MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

FURTHER CONCEPTS FOR ADVANCED MATHEMATICS, FP1

Practice Paper FP1-A

Additional materials: Answer booklet/paper
Graph paper
MEI Examination formulae and tables (MF12)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You **may** use a graphical calculator in this paper.

INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that you may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (36 marks)

1 Solve the equation $x^2 + 3x + 4 = 0$. [3]

2 Express $\dfrac{3+2\mathrm{j}}{4-\mathrm{j}}$ in the form $x + y\mathrm{j}$. [3]

3 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}$$

Find

(i) $\mathbf{A}^2$, [2]

(ii) $\mathbf{A}^{-1}$. [1]

4 The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 & -2 \\ 0 & 1 \end{pmatrix}$

(i) Use **A** and **B** to prove that matrix multiplication is not commutative. [2]

(ii) Use **A**, **B** and **C** to give an example in which matrix multiplication is associative. [3]

5 **(i)** Show that $\dfrac{8}{x+1} - \dfrac{3}{x-4} \equiv \dfrac{5(x-7)}{(x+1)(x-4)}$ [2]

(ii) Hence solve the inequality $\dfrac{8}{x+1} - \dfrac{3}{x-4} > 5$. [5]

6 The cubic equation $x^3 + px^2 + qx + r = 0$ has roots α, β and γ.

Find

(i) $\alpha^2 + \beta^2 + \gamma^2$. [4]

(ii) The equation which has roots $-\alpha$, $-\beta$ and $-\gamma$. [3]

7 Prove by induction that $\sum_{r=1}^{n} r^2 = \dfrac{1}{6}n(n+1)(2n+1)$. [8]

Section B (36 marks)

8 **(i)** You are given that $\dfrac{(x+2)(x-3)}{(x-1)} \equiv Ax + B + \dfrac{C}{(x-1)}, \quad x \neq 1.$

Show that $B = 0$ and find A and C. [6]

(ii) Fig. 8 shows a sketch graph of $y = \dfrac{(x+2)(x-3)}{(x-1)}$.

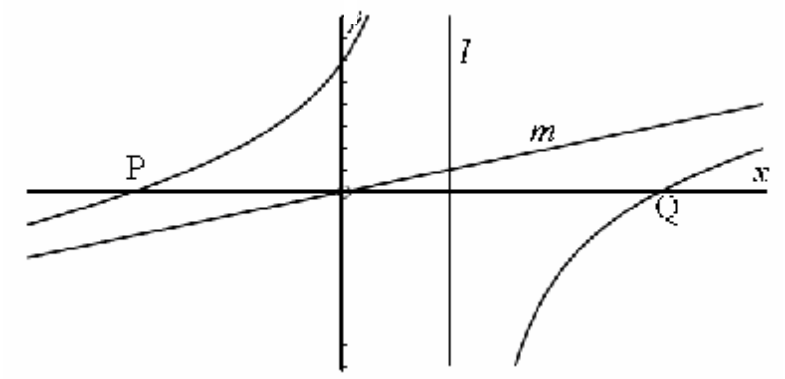

Fig. 8

Write down the coordinates of P and Q, the points where the graph cuts the x axis, and the equation of the line l. [3]

(iii) The line m has equation $y = x$. Prove that the curve does not cross the line m. [3]

9 The matrices $\mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ define transformations in the (x,y)-plane.

(i) (A) Under **P**, the point M (3, 2) is transformed to M′. Find the coordinates of M′. [1]

(B) Find $\mathbf{P}^2$. [2]

(C) Hence describe the transformation represented by **P**. [1]

(ii) (A) Under **Q**, the point M (3, 2) is transformed to M″. Find the coordinates of M″. [1]

(B) Find $\mathbf{Q}^4$. [3]

(C) Hence describe the transformation represented by **Q**. [1]

(iii) Find **PQ** and describe the transformation that it represents. [3]

10 You are given the complex numbers $z_1 = \sqrt{3} + 2\mathrm{j}$ and $z_2 = 4\left(\cos\frac{\pi}{6} + \mathrm{j}\sin\frac{\pi}{6}\right)$.

(i) Find the modulus and argument of z_1. [2]

(ii) Write z_2 in the form $a + b\mathrm{j}$ where a and b are to be given exactly. [2]

(iii) Illustrate z_1 and z_2 on an Argand diagram. [2]

(iv) Find $z_2 - z_1$ and indicate this on your Argand diagram. [2]

(v) Describe the locus of the points, z, on the Argand diagram for which $|z - z_1| = \sqrt{3}$. Sketch the locus on your diagram. [4]