction

6(i)	$u_2 = \frac{2}{1+2} = \frac{2}{3}$, $u_3 = \frac{\frac{2}{3}}{1+\frac{2}{3}} = \frac{2}{5}$	M1 A1 **[2]**	Use of inductive definition c.a.o.
6(ii)	When $n = 1$, $\frac{2}{2\times 1-1} = 2$, so true for $n = 1$	B1	Showing use of $u_n = \frac{2}{2n-1}$
	Assume $u_k = \frac{2}{2k-1}$	E1	Assuming true for k
	$\Rightarrow u_{k+1} = \frac{\frac{2}{2k-1}}{1+\frac{2}{2k-1}}$	M1	u_{k+1}
	$= \frac{\frac{2}{2k-1}}{\frac{2k-1+2}{2k-1}} = \frac{2}{2k+1}$ $= \frac{2}{2(k+1)-1}$	A1	Correct simplification
	But this is the given result with $k + 1$ replacing k. Therefore if it is true for k it is also true for $k + 1$.	E1	Dependent on A1 and previous E1
	Since it is true for $k = 1$, it is true for all positive integers.	E1 **[6]**	Dependent on B1 and previous E1
			Section A Total: 36

Section B

7(i)	$\left(0,\ -\frac{1}{2}\right)$	B1	
	$(-3, 0)$, $\left(\frac{1}{2}, 0\right)$	B1 **[2]**	For both
7(ii)	$x=3, x=2$ and $y=2$	B1 B1 B1 **[3]**	
7(iii)	Large positive x, $y \to 2^+$ (e.g. substitute $x=100$ to give 2.15…, or convincing algebraic argument)	M1 A1	Must show evidence of method A0 if no valid method
		B1 **[3]**	Correct RH branch
7(iv)	$\frac{(2x-1)(x+3)}{(x-3)(x-2)}=2$ $\Rightarrow (2x-1)(x+3)=2(x-3)(x-2)$	M1	Or other valid method to find intersection with horizontal asymptote
	$\Rightarrow x=1$	A1	
	From graph $x<1$ or $2<x<3$	B1 B1 **[4]**	For $x<1$ For $2<x<3$

8(i)	$\arg\alpha = \frac{\pi}{6}$, $\lvert\alpha\rvert = 2$	B1	Modulus of α
		B1	Argument of α (allow 30°)
	$\arg\beta = \frac{\pi}{2}$, $\lvert\beta\rvert = 3$	B1	Both modulus and argument of β (allow 90°)
		[3]	
8(ii)	$\alpha\beta = \left(\sqrt{3}+\mathrm{j}\right)3\mathrm{j} = -3+3\sqrt{3}\mathrm{j}$	M1	Use of $\mathrm{j}^2 = -1$
		A1	Correct
	$\frac{\beta}{\alpha} = \frac{3\mathrm{j}}{\sqrt{3}+\mathrm{j}} = \frac{3\mathrm{j}\left(\sqrt{3}-\mathrm{j}\right)}{\left(\sqrt{3}+\mathrm{j}\right)\left(\sqrt{3}-\mathrm{j}\right)}$	M1	. Correct use of conjugate of denominator
	$= \frac{3+3\sqrt{3}\mathrm{j}}{4} = \frac{3}{4}+\frac{3\sqrt{3}\mathrm{j}}{4}$	A1	Denominator = 4
		A1	All correct
		[5]	
8(iii)		M1	Argand diagram with at least one correct point
		A1(ft)	Correct relative positions with appropriate labelling
		[2]	

Qu	Answer	Mark	Comment
Section B (continued)			
9(i)	P is a rotation through 90 degrees about the origin in a clockwise direction. Q is a stretch factor 2 parallel to the x-axis	B1 B1 B1 B1 **[4]**	Rotation about origin 90 degrees clockwise, or equivalent Stretch factor 2 Parallel to the x-axis
9(ii)	$\mathbf{QP} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix}$	M1 A1 **[2]**	Correct order c.a.o.
9(iii)	$\begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 4 & 2 \\ -2 & -1 & -3 \end{pmatrix}$ $A' = (0,\ -2)$, $B' = (4,\ -1)$, $C' = (2,\ -3)$	M1 A1(ft) **[2]**	Pre-multiply by their **QP** - may be implied For all three points
9(iv)	$\mathbf{R} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1 B1 **[2]**	One for each correct column
9(v)	$\mathbf{RQP} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$ $(\mathbf{RQP})^{-1} = \frac{-1}{2}\begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$	M1 A1(ft) M1 A1 **[4]**	Multiplication of their matrices in correct order Attempt to calculate inverse of their **RQP** c.a.o.
			Section B Total: 36
			Total: 72

GCE

Mathematics

Advanced GCE

Unit **4725:** Further Pure Mathematics 1

Mark Scheme for January 2011

Oxford Cambridge and RSA Examinations

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1	**(i)**	$\begin{pmatrix}7 & 9\end{pmatrix}$	B1B1 **2**	Each element correct **SC** (7,9) scores B1
	(ii)	$\begin{pmatrix}18\end{pmatrix}$	B1* depB1 **2**	Obtain correct value Clearly given as a matrix
	(iii)	$\begin{pmatrix}12 & -4\\ 6 & -2\end{pmatrix}$	M1 A1 A1 **3** **7**	Obtain 2×2 matrix Obtain 2 correct elements Obtain other 2 correct elements
2.	**(i)**	$-12+13\mathrm{i}$	B1B1 **2**	Real and imaginary parts correct
	(ii)	$\frac{27}{37}-\frac{14}{37}\mathrm{i}$	B1 M1 A1 A1 **4** **6**	z^* seen Multiply by w^* Obtain correct real part or numerator Obtain correct imaginary part or denom. Sufficient working must be shown
3			B1* M1* A1* depA1 **4** **4**	Establish result true for n = 1 or 2 Use given result in recurrence relation in a relevant way Obtain $2^n + 1$ correctly Specific statement of induction conclusion
4		***Either*** $\frac{a}{4}n^2(n+1)^2+\frac{bn}{2}(n+1)$ $a=4 \quad b=-4$	B1 M1 A1 M1 A1 A1 **6**	Correct value for $\sum r$ stated or used Express as sum of two series Obtain correct unsimplified answer Compare coefficients or substitute values for n Obtain correct answers
		Or $a+b=0 \quad 4a+b=12$ $a=4 \quad b=-4$	M1 A1 A1 M1 A1 A1 **6**	Use 2 values for n Obtain correct equations Solve simultaneous equations Obtain correct answers
5		$\mathbf{A}^2$	B1 M1 A1cao **3** **3**	$(\mathbf{A}^{-1})^{-1} = \mathbf{A}$ seen or implied Use product inverse correctly Obtain correct answer

6 (i) (a)		B1*		Vertical line
		depB1	**2**	Clearly through (4, 0)
(b)		B1		Sloping line with +ve slope
		B1		Through (0, -2)
		B1ft	**3**	Half line starting on *y*-axis 45° shown convincingly
(ii)		B1ft		Shaded to left of their **(i) (a)**
		B1ft		Shaded below their **(i) (b)** must be +ve slope
		B1ft	**3**	Shaded above horizontal through their (0, -2)
				NB These 3 marks are independent, but 3/3 only for fully correct answer.
		8		
7 (i)	$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$	B1 B1	**2**	Each column correct
(ii)		B1*		Enlargement or stretch in x **and** y axes
		depB1	**2**	Scale factor $\sqrt{3}$
(iii) (a)		B1		$(2,0),(6,2)$ indicated
		B1		(8, 2) seen
		B1	**3**	Accurate diagram, including unit square
(b)	det**C** = 4	B1		Correct value found
		B1	**2**	Scale factor for area
		9		
8 (i) *Either*				
	$\alpha+\beta=\frac{1}{2}, \alpha\beta=\frac{3}{2}$	B1		State or use both correct results in (i) or (ii)
	$\alpha+\beta+\frac{\alpha+\beta}{\alpha\beta}$ or $\alpha+\beta+\frac{2}{3}(\alpha+\beta)$	M1		Express sum of new roots in terms of $\alpha+\beta$ and $\alpha\beta$
		M1		Substitute their values into their expression
	$p=\frac{5}{6}$	A1	**4**	Obtain **given** answer correctly
Or				
	$3u^2-u+2(=0)$	B1		Substitute $x=\frac{1}{u}$ and obtain correct quadratic (equation)
		M1		Use sum of roots of new equation
		M1		Substitute their values into their expression
	$p=\frac{5}{6}$	A1		Obtain **given** answer correctly

(ii)	$\alpha'\beta' = \alpha\beta + \frac{1}{\alpha\beta} + \frac{\beta}{\alpha} + \frac{\alpha}{\beta}$	B1		Correct expansion
	$\frac{\beta}{\alpha} + \frac{\alpha}{\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$	M1		Show how to deal with $\alpha^2 + \beta^2$
		A1		Obtain correct expression
	$q = \frac{1}{3}$	M1		Substitute their values into $\alpha'\beta'$
		A1	5	Obtain correct answer a.e.f.
		9		
9 (i)		M1		Show correct expansion process for 3 x 3
		M1		Correct evaluation of any 2 x 2
	det **M** = $a^2 - 7a + 6$	A1	3	correct answer
(ii)		M1		Solve det**M** = 0
	$a = 1$ or 6	A1A1	3	Obtain correct answer, ft their (i)
(iii)		M1		Attempt to eliminate one variable
		A1		Obtain 2 correct equations in 2 unknowns
		A1	3	Justify infinite number of solutions **SC** 3/3 if unique solution conclusion consistent with their (i) or (ii)
		9		
10 (i)		M1		Use correct denominator
		A1	2	Obtain **given** answer correctly
(ii)		M1		Express terms as differences using (i)
		M1		Do this for at least 3 terms
		A1		First 3 terms all correct
		A1		Last 2 terms all correct
	$\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$	M1		Show relevant cancelling
		A1	6	Obtain correct answer a.e.f.
(iii)	$\frac{1}{2}$	B1ft		S_∞ stated or start at $n + 1$ as in (ii)
	$\frac{1}{n+1} - \frac{1}{n+2}$	M1		S_∞ - their (ii) or show correct cancelling
	$\frac{1}{(n+1)(n+2)}$	A1	3	Obtain **given** answer correctly
		11		

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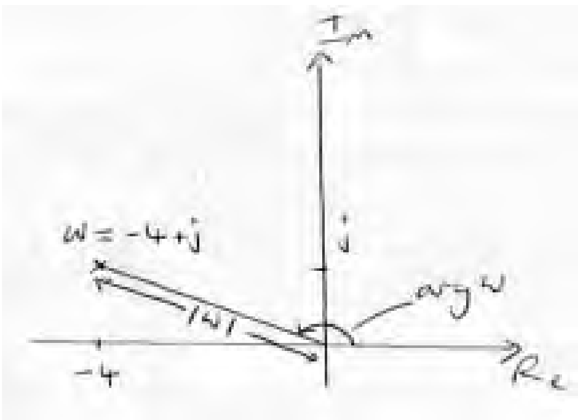
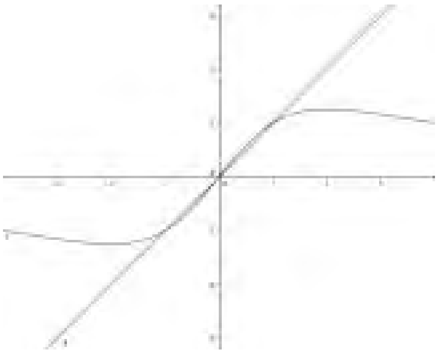
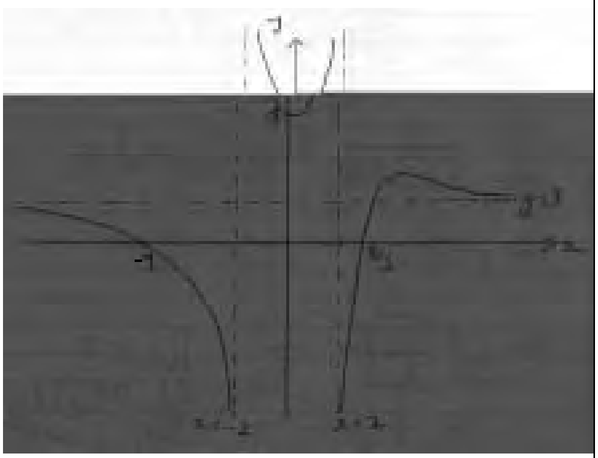
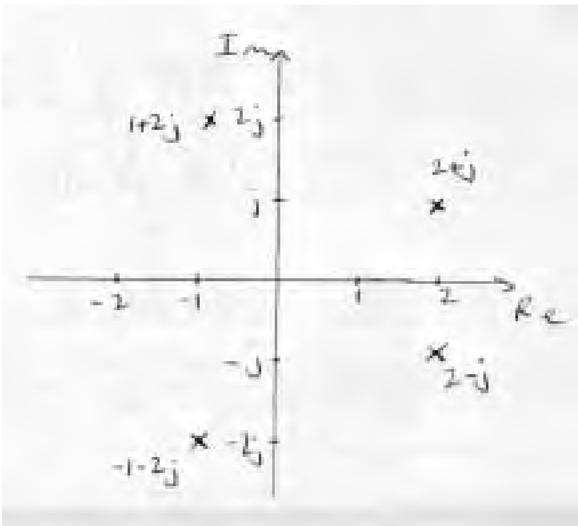
Qu	Answer	Mark	Comment
Section A			
1(i)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	B1	Accept expressions in sin and cos
1(ii)	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	B1	
1(iii)	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	M1 A1ft	Ans (ii) x Ans (i) attempt evaluation
1(iv)	Reflection in the x axis	B1 **[5]**	
2(i)	$\frac{z+w}{w}=\frac{-1-\text{j}}{-4+\text{j}}\times\frac{-4-\text{j}}{-4-\text{j}}$	M1	Multiply top and bottom by -4 - j
	$=\frac{3+5\text{j}}{17}=\frac{3}{17}+\frac{5}{17}\text{j}$	A1 A1 **[3]**	Denominator = 17 Correct numerators
2(ii)	$\lvert w\rvert=\sqrt{17}$	B1	
	$\arg w=\pi-\arctan\frac{1}{4}=2.90$	B1	Not degrees
	$w=\sqrt{17}\left(\cos 2.90+\text{j}\sin 2.90\right)$	B1 **[3]**	c.a.o. Accept $\left(\sqrt{17},\ 2.90\right)$ Accept 166 degrees
2(iii)		B1 B1 **[2]**	Correct position Mod w and Arg w correctly shown
3	$\alpha+\beta+\gamma=4=-p$ $p=-4$	M1 A1	May be implied
	$\left(\alpha+\beta+\gamma\right)^2=\alpha^2+\beta^2+\gamma^2+2\left(\alpha\beta+\alpha\gamma+\beta\gamma\right)$	M1	Attempt to use $\left(\alpha+\beta+\gamma\right)^2$
	$\Rightarrow 16=6+2q$ $\Rightarrow q=5$	A1 A1 **[5]**	o.e. Correct c.a.o.

4	$\frac{5x}{x^2+4} < x$ $\Rightarrow 5x < x^3 + 4x$ $\Rightarrow 0 < x^3 - x$ $\Rightarrow 0 < x(x+1)(x-1)$ $\Rightarrow x > 1, \; -1 < x < 0$ 	M1* A1 A1 M1dep* A1 A1 **[6]**	Method attempted towards factorisation to find critical values $x = 0$ $x = 1, x = -1$ Valid method leading to required intervals, graphical or algebraic $x > 1$ $-1 < x < 0$ SC B2 No valid working seen $x > 1$ $-1 < x < 0$
5	$\sum_{r=1}^{20} \frac{1}{(3r-1)(3r+2)} \equiv \frac{1}{3}\sum_{r=1}^{20}\left[\frac{1}{3r-1} - \frac{1}{3r+2}\right]$ $= \frac{1}{3}\left[\left(\frac{1}{2} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{8}\right) + ... + \left(\frac{1}{59} - \frac{1}{62}\right)\right]$ $= \frac{1}{3}\left(\frac{1}{2} - \frac{1}{62}\right) = \frac{5}{31}$	M1 A1 A1 M1 A1 **[5]**	Attempt to use identity – may be implied Correct use of 1/3 seen Terms in full (at least first and last) Attempt at cancelling c.a.o.

6	When $n = 1$, $\frac{1}{4}n^2(n+1)^2 = 1$, so true for $n = 1$	B1	
	Assume true for $n = k$ $\sum_{r=1}^{k} r^3 = \frac{1}{4}k^2(k+1)^2$	E1	Assume true for k
	$\Rightarrow \sum_{r=1}^{k+1} r^3 = \frac{1}{4}k^2(k+1)^2 + (k+1)^3$	M1	Add $(k+1)$th term to both sides
	$= \frac{1}{4}(k+1)^2[k^2 + 4(k+1)]$	M1	Factor of $\frac{1}{4}(k+1)^2$
	$= \frac{1}{4}(k+1)^2[k^2 + 4k + 4]$		
	$= \frac{1}{4}(k+1)^2(k+2)^2$	A1	c.a.o. with correct simplification
	$= \frac{1}{4}(k+1)^2((k+1)+1)^2$		
	But this is the given result with $k + 1$ replacing k. Therefore if it is true for k it is true for $k + 1$.	E1	Dependent on A1 and previous E1
	Since it is true for $n = 1$, it is true for $n = 1, 2, 3$ and so true for all positive integers.	E1	Dependent on B1 and previous E1 and correct presentation
		[7]	
			Section A Total: 36

Section B

7(i)	$(0, 18)$ $(-9, 0)$, $\left(\frac{8}{3}, 0\right)$	B1 B1 B1 **[3]**	
7(ii)	$x = 2$, $x = -2$ and $y = 3$	B1 B1 B1 **[3]**	
7(iii)	Large positive x, $y \to 3^+$ from above Large negative x, $y \to 3^-$ from below (e.g. consider $x = 100$, or convincing algebraic argument)	B1 B1 M1 **[3]**	 Must show evidence of working
7(iv)		B1 B1 B1 **[3]**	3 branches correct Asymptotes correct and labelled Intercepts correct and labelled

8(i)	Because a cubic can only have a maximum of two complex roots, which must form a conjugate pair.	E1 [1]	.
8(ii)	$2+\text{j},\ -1-2\text{j}$	B1 B1	
	$\text{P}(z)=(z-(2-\text{j}))(z-(2+\text{j}))(z-(-1+2\text{j}))(z-(-1-2\text{j}))$	M1	Use of factor theorem
	$=\left((z-2)^2+1\right)\left((z+1)^2+4\right)$	M1	Attempt to multiply out factors
	$=\left(z^2-4z+5\right)\left(z^2+2z+5\right)$ $=z^4-2z^3+2z^2-10z+25$	A4	-1 for each incorrect coefficient
	OR		
	$\alpha+\beta+\gamma+\delta=2\Rightarrow a=-2$ $\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta=2\Rightarrow b=2$ $\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta=10\Rightarrow c=-10$ $\alpha\beta\gamma\delta=25\Rightarrow d=25$ $\Rightarrow \text{P}(z)=z^4-2z^3+2z^2-10z+25$	M2 B1 A3	M1 for attempt to use all 4 root relationships. M2 for all correct a = -2 b, c, d correct -1 for each incorrect -1 for P(z) not explicit, following A4 or B1A3
		[8]	
8(iii)		B1	All correct with annotation on axes or labels
	$\|z\|=\sqrt{5}$	B1 [2]	

Qu	Answer	Mark	Comment
Section B (continued)			
9(i)	$\mathbf{M}=\begin{pmatrix}2 & -1\\ 3 & k\end{pmatrix}$	B2 **[2]**	- 1 each error
9(ii)	$\mathbf{M}^{-1}$ does not exist for $2k+3=0$	M1	May be implied
	$k=\frac{-3}{2}$	A1	
	$\mathbf{M}^{-1}=\frac{1}{2k+3}\begin{pmatrix}k & 1\\ -3 & 2\end{pmatrix}$	B1	Correct inverse
	$\frac{1}{13}\begin{pmatrix}5 & 1\\ -3 & 2\end{pmatrix}\begin{pmatrix}1\\ 21\end{pmatrix}$	M1	Attempt to pre-multiply by their inverse
	$=\begin{pmatrix}2\\ 3\end{pmatrix}$	A1ft A1	Correct matrix multiplication c.a.o.
	$\Rightarrow x=2,\ y=3$	A1ft **[7]**	At least one correct
9(iii)	There are no unique solutions	B1 **[1]**	
9(iv)	(A) Lines intersect (B) Lines parallel (C) Lines coincident	B1 B1 B1 **[3]**	
			Section B Total: 36
			Total: 72

RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4755:** Further Concepts for Advanced Mathematics

Mark Scheme for January 2012

Oxford Cambridge and RSA Examinations

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

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Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

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A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

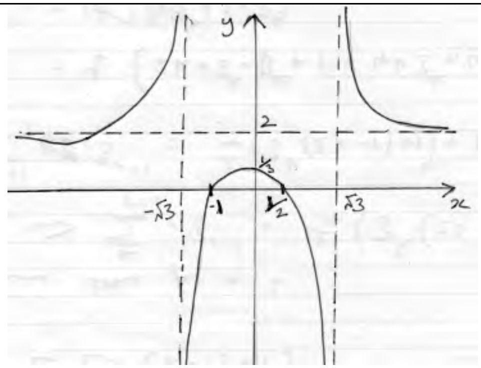
Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

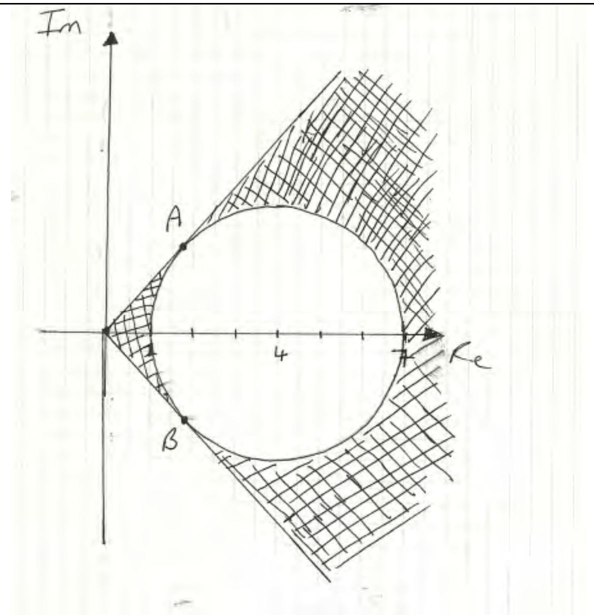
Question			Answer	Marks	Guidance
1	(i)		$\mathbf{AB}=\begin{pmatrix}2 & -1 & 1\\ 0 & p & -4\end{pmatrix}\begin{pmatrix}0 & q\\ 2 & -2\\ 1 & -3\end{pmatrix}=\begin{pmatrix}-1 & 2q-1\\ 2p-4 & -2p+12\end{pmatrix}$	M1 A2 **[3]**	Attempt to multiply in correct order Correct and simplified -1 each error
1	(ii)		$\mathbf{BA}=\begin{pmatrix}0 & q\\ 2 & -2\\ 1 & -3\end{pmatrix}\begin{pmatrix}2 & -1 & 1\\ 0 & p & -4\end{pmatrix}=\begin{pmatrix}* & * & *\\ * & * & *\\ * & * & *\end{pmatrix}$ $\mathbf{BA}\neq\mathbf{AB}$ hence not commutative	M1 A1 **[2]**	Valid method to compare products Reason for conclusion stated
2			$2x^3-3\equiv(x+3)(Ax^2+Bx+C)+D$ $B=-6,\ C=18, D=-57$	B1 M1 A3 **[5]**	$A=2$ Evidence of comparing coefficients or other valid method (may be implied) 1 mark each for B, C and D, c.a.o.
3			$6^3-10\times6^2+37\times6+p=0$ $\Rightarrow p=-78$ $z^3-10z^2+37z-78=(z-6)(z^2-4z+13)$ $z=\dfrac{4\pm\sqrt{16-52}}{2}=2\pm3\text{j}$ So other roots are $2+3\text{j}$ and $2-3\text{j}$	M1 A1 M1 A1 M1 A1 **[6]**	Substituting in 6, or other valid method cao Valid attempt to factorise Correct quadratic factor Valid method for solution of their 3 term quadratic One mark for both cao

Question			Answer	Marks	Guidance
4			$\sum_{r=1}^{n} r^2(r-1) = \sum_{r=1}^{n} r^3 - \sum_{r=1}^{n} r^2$	M1*	Attempt to split into two summations.
			$= \frac{1}{4}n^2(n+1)^2 - \frac{1}{6}n(n+1)(2n+1)$	M1 A1	Attempt to use at least one standard result appropriately Correct
			$= \frac{1}{12}n(n+1)(3n^2 - n - 2)$, oe or $\frac{1}{12}n(n-1)(3n^2 + 5n + 2)$, oe	M1 dep *	Attempt to factorise using either $n(n-1)$ or $n(n+1)$
			$= \frac{1}{12}n(n+1)(n-1)(3n+2)$	A2 **[6]**	All correct SC A1 correct but $(3kn + 2k) / 12k$ seen

Question			Answer	Marks	Guidance
5			$\omega = \frac{z}{2} + 1 \Rightarrow z = 2(\omega - 1)$	B1	Substitution
			$\left(2(\omega-1)\right)^3 - 5\left(2(\omega-1)\right)^2 + 3\left(2(\omega-1)\right) - 4 = 0$	M1	Substitute their expression for z into cubic and attempt to expand
			$\Rightarrow 4\omega^3 - 22\omega^2 + 35\omega - 19 = 0$	A4	Minus 1 each error (allow integer multiples)
				[6]	
			OR	OR	
			$\alpha + \beta + \gamma = 5$ $\alpha\beta + \alpha\gamma + \beta\gamma = 3$ $\alpha\beta\gamma = 4$	B1	Correct sums and products of roots
			Let new roots be k, l, m then $k + l + m = \frac{1}{2}(\alpha + \beta + \gamma) + 3 = \frac{11}{2} = \frac{-B}{A}$ $kl + km + lm = \frac{1}{4}(\alpha\beta + \alpha\gamma + \beta\gamma) +$ $(\alpha + \beta + \gamma) + 3 = \frac{35}{4} = \frac{C}{A}$ $klm = \frac{1}{8}\alpha\beta\gamma + \frac{1}{4}(\alpha\beta + \beta\gamma + \beta\gamma)$ $+ \frac{1}{2}(\alpha + \beta + \gamma) + 1 = \frac{19}{4} = \frac{-D}{A}$	M1	Attempt to use root relations of original equation to find all three sums and products of roots in related equation
			$\Rightarrow \omega^3 - \frac{11}{2}\omega^2 + \frac{35}{4}\omega - \frac{19}{4} = 0$		(*)
			$\Rightarrow 4\omega^3 - 22\omega^2 + 35\omega - 19 = 0$	A4	SC (*) A3 Minus 1 each error (allow integer multiples)
				[6]	

Question			Answer	Marks	Guidance
6			When $n = 1$, $\sum_{r=1}^{n} r3^{r-1} = 1\times 3^0 = 1$		
			and $\frac{1}{4}\left[3^n(2n-1)+1\right] = \frac{1}{4}\left[3\times(2-1)+1\right]=1$, so true for $n = 1$	B1	
			Assume $\sum_{r=1}^{k} r3^{r-1} = \frac{1}{4}\left[3^k(2k-1)+1\right]$	E1	Assuming true for k
			$\sum_{r=1}^{k+1} r3^{r-1} = \frac{1}{4}\left[3^k(2k-1)+1\right]+(k+1)3^{k+1-1}$	M1*	Adding $(k+1)$th term (incorrect expressions on LHS lose final E1)
			$= \frac{1}{4}\left[3^k(2k-1)+1+4(k+1)3^k\right]$	M1 dep*	Attempt to obtain factor of $\frac{1}{4}$
			$= \frac{1}{4}\left[3^k(2k-1+4(k+1))+1\right]$	M1dep*	For $\left[3^k(ak+b)+c\right]$ $c \neq 0$
			$= \frac{1}{4}\left[3^k(6k+3)+1\right]$		
			$= \frac{1}{4}\left[3^{k+1}(2k+1)+1\right]$	A1	
			$= \frac{1}{4}\left[3^{k+1}(2(k+1)-1)+1\right]$		Or target seen
			Therefore if true for $n = k$ it is also true for $n = k+1$.	E1	Dependent on A1 and previous E1
			Since it is true for $k = 1$, it is true for all positive integers.	E1	Dependent on B1 and previous E1
				[8]	
7	(i)		$(-1,\ 0)$, $(\frac{1}{2},\ 0)$	B1	Both x-intercepts
			$(0,\ \frac{1}{3})$	B1	y-intercept
				[2]	
7	(ii)		$x=-\sqrt{3},\ x=\sqrt{3},\ y=2$	B1,B1,B1*	
				[3]	

Question			Answer	Marks	Guidance
7	(iii)		Evidence of method needed e.g. evaluation for 'large' values or convincing algebraic argument (A)Large positive x, $y \to 2^+$ so from above (B) Large negative x, $y \to 2^-$ so from below	M1 A1 dep* A1 dep* **[3]**	Allow if $y = 2$ indicated but not explicit in (ii) SC B1 dep* Correct (A) and (B) following M0
7	(iv)			B1 B1 B1 **[3]**	Correct asymptotes shown and labelled Correct central branch with intercepts labelled Correct shape. Allow asymptotes at $x = \pm 3$ and $y = k$, $k > 0$. asymptotic behaviour shown with clear minimum in the LH branch.
7	(v)		$(x+1)(2x-1) = 2(x^2-3)$ $x = -5$ $x < -5$ or $-\sqrt{3} < x < \sqrt{3}$	M1 B1 B1 **[3]**	Finding where curve cuts $y = 2$ (or valid solution of an inequality)

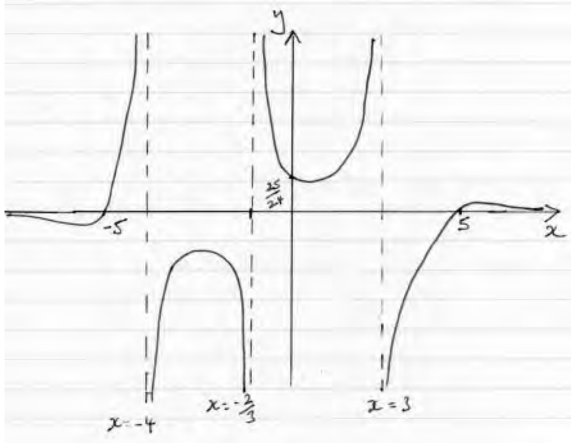
Question		Answer	Marks	Guidance
8	(i)		B3 **[3]**	Circle, B1; centre 4, B1; radius 3 with evidence of scale B1;
8	(ii)		B1 B1 **[2]**	Tangent OA Tangent OB
8	(iii)		B1 B1 **[2]**	Region outside their circle indicated Correct region shown
8	(iv)	$\alpha = \arcsin\frac{3}{4}$ $\alpha = 0.848$ $\beta = -0.848$	M1 A2 ft **[3]**	Valid method ft their tangents if circle centred on any axis One for each; accept 48.6° and –48.6° A1 max if $\alpha < \beta$
9	(i)	**R** represents a rotation through 90° $\mathbf{R}^4$ represents 4 successive rotations through 90°, making 360°, which is a full turn, which is equivalent to the identity	B1 B1 E1 **[3]**	 4 successive rotations Interpretation of $\mathbf{R}^4$ and **I** required
9	(ii)	$\mathbf{R}^{-1}$ represents a rotation of 90° clockwise about the origin. $\mathbf{R}^{-1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	B1 B1 **[2]**	Rotation, angle, centre and sense

Question			Answer	Marks	Guidance
9	(iii)		$\mathbf{S} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$	B2 **[2]**	One mark for each correct column (allow 3sf)
9	(iv)		$m = 3$ $n = 2$ $\mathbf{S}^3 = \mathbf{R}^2$ because both represent a rotation through 180°	B1 E1 **[2]**	$m = 3$ and $n = 2$
9	(v)		$\mathbf{RS} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$	M1 A1ft	ft their **S** -1 each error
			$\mathbf{RS} = \mathbf{SR}$ because $\mathbf{RS}$ represents a 60° rotation anticlockwise about the origin followed by a 90° rotation anticlockwise about the origin, making a total rotation of 150° anticlockwise about the origin. $\mathbf{SR}$ represents these two rotations in the opposite order, but the net effect is still a rotation of 150° anticlockwise about the origin.	E1 **[3]**	Convincing explanation, correct, no ft

Question			Answer	Marks	Guidance
1	(i)		Transformation A is a reflection in the y-axis. Transformation B is a rotation through 90° clockwise about the origin.	B1 B1 **[2]**	
1	(ii)		$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	M1 A1 **[2]**	Attempt to multiply in correct order cao
1	(iii)		Reflection in the line $y = x$	B1 **[1]**	
2	(i)		$\|z_1\| = \sqrt{3^2 + \left(3\sqrt{3}\right)^2} = 6$ $\arg(z_1) = \arctan\frac{3\sqrt{3}}{3} = \frac{\pi}{3}$	M1 A1 M1 A1 **[4]**	Use of Pythagoras cao cao
2	(ii)		$z_2 = \frac{5}{2} + \frac{5\sqrt{3}}{2}\text{j}$	M1 A1 **[2]**	May be implied cao
2	(iii)		Because z_1 and z_2 have the same argument	E1 **[1]**	Consistent with (i)
3			$\alpha + \frac{\alpha}{6} + \alpha - 7 = \frac{-8}{3} \Rightarrow \alpha = 2$ Other roots are -5 and $\frac{1}{3}$ Product of roots = $\frac{-q}{3} = \frac{-10}{3} \Rightarrow q = 10$ Sum of products in pairs $= \frac{p}{3} = -11 \Rightarrow p = -33$	M1 A1 M1 A1 M1 A1	Attempt to use sum of roots Value of α (cao) Attempt to use product of roots $q = 10$ c.a.o. Attempt to use sum of products of roots in pairs $p = -33$ cao

Question			Answer	Marks	Guidance
			OR, for final four marks		
			$(x-2)(x+5)(3x-1)$	M1	Express as product of factors
			$=3x^3+8x^2-33x+10$	M1	Expanding
			$\Rightarrow p=-33$ and $q=10$	A1	$p=-33$ cao
				A1	$q=10$ cao
				[6]	
4			$\frac{3}{x-4}>1\Rightarrow 3(x-4)>(x-4)^2$	M1*	Multiply through by $(x-4)^2$
			$\Rightarrow 0>x^2-11x+28$		
			$\Rightarrow 0>(x-4)(x-7)$	M1dep*	Factorise quadratic
			$\Rightarrow 4<x<7$	B2	One each for $4<x$ and $x<7$
			OR		
			$\frac{3}{x-4}-1>0 \Rightarrow \frac{7-x}{x-4}>0$	M1*	Obtain single fraction >0
			Consideration of graph sketch or table of values/signs	M1dep*	
			$\Rightarrow 4<x<7$	B2	One each for $4<x$ and $x<7$
			OR		
			$3=x-4\Rightarrow x=7$ (each side equal)		
			$x=4$ (asymptote)		
			Critical values at $x=7$ and $x=4$	M1*	Identification of critical values at $x=7$ and $x=4$
			Consideration of graph sketch or table of values/signs	M1dep*	
			$4<x<7$	B2	One each for $4<x$ and $x<7$
			OR		
			Consider inequalities arising from both $x<4$ and $x>4$	M1*	
			Solving appropriate inequalities to their $x>7$ and $x<7$	M1dep*	
			$4<x<7$	B2	One for each $4<x$ and $x<7$, and no other solutions
				[4]	

Question			Answer	Marks	Guidance
5	(i)		$\frac{1}{2r+1}-\frac{1}{2r+3}=\frac{2r+3-(2r+1)}{(2r+1)(2r+3)}=\frac{2}{(2r+1)(2r+3)}$	M1 A1 **[2]**	Attempt at common denominator
5	(ii)		$\sum_{r=1}^{30}\frac{1}{(2r+1)(2r+3)}=\frac{1}{2}\sum_{r=1}^{30}\left[\frac{1}{2r+1}-\frac{1}{2r+3}\right]$ $=\frac{1}{2}\left[\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\ldots+\left(\frac{1}{59}-\frac{1}{61}\right)+\left(\frac{1}{61}-\frac{1}{63}\right)\right]$ $=\frac{1}{2}\left(\frac{1}{3}-\frac{1}{63}\right)=\frac{10}{63}$	M1 M1 M1 A1 A1 **[5]**	Use of (i); do not penalise missing factor of $\frac{1}{2}$ Sufficient terms to show pattern Cancelling terms Factor ½ used oe cao
6	(i)		$a_2=3\times2=6, a_3=3\times7=21$	B1 **[1]**	cao
6	(ii)		When $n = 1$, $\frac{5\times3^0-3}{2}=1$, so true for $n = 1$ Assume $a_k=\frac{5\times3^{k-1}-3}{2}$ $\Rightarrow a_{k+1}=3\left(\frac{5\times3^{k-1}-3}{2}+1\right)$ $=\frac{5\times3^k-9}{2}+3=\frac{5\times3^k-9+6}{2}$ $=\frac{5\times3^k-3}{2}=\frac{5\times3^{(k+1)-1}-3}{2}$ But this is the given result with $k + 1$ replacing k. Therefore if it is true for $n = k$ it is also true for $n = k+1$. Since it is true for $n = 1$, it is true for all positive integers.	B1 E1 M1 A1 E1 E1 **[6]**	Showing use of $a_n=\frac{5\times3^{n-1}-3}{2}$ Assuming true for $n = k$ a_{k+1}, using a_k and attempting to simplify Correct simplification to left hand expression. May be identified with a 'target' expression using $n=k+1$ Dependent on A1 and previous E1 Dependent on B1 and previous E1

Question			Answer	Marks	Guidance
7	(i)		$(-5,\ 0),\ (5,\ 0),\left(0,\ \frac{25}{24}\right)$	B1 B1 B1 **[3]**	-1 for each additional point
7	(ii)		$x=3$, $x=-4$, $x=-\frac{2}{3}$ and $y=0$	B1 B1 B1 B1 **[4]**	
7	(iii)		Some evidence of method needed e.g. substitute in 'large' values or argument involving signs Large positive x, $y \to 0^+$ Large negative x, $y \to 0^-$	M1 B1 B1 **[3]**	
7	(iv)			B1* B1dep* B1 B1 **[4]**	4 branches correct Asymptotic approaches clearly shown Vertical asymptotes correct and labelled Intercepts correct and labelled

Question			Answer	Marks	Guidance
8	(i)		$3(1+3j)^3-2(1+3j)^2+22(1+3j)+40$ $=3(-26-18j)-2(-8+6j)+22(1+3j)+40$ $=(-78+16+22+40)+(-54-12+66)j$ $=0$ So $z=1+3j$ is a root	M1 A1 A1 A1 **[4]**	Substitute $z=1+3j$ into cubic $(1+3j)^2=-8+6j$, $(1+3j)^3=-26-18j$ Simplification (correct) to show that this comes to 0 and so $z=1+3j$ is a root
8	(ii)		All cubics have 3 roots. As the coefficients are real, the complex conjugate is also a root. This leaves the third root, which must therefore be real.	E1 **[1]**	Convincing explanation
8	(iii)		$1-3j$ must also be a root Sum of roots $=-\frac{-2}{3}=\frac{2}{3}$ **OR** product of roots $=-\frac{40}{3}$ **OR** $\sum\alpha\beta=\frac{22}{3}$ $(1+3j)+(1-3j)+\alpha=\frac{2}{3}$ **OR** $(1+3j)(1-3j)\alpha=-\frac{40}{3}$ **OR** $(1-3j)(1+3j)+(1-3j)\alpha+(1+3j)\alpha=\frac{22}{3}$ $\Rightarrow \alpha=\frac{-4}{3}$ is the real root	B1 M1 A2,1,0 A1	 Attempt to use one of $\sum\alpha, \alpha\beta\gamma, \sum\alpha\beta$ Correct equation Cao
			OR $1-3j$ must also be a root $(z-1+3j)(z-1-3j)=z^2-2z+10$ $3z^3-2z^2+22z+40\equiv(z^2-2z+10)(3z+4)=0$ $\Rightarrow z=\frac{-4}{3}$ is the real root	 B1 M1 A1 A1 A1 **[5]**	 Use of factors Correct quadratic factor Correct linear factor (by inspection or division) Cao

Question			Answer	Marks	Guidance
9	(i)		$p=7\times(-4)+(-1)\times(-19)+(-1)\times(-9)=0$	E1	AG must see correct working
			$q=2\times11+1\times(-7)+k\times(2-k)$	M1	
			$\Rightarrow q=15+2k-k^2$	A1	AG Correct working
				[3]	
9	(ii)		$\mathbf{AB}=\begin{pmatrix}79&0&0\\0&79&0\\0&0&79\end{pmatrix}$	B2	-1 each error
			$\mathbf{A}^{-1}=\frac{1}{79}\begin{pmatrix}-4&-5&11\\-19&-4&-7\\-9&-31&5\end{pmatrix}$	M1	Use of **B**
				B1	$\frac{1}{79}$
				A1	Correct inverse
				[5]	
9	(iii)		$\begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{1}{79}\begin{pmatrix}-4&-5&11\\-19&-4&-7\\-9&-31&5\end{pmatrix}\begin{pmatrix}14\\-23\\9\end{pmatrix}=\begin{pmatrix}2\\-3\\8\end{pmatrix}$	M1	Attempt to pre-multiply by their $\mathbf{A}^{-1}$
			$\Rightarrow x=2,\ y=-3,\ z=8$	A1	SC A2 for x, y, z unspecified
				A1	sSC B1 for A^{-1} not used or incorrectly placed.
				A1	
				[4]	

RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4755:** Further Concepts for Advanced Mathematics

Mark Scheme for January 2013

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations

Annotation	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

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A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

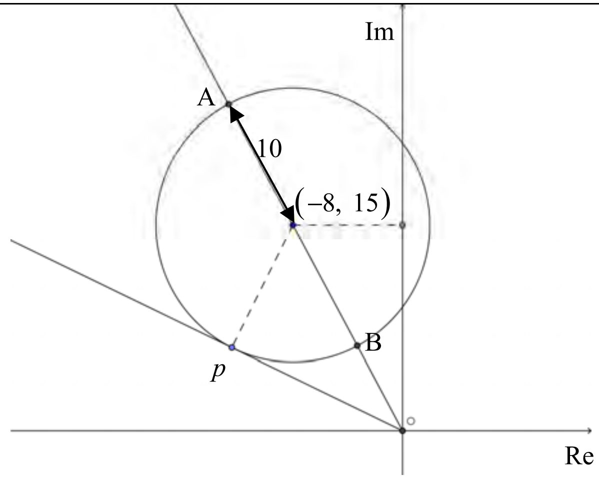
Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		A is a reflection in the line $y = x$	B1		
			B is a two way stretch, (scale) factor 2 in the x-direction and (scale) factor 3 in the y-direction	B1 B1 **[3]**	Stretch, with attempt at details. Details correct.	
1	(ii)		$\mathbf{BA} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 3 & 0 \end{pmatrix}$	M1 A1 **[2]**	Attempt to multiply in correct order	
2			$\frac{z}{z^*} = \frac{a+b\mathrm{j}}{a-b\mathrm{j}} = \frac{(a+b\mathrm{j})^2}{(a-b\mathrm{j})(a+b\mathrm{j})}$	M1	Multiply top and bottom by $a + b\mathrm{j}$ and attempt to simplify	
			$= \frac{a^2 + 2ab\mathrm{j} - b^2}{a^2 + b^2}$	M1	Using $j^2 = -1$	
			$\Rightarrow \mathrm{Re}\left(\frac{z}{z^*}\right) = \frac{a^2 - b^2}{a^2 + b^2}$ and $\mathrm{Im}\left(\frac{z}{z^*}\right) = \frac{2ab}{a^2 + b^2}$	A1 A1 **[4]**	cao correctly labelled cao correctly labelled	
3			$z = 2 - \mathrm{j}$ is also a root	B1	Stated, not just used.	
			$\alpha\beta\gamma = \frac{15}{2}$, or $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{22}{2}$, with $\alpha\beta = (2 + j)(2 - j) = 5$ used.	M1 A1	Attempt to use roots in a relationship Correct equation obtained for γ .	Allow incorrect signs
			OR $(az + b)(z - 2 + j)(z - 2 - j) = 2z^3 + pz^2 + 22z - 15$ $\Rightarrow (az + b)(z^2 - 4z + 5) = 2z^3 + pz^2 + 22z - 15$	M1 A1	Attempt use of complex factors. Correct complex factors; one pair of factors correctly multiplied	Allow incorrect signs (z+…)
			OR $2(2 + 11j) + p(3 + 4j) + 22(2 + j) - 15 = 0$	M1 A1	Substitution correct equation	Allow an incorrect sign
			Complete valid method for then obtaining the other unknown.	M1	Root relation, obtaining linear factor, equating real and imaginary parts	Signs correct
			real root $= \frac{3}{2}$, $p = -11$	A1 A1 **[6]**	FT one value	

Question			Answer	Marks	Guidance	
4	(i)		$x^2 - x + 2$ has discriminant -7, so $x^2 - x + 2 \neq 0$ and when e.g. $x = 0$, $x^2 - x + 2 > 0$ so positive for all x	E2,1,0	Discriminant < 0 shown **and** sign of $x^2 - x + 2$ or curve position discussed.	Allow complex roots found, with discussion
			OR			
			$x^2 - x + 2 = (x - \frac{1}{2})^2 + \frac{7}{4} \geq \frac{7}{4} > 0$ for all x.	E2,1,0	Completing square and minimum value discussed	
			OR using $y = x^2 - x + 2$			
			$\frac{dy}{dx} = 2x - 1 = 0$ when $x = \frac{1}{2}$ and $y = \frac{7}{4}$; $\frac{d^2y}{dx^2} = 2 > 0$	E2,1,0	Calculus, showing minimum value>0.	
			Hence y has minimum value, and $y \geq \frac{7}{4} > 0$ for all x.			
				[2]		
4	(ii)		$\frac{2x}{x^2 - x + 2} > x$			
			$\Rightarrow 2x > x^3 - x^2 + 2x$	M1	Valid attempt to eliminate fraction	Or combine to one fraction > or<0
			$\Rightarrow 0 > x^3 - x^2 \Rightarrow 0 > x^2(x-1)$	M1	Simplification and factors	In numerator
			0, 1 critical values	A1	Both, no other values given.	
			$x < 1$	A1		
			$\Rightarrow x < 0$ or $0 < x < 1$ or $x < 1, x \neq 0$	A1 **[5]**	cao	
			OR Graphical approach by sketching			
			$y = \frac{2x}{x^2 - x + 2}$ and $y = x$ or $y = \frac{2x}{x^2 - x + 2} - x$	M2,1,0	Accuracy of sketch	
			Critical values 0 and 1	A1	Both	
			$x < 1$	A1		
			$\Rightarrow x < 0$ or $0 < x < 1$ or $x < 1, x \neq 0$	A1 **[5]**	cao	

Question			Answer	Marks	Guidance	
5	(i)		$\sum_{r=1}^{100}\frac{1}{(5+3r)(2+3r)}=k\sum_{r=1}^{100}\left[\frac{1}{2+3r}-\frac{1}{5+3r}\right]$	M1		
			$=k[\left(\frac{1}{5}-\frac{1}{8}\right)+\left(\frac{1}{8}-\frac{1}{11}\right)+\ldots$	M1	Write out terms (at least first and last terms in full)	
			$+\left(\frac{1}{302}-\frac{1}{305}\right)]$	A1		
			$=k\left(\frac{1}{5}-\frac{1}{305}\right)$	M1	Cancelling inner terms	
			$=\frac{20}{305}=\frac{4}{61}$, oe	A1 [5]	cao	
	(ii)		$\frac{1}{15}$	B1 [1]		
6			When $n=1$, $(-1)^0\frac{1\times 2}{2}=1$ and $1^2=1$, so true for $n=1$	B1		
			Assume true for $n=k$ $\Rightarrow 1^2-2^2+3^2-\ldots..+(-1)^{k-1}k^2=(-1)^{k-1}\frac{k(k+1)}{2}$	E1	Assuming true result for some n.	Condone series shown incomplete
			$\Rightarrow 1^2-2^2+3^2-\ldots..+(-1)^{k-1}k^2+(-1)^{k+1-1}(k+1)^2$ $=(-1)^{k-1}\frac{k(k+1)}{2}+(-1)^{k+1-1}(k+1)^2$	M1*	Adding $(k+1)$th term to both sides.	
			$=(-1)^k\left[\frac{-k(k+1)}{2}+(k+1)^2\right]$	M1 Dep*	Attempt to factorise (at least one valid factor)	
			$=(-1)^k(k+1)\left(\frac{-k}{2}+k+1\right)$	A1	Correct factorisation Accept $(-1)^{k\pm m}$ provided expression correct.	

Question			Answer	Marks	Guidance	
			$=(-1)^k(k+1)\left(\frac{k+2}{2}\right)$	A1	Valid simplification with $(-1)^k$	
			$=(-1)^{[n-1]}\frac{n(n+1)}{2},\ n=k+1$	E1	Or target seen	
			Therefore if true for $n = k$ it is also true for $n = k+1$ Since it is true for $n = 1$, it is true for all positive integers, n.	E1 **[8]**	Dependent on A1 and previous E1 Dependent on B1 and previous E1	
7	(i)		Asymptotes $y = 0$, $x = 5, x = 8$ Crosses axes at $(4,\ 0)$, $(0, -\frac{1}{10})$ $\frac{x-4}{(x-5)(x-8)}>0 \Rightarrow x>8$ or $4<x<5$	B1 B1 B1 B1 B1 B1 **[6]**	both	
7	(ii)		$\frac{x-4}{(x-5)(x-8)}=k \Rightarrow x-4=kx^2-13kx+40k$	M1	Attempt to remove fraction and simplify	
			$\Rightarrow kx^2-(13k+1)x+40k+4=0$	A1	3 term quadratic (= 0)	
			$b^2-4ac=(13k+1)^2-4k(40k+4)$	M1	Attempt to use discriminant	
			$=9k^2+10k+1$	A1	Correct 3-term quadratic	
			Critical values –1, –1/9	A1	Roots found or factors shown	
			For no solutions to exist, $9k^2+10k+1<0$ $\Rightarrow -1<k<-\frac{1}{9}$	E1		
			No point on the graph has a y coordinate in the range $\Rightarrow -1<y<-\frac{1}{9}$	E1 **[7]**	Accept equivalent statement	

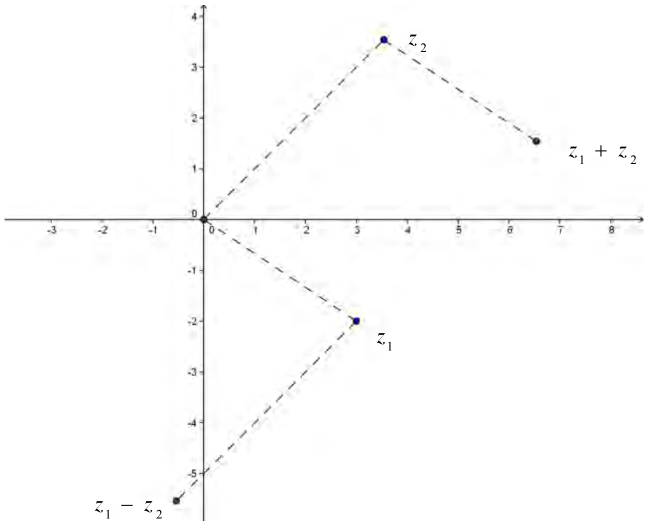
Question			Answer	Marks	Guidance	
8	(i)					The circle should be reasonably circular. The radius should be shown to be 10 by annotation as in the diagram or by other positions marked. The centre point should be indicated and correct. The region should be shown by a key or by description. Accept a "dotty" outline to a shaded interior.
			The set of points for which $\left\|z-\left(-8+15\mathrm{j}\right)\right\|<10$ is all points inside the circle, radius 10, centre $\left(-8,\ 15\right)$, excluding the points on the circumference.	B4 **[4]**	Circle, B1; radius 10, B1; centre $\left(-8,\ 15\right)$, B1;all points inside but not on circumference of the correctly placed circle, B1	Correctly placed: the "circle" must lie above the Re axis and intersect the Im axis twice as in the diagram.

Question			Answer	Marks	Guidance	
8	(ii)		Origin to centre of circle $=\sqrt{(-8)^2+15^2}=17$.	M1		Allow centre at $\pm 8 \pm 15j$ and FT
			Origin to centre of the circle $\pm$ 10	M1	Use of radius of circle	
			Point A is the point on the circle furthest from the origin. Since the radius of the circle is 10, OA = 27. Point B is the point on the circle closest to the origin. Since the radius of the circle is 10, OB=7. Hence for z in the circle $7<\|z\|<27$	E1 **[3]**	Correct explanation for both	
8	(iii)		P is the point where a line from the origin is a tangent to the circle giving the greatest argument $\theta,\ -\pi<\theta\le\pi$	B1	Correctly positioned on circle	Allow circles centred as in (ii)
			$\|p\|=\sqrt{17^2-10^2}=\sqrt{189}=13.7$ (3 s.f.)	B1	Accept $\sqrt{189}$ or $3\sqrt{21}$ or 13.7	
			$\arg p=\frac{\pi}{2}+\arcsin\frac{8}{17}+\arcsin\frac{10}{17}$	M1	Attempt to calculate the correct angle.	Correct circle only
			$=2.69$ (3 s.f.)	A1 **[4]**	cao Accept 154°	
9	(i)		$(8\times4)-(7\times5)-(12\times1)=-15$	M1	Any valid method soi	
			$\Rightarrow k=-\frac{1}{15}$	A1 **[2]**	No working or wrong working SC B1	
9	(ii)		$\begin{pmatrix}x\\y\\z\end{pmatrix}=-\frac{1}{15}\begin{pmatrix}4&2&3\\5&4&0\\1&-1&2\end{pmatrix}\begin{pmatrix}14\\-25\\3\end{pmatrix}=\begin{pmatrix}-1\\2\\-3\end{pmatrix}$	B1	Use of $\mathbf{A}^{-1}$ in correct position(s)	Condone missing k
				M1	Attempt to multiply matrices to obtain column vector	
			$x=-1,\ y=2,\ z=-3$	A2 **[4]**	–1 each error	
9	(iii)		$(1\times a)+(-8\times-4)+(-21\times2)=0\Rightarrow a=10$	M1	Attempt to multiply $\mathbf{BB}^{-1}$ matrices to find a or b soi	
			$(-7\times5)+(5\times1)+(15\times b)=0\Rightarrow b=2$	A1 **[2]**	For both	

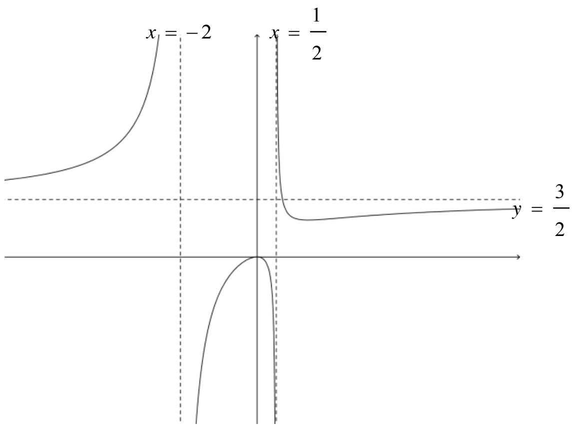
Question			Answer	Marks	Guidance	
9	(iv)		$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$	B1	By notation or explicitly	
			$= \frac{1}{3}\begin{pmatrix} 1 & 0 & 5 \\ -4 & -3 & 1 \\ 2 & 1 & 2 \end{pmatrix} \times -\frac{1}{15}\begin{pmatrix} 4 & 2 & 3 \\ 5 & 4 & 0 \\ 1 & -1 & 2 \end{pmatrix}$	M1	Attempt to multiply in correct sequence, may be implied by the answer (at least 7 elements correct)	Must include k
			$= -\frac{1}{45}\begin{pmatrix} 9 & -3 & 13 \\ -30 & -21 & -10 \\ 15 & 6 & 10 \end{pmatrix}$	A2	–1 each error FT their value of b.	
				[4]		

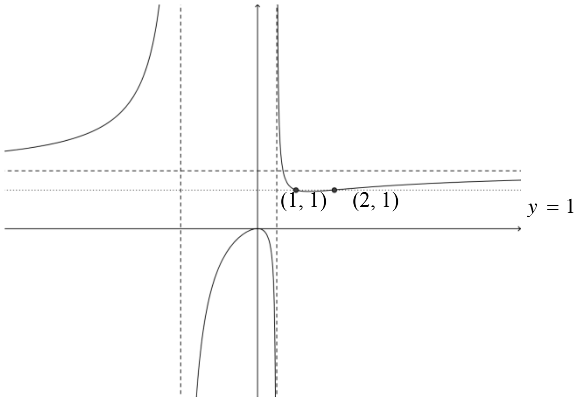
Question			Answer	Marks	Guidance	
1			$2x(x^2-5) \equiv (x-2)(Ax^2+Bx+C)+D$	M1	Evidence of comparing coefficients, or multiplying out the RHS, or substituting. May be implied by A = 2 or D = - 4	
			Comparing coefficients of x^3, $A=2$	B1		
			Comparing coefficients of x^2, $B-2A=0 \Rightarrow B=4$	B1		
			Comparing coefficients of x, $C-2B=-10 \Rightarrow C=-2$	B1		
			Comparing constants, $D-2C=0 \Rightarrow D=-4$	B1	Unidentified, max 4 marks.	
				[5]		
2			$z=\frac{3}{2}$ is a root $\Rightarrow (2z-3)$ is a factor.	M1	Use of factor theorem, accept $2z+3, z\pm\frac{3}{2}$	
			$\Rightarrow (2z-3)(z^2+bz+c)=(2z^3+9z^2+2z-30)$	M1	Attempt to factorise cubic to linear x quadratic	
			Other roots when $z^2+6z+10=0$	M1	Compare coefficients to find quadratic (or other valid complete method leading to a quadratic)	
				A1	Correct quadratic	
			$z=\frac{-6\pm\sqrt{36-40}}{2}$	M1	Use of quadratic formula (or other valid method) in their quadratic	
			$=-3+\mathrm{j}$ or $-3-\mathrm{j}$	A1	oe for both complex roots FT their 3-term quadratic provided roots are complex.	
			OR $\frac{3}{2}+\beta+\gamma=-\frac{9}{2}, \frac{3}{2}\beta\gamma=15$, **or** $\frac{3}{2}\beta+\beta\gamma+\frac{3}{2}\gamma=1$	M1	Two root relations (may use α)	
			$\beta+\gamma=-6, \beta\gamma=10$	M1	leading to sum and product of unknown roots	
			$z^2+6z+10=0$,	M1	and quadratic equation	
				A1	which is correct	
			$z=\frac{-6\pm\sqrt{36-40}}{2}$	M1	Use of quadratic formula (or other valid method) in their quadratic	
			$=-3+\mathrm{j}$ or $-3-\mathrm{j}$	A1	oe For both complex roots FT their 3-term quadratic provided roots are complex.	
			or roots must be complex, so $a\pm bj, 2a=-6, 9+b^2=10$	M1	SCM0B1 if conjugates not justified	
			$z=-3+j, z=-3-j$	A1		
				[6]		

Question			Answer	Marks	Guidance	
3	(i)		$-2-4p=0$	M1	Any valid row x column leading to p	
			$\Rightarrow\ p=-\frac{1}{2}$	B1		
				[2]		
3	(ii)		$\begin{pmatrix}x\\y\\z\end{pmatrix}=\mathbf{N}^{-1}\begin{pmatrix}-39\\5\\22\end{pmatrix}$	M1	Attempt to use $\mathbf{N}^{-1}$	Correct solution by means of simultaneous equations can earn full marks.
			$=\begin{pmatrix}1&0&2\\2&1&3\\\frac{-7}{2}&\frac{-1}{2}&-6\end{pmatrix}\begin{pmatrix}-39\\5\\22\end{pmatrix}$	M1	Attempt to multiply matrices (implied by 3x1 result)	M1 elimination of one unknown, M1 solution for one unknown
			$=\begin{pmatrix}5\\-7\\2\end{pmatrix}$	A1	One element correct	A1 one correct, A1 all correct
				A1	All 3 correct. FT their p	
				[4]		
4	(i)		$z_2=5\left(\cos\frac{\pi}{4}+\mathrm{j}\sin\frac{\pi}{4}\right)$	M1	May be implied	
			$=\frac{5\sqrt{2}}{2}+\frac{5\sqrt{2}}{2}\mathrm{j}$	A1	oe (exact numerical form)	
				[2]		

Question		Answer	Marks	Guidance
4	(ii)	$z_1 + z_2 = 3 + \frac{5\sqrt{2}}{2} + \left(-2 + \frac{5\sqrt{2}}{2}\right)\mathrm{j} = 6.54 + 1.54\mathrm{j}$ $z_1 - z_2 = 3 - \frac{5\sqrt{2}}{2} + \left(-2 - \frac{5\sqrt{2}}{2}\right)\mathrm{j} = -0.54 - 5.54\mathrm{j}$	M1	Attempt to add and subtract z_1 and their z_2 - may be implied by Argand diagram
		 	B3	For points cao, -1 each error – dotted lines not needed.
			[4]	
5		$\sum_{r=1}^{n} \frac{1}{(4r-3)(4r+1)} = \frac{1}{4}\sum_{r=1}^{n}\left[\frac{1}{4r-3} - \frac{1}{4r+1}\right]$	M1	For splitting summation into two. Allow missing 1/4
		$= \frac{1}{4}\left[\left(\frac{1}{1} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{9}\right) + \ldots + \left(\frac{1}{4n-3} - \frac{1}{4n+1}\right)\right]$	M1	Write out terms (at least first and last terms in full)
			A1	Allow missing 1/4
		$= \frac{1}{4}\left[1 - \frac{1}{4n+1}\right]$	M1	Cancelling inner terms; SC insufficient working shown above,M1M0M1A1 (allow missing 1/4)
			A1	Inclusion of 1/4 justified
		$= \frac{1}{4}\left[\frac{4n+1-1}{4n+1}\right] = \frac{n}{4n+1}$	A1	Honestly obtained (AG)
			[6]	

Question			Answer	Marks	Guidance	
6			$w = \frac{x}{3} + 1 \Rightarrow 3(w-1) = x$	M1		
			$x^3 - 5x^2 + 3x - 6 = 0$			
			$\Rightarrow (3(w-1))^3 - 5(3(w-1))^2 + 3(3(w-1)) - 6 = 0$	M1	Substituting	
				A1	Correct	
			$\Rightarrow 27(w^3 - 3w^2 + 3w - 1) - 45(w^2 - 2w + 1) + 9w - 15 = 0$			
			$\Rightarrow 27w^3 - 126w^2 + 180w - 87 = 0$	A3	FT $x = 3w + 3, 3w \pm 1$, -1 each error	
			$\Rightarrow 9w^3 - 42w^2 + 60w - 29 = 0$	A1	cao	
			OR			
			In original equation $\sum \alpha = 5, \sum \alpha\beta = 3, \alpha\beta\gamma = 6$	M1A1	all correct for A1	
			New roots $\mathrm{A}, \mathrm{B}, \Gamma$			
			$\sum \mathrm{A} = \frac{\sum \alpha}{3} + 3, \sum \mathrm{AB} = \frac{\sum \alpha\beta}{9} + \frac{2}{3}\sum \alpha + 3$			
			$\mathrm{AB}\Gamma = \frac{\alpha\beta\gamma}{27} + \frac{\sum \alpha\beta}{9} + \frac{\sum \alpha}{3} + 1$	M1 A3	At least two relations attempted Correct -1 each error FT their 5,3,6	
			Fully correct equation	A1 **[7]**	Cao, accept rational coefficients here	

Question		Answer	Marks	Guidance	
7	(i)	Vertical asymptotes at $x=-2$ and $x=\frac{1}{2}$ occur when	M1	Some evidence of valid reasoning – may be implied	
		$(bx-1)(x+a)=0$			
		$\Rightarrow a=2$ and $b=2$	A1 A1		
		Horizontal asymptote at $y=\frac{3}{2}$ so when x gets very large,	A1		
		$\frac{cx^2}{(2x-1)(x+2)}\to\frac{3}{2}\Rightarrow c=3$	[4]		
7	(ii)	Valid reasoning seen	M1	Some evidence of method needed e.g. substitute in 'large' values with result	
		Large positive x, $y\to\frac{3}{2}$ from below			
		Large negative x, $y\to\frac{3}{2}$ from above	A1	Both approaches correct (correct b,c)	
			B1 B1	LH branch correct RH branch correct Each one carefully drawn.	
			[4]		

Question			Answer	Marks	Guidance	
7	(iii)		$\frac{3x^2}{(2x-1)(x+2)}=1 \Rightarrow 3x^2=(2x-1)(x+2)$	M1	Or other valid method, to values of x (allow valid solution of inequality)	
			$\Rightarrow 0=(x-2)(x-1)$		Explicit values of x	
			$\Rightarrow x=1 \text{ or } x=2$	A1		
			From the graph $\frac{3x}{(2x-1)(x+2)}<1$			
			for $-2<x<\frac{1}{2}$ or $1<x<2$	B1 B1	FT their x=1,2 provided >1/2.	
				[4]		

Question			Answer	Marks	Guidance	
8	(i)		$\sum_{r=1}^{n}[r(r-1)-1]=\sum_{r=1}^{n}r^2-\sum_{r=1}^{n}r-n$	M1	Split into separate sums	
			$=\frac{1}{6}n(n+1)(2n+1)-\frac{1}{2}n(n+1)-n$	M1	Use of at least one standard result (ignore 3rd term)	
				A1	Correct	
			$=\frac{1}{6}n[(n+1)(2n+1)-3(n+1)-6]$	M1	Attempt to factorise. If more than two errors, M0	
			$=\frac{1}{6}n[2n^2-8]$			
			$=\frac{1}{3}n[n^2-4]$	A1	Correct with factor $\frac{1}{3}n$ oe	
			$=\frac{1}{3}n(n+2)(n-2)$		Answer given	
				[5]		
8	(ii)		When $n=1$,			
			$\sum_{r=1}^{n}[r(r-1)-1]=(1\times 0)-1=-1$			
			and $\frac{1}{3}n(n+2)(n-2)=\frac{1}{3}\times 1\times 3\times -1=-1$			
			So true for $n=1$	B1		
			Assume true for $n=k$	E1	Or "if true for n=k, then…"	
			$\sum_{r=1}^{k}[r(r-1)-1]=\frac{1}{3}k(k+2)(k-2)$			
			$\Rightarrow\sum_{r=1}^{k+1}[r(r-1)-1]=\frac{1}{3}k(k+2)(k-2)+(k+1)k-1$	M1*	Add (k + 1)th term to both sides	
			$=\frac{1}{3}k^3+k^2-\frac{4}{3}k+k-1$			
			$=\frac{1}{3}(k^3+3k^2-k-3)$			

Question			Answer	Marks	Guidance	
			$= \frac{1}{3}(k+1)(k^2+2k-3)$	M1dep *	Attempt to factorise a cubic with 4 terms	
			$= \frac{1}{3}(k+1)(k+3)(k-1)$	A1		
			$= \frac{1}{3}(k+1)((k+1)+2)((k+1)-2)$		Or $= \frac{1}{3}n(n+2)(n-2)$ where $n = k+1$; or target seen	
			But this is the given result with $n = k + 1$ replacing $n = k$. Therefore if the result is true for $n = k$, it is also true for $n = k+1$.	E1	Depends on A1 and first E1	
			Since it is true for $n = 1$, it is true for all positive integers, n.	E1 **[7]**	Depends on B1 and second E1	
9	**(i)**		**Q** represents a rotation 90 degrees clockwise about the origin	B1 B1 **[2]**	Angle, direction and centre	
9	**(ii)**		$\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ $P = (-2, 2)$	M1 A1 **[2]**	Allow both marks for P(-2, 2) www	
9	**(iii)**		$\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ y \end{pmatrix}$ l is the line $y = -x$	M1 A1 **[2]**	Or use of a minimum of two points Allow both marks for $y = -x$ www	
9	**(iv)**		$\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 6 \end{pmatrix}$ n is the line $y = 6$	M1 B1 **[2]**	Use of a general point or two different points leading to $\begin{pmatrix} -6 \\ 6 \end{pmatrix}$ y=6; if seen alone M1B1	

Question			Answer	Marks	Guidance	
9	(v)		$\det \mathbf{M} = 0 \Rightarrow \mathbf{M}$ is singular (or 'no inverse'). The transformation is many to one.	B1 E1 **[2]**	www Accept area collapses to 0, or other equivalent statements	
9	(vi)		$\mathbf{R} = \mathbf{Q}\mathbf{M} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ y \end{pmatrix}$ q is the line $y = x$	M1 A1 **[2]**	Attempt to multiply in correct order Or argue by rotation of the line $y = -x$ $y = x$ SC B1 following M0	

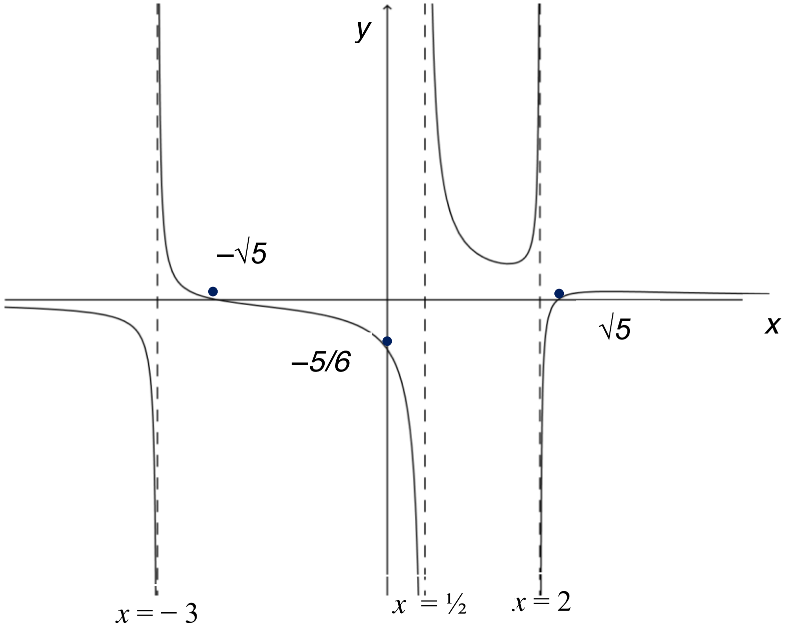
Question			Answer	Marks	Guidance
1			$\sum_{\gamma=1}^{n} r(r-2) = \sum_{\gamma=1}^{n} r^2 - 2\sum_{\gamma}^{n} r$	M1	Separate sum (may be implied)
			$= \frac{1}{6}n(n+1)(2n+1) - n(n+1)$	A1,A1	1 mark for each part oe
			$= \frac{1}{6}n(n+1)[(2n+1)-6]$	M1	$n(n+1)$(linear factor) seen
			$= \frac{1}{6}n(n+1)(2n-5)$	A1	Or $n(n+1)(2n-5)/6$ only, ie 1/6 must be a factor
				[5]	
2	(i)		$\begin{pmatrix} -3 & -2 \\ -2 & 1 \end{pmatrix}$	B1,B1 [2]	1 mark for each column. Must be a 2×2 matrix Condone lack of brackets throughout
2	(ii)		$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	B1 [1]	
2	(iii)		$\begin{pmatrix} -3 & -2 \\ 2 & -1 \end{pmatrix}$	B1,B1 [2]	1 mark for each column (no ft). Must be a 2×2 matrix

Question			Answer	Marks	Guidance
3			$z = 2 - 3\text{j}$ is also a root	B1	
			Either		
			$(z-(2+3\text{j}))(z-(2-3\text{j})) = ((z-2)+3\text{j})((z-2)-3\text{j})$	M1	Condone (z+ 2+ 3j)(z+ 2–3j)
			$= z^2 - 4z + 13$	A1	Correct quadratic
			$z^4 - 5z^3 + 15z^2 - 5z - 26 = (z^2 - 4z + 13)(z^2 - z - 2)$	M1 A1	Valid method to find the other quadratic factor. Correct quadratic
			$(z^2 - z - 2) = (z-2)(z+1)$		
			So the other roots are 2 and –1	A1,A1 **[7]**	1 mark for each root, cao
			Or		
			$2 + 3j + 2 - 3j + \gamma + \delta = 5$ oe	B1	Sum of roots with substitution of roots $2 \pm 3j$ for α and β
			$(2+3j)(2-3j)\gamma\delta = -26$ $\gamma\delta = -2$	M1	Attempt to obtain equation in $\gamma\delta$ using a root relation and $2 \pm 3j$
			$\Rightarrow 4 + \gamma + \delta = 5 \Rightarrow \gamma = 1 - \delta$		
			and $13\gamma\delta = -26 \Rightarrow \gamma\delta = -2$	M1	Eliminating γ or δ leading to a quadratic equation
			$\Rightarrow \delta(1-\delta) = -2 \Rightarrow \delta^2 - \delta - 2 = 0$	A1	Correct equation obtained
			$\Rightarrow (\delta+1)(\delta-2) = 0$		
			So the other roots are – 1 and 2.	A1,A1 **[7]**	1 mark for each, cao If 2, –1 guessed from $\gamma + \delta = 1$ and $\gamma\delta = -2$ give A1 A1 for these equations and A1A1 for the roots. **SC** factor theorem used. M1 for substitution of $z = -1$ (or 2) or division by (z + 1) (or by z – 2), A1 if zero obtained, B1 for the root stated to be – 1(or 2). For the other root, similarly but M1A1A1 Max [**7/7**] Answers only get M0M0, max [**1/7**]

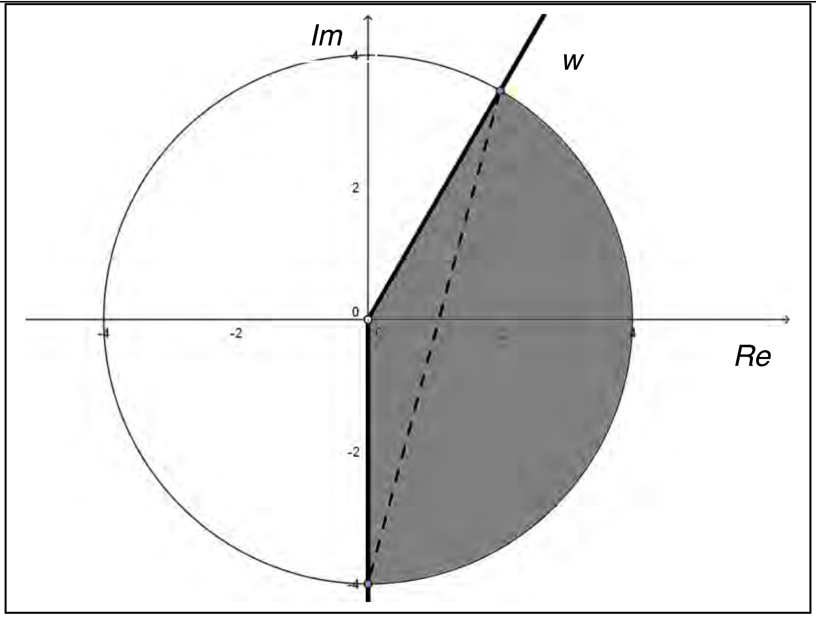
Question			Answer	Marks	Guidance
4			$\sum_{r=1}^{n}\frac{1}{(2r+3)(2r+5)}=\frac{1}{2}\sum_{r=1}^{n}\left[\frac{1}{2r+3}-\frac{1}{2r+5}\right]$	M1	Split to partial fractions. Allow missing ½
			$=\frac{1}{2}\left[\left(\frac{1}{5}-\frac{1}{7}\right)+\left(\frac{1}{7}...\right)+.....+\left(...-\frac{1}{2n+5}\right)\right]$	M1 A1	Expand to show pattern of cancelling, at least 4 fractions All correct, allow missing ½, condone r
			$=\frac{1}{2}\left[\frac{1}{5}-\frac{1}{2n+5}\right]=\frac{n}{5(2n+5)}$	M1 A1	Cancel to first minus last term must be in terms of n. oe single fraction
				[5]	

Question			Answer	Marks	Guidance
5			**Either**		
			$y=3x-1 \Rightarrow x=\frac{y+1}{3}$	M1*	Change of variable, condone $\frac{y-1}{3}, \frac{y}{3}\pm 1$.
			$\Rightarrow 3\left(\frac{y+1}{3}\right)^3-9\left(\frac{y+1}{3}\right)^2+\left(\frac{y+1}{3}\right)-1=0$	M1dep* A1	Substitute into cubic expression Correct
			Correct coefficients in cubic expression (may be fractions)	A3ft	ft their substitution (-1 each error)
			$\Rightarrow y^3-6y^2-12y-14=0$	A1	cao. Must be an equation with integer coefficients
				[7]	
			Or		
			$\alpha+\beta+\gamma=\frac{9}{3}=3$ $\alpha\beta+\alpha\gamma+\beta\gamma=\frac{1}{3}$	M1	All three root relations, condone incorrect signs
			$\alpha\beta\gamma=\frac{1}{3}$	A1	All correct
			Let new roots be k, l, m then $k+l+m=3(\alpha+\beta+\gamma)-3=6$ $kl+km+lm=9(\alpha\beta+\alpha\gamma+\beta\gamma)-6(\alpha+\beta+\gamma)+3=-12$	M1	Using (3α-1) etc in $\sum k, \sum kl, klm$, at least two attempted, and using $\sum\alpha, \sum\alpha\beta, \alpha\beta\gamma$
			$klm=27\alpha\beta\gamma-9(\alpha\beta+\beta\gamma+\beta\gamma)+3(\alpha+\beta+\gamma)-1=14$	A3ft	One each for 6, – 12, 14, ft their $3, \frac{1}{3}, \frac{1}{3}$.
			$\Rightarrow y^3-6y^2-12y-14=0$	A1	cao. Must be an equation with integer coefficients
				[7]	

Question			Answer	Marks	Guidance
6			When $n = 1$, $\frac{1}{1\times 3} = \frac{1}{3}$ and $\frac{n}{2n+1} = \frac{1}{3}$, so true for $n = 1$	B1	Condone eg "$\frac{1}{3} = \frac{1}{3}$ "
			Assume true for $n = k$	E1	Assuming true for k, (some work to follow) If in doubt look for unambiguous "if…then" at next E1 Statement of assumed result not essential but further work should be seen
			Sum of $k + 1$ terms		NB "last term = sum of terms" seen anywhere earns final E0
			$= \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}$	M1	Adding correct $(k + 1)$th term to sum for k terms
			$= \frac{k(2k+3)+1}{(2k+1)(2k+3)}$	M1	Combining their fractions
			$= \frac{2k^2+3k+1}{(2k+1)(2k+3)}$		
			$= \frac{(k+1)(2k+1)}{(2k+1)(2k+3)} = \frac{k+1}{2k+3}$	A1	Complete accurate work
			which is $\frac{n}{2n+1}$ with $n = k+1$		May be shown earlier
			Therefore if true for $n = k$ it is also true for $n = k+1$.	E1	Dependent on A1 and previous E1.
			Since it is true for $n = 1$, it is true for all positive integers, n.	E1	Dependent on B1 and previous E1 E0 if "last term"= "sum of terms " seen above
				[7]	

Question			Answer	Marks	Guidance
7	(i)		$\left(0, -\frac{5}{6}\right)$	B1	Allow for both $x=0$ and $y=-\frac{5}{6}$ seen
			$(\sqrt{5},\ 0),\ (-\sqrt{5},\ 0)$	B1	(both) Allow $(\pm\sqrt{5},0)$ or for both $y=0$ and $x=\pm\sqrt{5}$ seen
				[2]	
7	(ii)		$a=2$	B1	
			$y=0$	B1	
			$x=-3$, $x=2$	B1	Must be two equations
				[3]	
7	(iii)			B1	Two outer branches correctly placed Inner branches correctly placed Correct asymptotes and intercepts labelled
				B1	For good drawing. Dep all 3 marks above
				B1	Look for a clear maximum point on the right-hand branch, (not really shown here).
				B1	Condone turning points in $-\sqrt{5}<x<\frac{1}{2}, y<0$
				[4]	
	(iv)		$-3<x<-\sqrt{5},\ \frac{1}{2}<x<2,\ x>\sqrt{5}$	B3	One mark for each. Strict inequalities. Allow 2.24 for $\sqrt{5}$ (if B3 then – 1 if more than 3 inequalities)
				[3]	

Question			Answer	Marks	Guidance
8	(i)		$\lvert w \rvert = \sqrt{\left(2^2 + \left(2\sqrt{3}\right)^2\right)} = 4$	B1	
			$\arg w = \arctan \frac{2\sqrt{3}}{2} = \frac{\pi}{3}$	M1	
			$w = 4\left(\cos\frac{\pi}{3} + \mathrm{j}\sin\frac{\pi}{3}\right)$	A1	Accept $\left(4, \frac{\pi}{3}\right)$, 1.05 rad , 60° in place of $\frac{\pi}{3}$, or $4e^{j\frac{\pi}{3}}$
				[3]	

Question		Answer	Marks	Guidance
8	(ii)		B1	Circle, or arc of circle, centre the origin
			B1	Radius 4
			B1	Half line from origin $\frac{\pi}{4} < \text{angle} < \frac{\pi}{2}$ with positive real axis or acute angle labelled as $\pi/3$
			B1	Use of negative Im axis clearly indicated
			B1	Correct region indicated. Dependent on first 4 B marks Ignore placing of w.
			B1	w at intersection of $\frac{\pi}{3}$ line and circle (dep 1st 3 B marks)
		Maximum $\lvert z-w\rvert = \sqrt{\left(2^2+\left(4+2\sqrt{3}\right)^2\right)}$ = 7.73 (3 s.f.)	B1	Maximum $\lvert z-w\rvert$ indicated by chord on diagram oe or sight of $-4j-(2+2\sqrt{3}j)$ oe
		Or 2x 4cos15□ = $2\sqrt{6}+2\sqrt{2}$	M1	Valid attempt to calculate maximum $\lvert z-w\rvert$
			A1	allow $\sqrt{32+16\sqrt{3}}$ oe (accept 2 s.f. or better)
			[9]	

Question			Answer	Marks	Guidance
9	(i)		$\beta = (-1)(3\alpha - 1) + 5\alpha + (-1)(2\alpha + 1)$	M1	multiply second row of **A** with first column of **B**
			$= -3\alpha + 1 + 5\alpha - 2\alpha - 1 = 0$	A1	Correct
				[2]	
9	(ii)		$\gamma = (1)(3\alpha - 1) + 15 + (-1)(2\alpha + 1)$	M1	Attempt to multiply relevant row of **A** with relevant column of **B.** Condone use of **BA** instead
			$= \alpha + 13$	A1	Correct
				[2]	
9	(iii)		When $\alpha = 2,\ \gamma = 15$		
			$\mathbf{A}^{-1} = \frac{1}{15}\begin{pmatrix} 5 & -8 & -1 \\ 5 & 1 & 2 \\ 5 & -5 & 5 \end{pmatrix}$	M1	Multiplication of **B** by $\frac{1}{\text{their } \gamma}$,(γ ≠ 1) using $\alpha = 2$ in both
				A1	Correct elements in matrix and correct γ.
			$\mathbf{A}^{-1}$ does not exist when $\alpha = -13$	B1ft	ft their $\gamma = 0$. Condone "$\alpha \neq -13$"
				[3]	
9	(iv)		$\frac{1}{15}\begin{pmatrix} 5 & -8 & -1 \\ 5 & 1 & 2 \\ 5 & -5 & 5 \end{pmatrix}\begin{pmatrix} 25 \\ 11 \\ -23 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$	M1	Set-up of pre-multiplication by their 3x3 $\mathbf{A}^{-1}$, or by **B** (using $\alpha = 2$)
			$= \frac{1}{15}\begin{pmatrix} 60 \\ 90 \\ -45 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ -3 \end{pmatrix}$	B1	$(60\ \ 90\ \ -45)'$ soi need not be fully evaluated
			$\Rightarrow x = 4,\ y = 6,\ z = -3$	A3	cao A1 for each explicit identification of x, y, z in a vector or a list. (-1 unidentified) Answers only or solution by other method, M0A0
				[5]	