MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

FURTHER CONCEPTS FOR ADVANCED MATHEMATICS, FP1

Practice Paper FP1-D

Additional materials: Answer booklet/paper
Graph paper
MEI Examination formulae and tables (MF12)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You **may** use a graphical calculator in this paper.

INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that you may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (36 marks)

1 Express the complex number $z = \frac{5\mathrm{j}}{3-\mathrm{j}}$ in the form $a + b\mathrm{j}$. [3]

2 Find the inverse of the matrix $\mathbf{M} = \begin{pmatrix} 2 & 3 \\ -1 & -4 \end{pmatrix}$.

Give your answer in simplified form. [4]

3 $z^4 + z^2 + 1 = 0$ has roots α, β, γ and δ.

(i) Write down the values of $\sum\alpha$ and $\sum\alpha\beta$. [2]

(ii) Show that $\sum\alpha^2 = -2$. [2]

4 **(i)** Draw an Argand diagram showing the set of points for which $\arg z = \frac{\pi}{4}$. [2]

(ii) Convert $z = 1 + \mathrm{j}$ to modulus argument form. [2]

(iii) Determine whether $z = 1 + \mathrm{j}$ lies on the locus of part (i). [1]

5 Solve the inequality $\frac{(x-3)(x+2)}{(x+1)} > 0$ for $x \neq -1$. [6]

6 **(i)** $z_1 = 2 + \mathrm{j}$ is one of the roots of the equation $z^2 - 4z + 5 = 0$. Find the other root z_2. [1]

(ii) Show that $\frac{1}{z_1} + \frac{1}{z_2} = \frac{4}{5}$. [3]

(iii) Show also that $\mathrm{Im}(z_1{}^2 + z_2{}^2) = 0$ and $\mathrm{Re}(z_1{}^2 - z_2{}^2) = 0$. [2]

7 Prove by induction that $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$ where n represents a positive integer. [8]

Section B (36 marks)

8 **(i)** Show that $\dfrac{1}{5r-2}-\dfrac{1}{5r+3}=\dfrac{5}{(5r-2)(5r+3)}$. [2]

(ii) Hence find, in terms of n, the sum of the series $\displaystyle\sum_{r=1}^{n}\frac{1}{(5r-2)(5r+3)}$,
giving your answer in its simplest form. [7]

(iii) Check that your answer is correct when $n = 2$. [2]

9 A curve has equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)=\dfrac{x^2+3x-4}{x^2-9}$.

(i) Find two values of x for which $\mathrm{f}(x)=0$. [2]

(ii) Write down the equation of the vertical asymptotes to $y=\mathrm{f}(x)$. [2]

(iii) Show that the equation of the curve can be written in the form $y=1+\dfrac{3x+5}{x^2-9}$.
Hence write down the equation of the horizontal asymptote. [3]

(iv) Sketch the curve $y=\mathrm{f}(x)$, using all the information from parts (i) – (iii). [3]

(v) Solve the inequality $\dfrac{x^2+3x-4}{x^2-9}\geq 1$. [3]

10 ABCD is a square with vertices at (0,0), (2,1), (1,3) and (-1,2) respectively.

The matrix $\mathbf{M}=\begin{pmatrix}3 & 4\\ 4 & -3\end{pmatrix}$ transforms ABCD to A'B'C'D'.

(i) Determine the coordinates of A', B', C' and D'. [4]

(ii) Draw a diagram to show the effect on ABCD of the transformation given by **M**. [2]

(iii) Find Det **M** and show that Area of A'B'C'D' = $|\text{Det } \mathbf{M}| \times$ Area of ABCD . [2]

(iv) The Matrix **M** may be written as the product **PQ** where **Q** is $\begin{pmatrix}0.6 & 0.8\\ 0.8 & -0.6\end{pmatrix}$.
Write down the matrix **P**. [1]

(v) Describe fully the transformation represented by **P** and **Q**. [3]