MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

FURTHER CONCEPTS FOR ADVANCED MATHEMATICS, FP1

Practice Paper FP1-C

Additional materials: Answer booklet/paper
Graph paper
MEI Examination formulae and tables (MF12)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You **may** use a graphical calculator in this paper.

INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that you may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (36 marks)

1 Find the sum of the first n terms of the series
$(2\times3) + (3\times4) + (4\times5) + \ldots + (n+1)(n+2)$. [5]

2 Solve the inequality $(x+2) < \dfrac{4x}{(x-3)}$. [5]

3 **(i)** If $\alpha + \beta + \gamma = -2$, $\alpha\beta + \beta\gamma + \gamma\alpha = 9$, and $\alpha\beta\gamma = -18$, write down the cubic equation with roots α, β and γ. [1]

(ii) Use the factor theorem to identify one root of the equation [1]

(iii) Show that the other two roots are imaginary. [3]

4 You are given that $z = a + b\mathrm{j}$ where a and b are real. z^* is the conjugate of z.
Find all possible values of z if $zz^* - 2\mathrm{j}z = 7 - 4\mathrm{j}$. [6]

5 If the roots of the equation $x^3 - 9x^2 + 3x - 39 = 0$ are α, β and γ, show that the equation whose roots are $\alpha - 3$, $\beta - 3$ and $\gamma - 3$ is $x^3 - 24x - 84 = 0$ [7]

6 Given that $\mathbf{M} = \begin{pmatrix} 5 & 8 \\ -2 & -3 \end{pmatrix}$, prove by induction that, for any positive integer n,

$$\mathbf{M}^n = \begin{pmatrix} 1+4n & 8n \\ -2n & 1-4n \end{pmatrix}.$$ [8]

Section B (36 marks)

7 A curve has equation $y = \dfrac{(4x+1)(x+16)}{(x^2-4)}$.

(i) Write down the co-ordinates of the points where the curve crosses the co-ordinate axes. [2]

(ii) Show that the equation can be written as $y = 4 + \dfrac{65x+32}{(x^2-4)}$ [3]

(iii) Hence write down the equations of the 3 asymptotes of the curve. [3]

(iv) Show that when $x > 2$, $y > 4$. [2]

(v) Sketch the curve, showing clearly the behaviour of the curve for large positive and negative values of x. [3]

8 The matrix $\mathbf{M} = \begin{pmatrix} 7 & -3 \\ -4 & 6 \end{pmatrix}$ defines a transformation in the (x,y)-plane.

A triangle S, with area 5 square units, is transformed by **M** into triangle T.

(i) Find the area of triangle T. [2]

(ii) Find the matrix that transforms T into S. [2]

Triangle U is obtained by rotating triangle S through 135° anticlockwise about the origin.

(iii) Find the matrix that transforms triangle S into triangle U, leaving the entries in surd form [3]

(iv) Find the matrix that transforms triangle T into triangle U. [4]

9 **(i)** Write down the sum of the roots of the cubic equation $3z^3 - 4z^2 + 8z + 8 = 0$ [1]

You are given that $\alpha = 1 + \sqrt{3}\mathrm{j}$ is a root of the equation.

(ii) Write down another complex root, β, and hence solve the equation. [3]

(iii) Describe the locus of points in the Argand diagram representing the complex numbers z for which $|z - \alpha| = \sqrt{3}$. Sketch this locus on an Argand diagram. [4]

(iv) Find $\dfrac{\alpha}{\beta}$ in the form $a + b\mathrm{j}$ and show that the point $z = \dfrac{\alpha}{\beta}$ lies on the locus in (iii). [4]