**FP1 questions from old P4, P5, P6 and FP1, FP2, FP3 papers (back to June 2002)**

Please note that the following pages contain questions from past papers which were not written at an AS standard and may be less accessible than those you will find on future AS FP1 papers from Edexcel.

Mark schemes are available on a separate document, originally sent with this one.

Where a question reference is marked with an asterisk (*), it is a partial version of the original.

Where a question reference is marked with a hash (#), the question has been reworded.

**1.** Given that $z = 22 + 4\text{i}$ and $\dfrac{z}{w} = 6 - 8\text{i}$, find

(*a*) $w$ in the form $a + b\text{i}$, where $a$ and $b$ are real,

**(3)**

(*b*) the argument of $z$, in radians to 2 decimal places.

**(2)**

[P4 January 2002 Qn 1]

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**2.** (*a*) Prove that $\sum_{r=1}^{n}(r+1)(r-1) = \frac{1}{6}n\,(n-1)(2n+5)$.

**(5)**

(*b*) Deduce that $n(n - 1)(2n + 5)$ is divisible by 6 for all $n > 1$.

**(2)**

[P4 January 2002 Qn 3]

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**3.**

$$f(x) = x^3 + x - 3.$$

The equation $f(x) = 0$ has a root, $\alpha$, between 1 and 2.

(*a*) By considering $f'(x)$, show that $\alpha$ is the only real root of the equation $f(x) = 0$.

**(3)**

(*b*) Taking 1.2 as your first approximation to $\alpha$, apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to $\alpha$. Give your answer to 3 significant figures.

**(2)**

(*c*) Prove that your answer to part (*b*) gives the value of $\alpha$ correct to 3 significant figures.

**(2)**

[P4 January 2002 Qn 4]

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**4.** Given that $2 + \text{i}$ is a root of the equation

$z^2 + bz + c = 0$, where $b$ and $c$ are real constants,

(i) write down the other root of the equation,

(ii) find the value of $b$ and the value of $c$.

**(5)**

[*P4 January 2002 Qn 5]

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**5.** Prove that

$$\sum_{r=1}^{n} 6(r^2 - 1) \equiv (n-1)n(2n+5).$$

**(4)**

[P4 June 2002 Qn 1]

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**6.** Given that $z = 3 + 4\text{i}$ and $w = -1 + 7\text{i}$,

(*a*) find $|w|$.

**(1)**

The complex numbers $z$ and $w$ are represented by the points $A$ and $B$ on an Argand diagram.

(*b*) Show points $A$ and $B$ on an Argand diagram.

**(1)**

(*c*) Prove that $\triangle OAB$ is an isosceles right-angled triangle.

**(5)**

(*d*) Find the exact value of $\arg\left(\dfrac{z}{w}\right)$.

**(3)**

[P4 June 2002 Qn 5]

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**7.** The point $P\left(2p, \dfrac{2}{p}\right)$ and the point $Q\left(2q, \dfrac{2}{q}\right)$, where $p \neq -q$, lie on the rectangular hyperbola with equation $xy = 4$.

The tangents to the curve at the points $P$ and $Q$ meet at the point $R$.

Show that at the point $R$,

$$x = \frac{4pq}{p+q} \text{ and } y = \frac{4}{p+q}.$$

**(8)**

[*P5 June 2002 Qn 7]

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**8.** For $n \in \mathbb{Z}^+$ prove that

($a$) $2^{3n+2} + 5^{n+1}$ is divisible by 3,

**(9)**

($b$) $\begin{pmatrix} -2 & -1 \\ 9 & 4 \end{pmatrix}^n = \begin{pmatrix} 1-3n & -n \\ 9n & 3n+1 \end{pmatrix}$.

**(7)**

[P6 June 2002 Qn 6]

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**9.** $$f(x) = 2\sin 2x + x - 2.$$

The root $\alpha$ of the equation $f(x) = 0$ lies in the interval $[2, \pi]$.

Using the end points of this interval find, by linear interpolation, an approximation to $\alpha$.

**(4)**

[*P4 January 2003 Qn 4]

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**10.** Given that $z = 3 - 3i$ express, in the form $a + ib$, where $a$ and $b$ are real numbers,

($a$) $z^2$,

**(2)**

($b$) $\frac{1}{z}$.

**(2)**

($c$) Find the exact value of each of $|z|$, $|z^2|$ and $\left|\frac{1}{z}\right|$.

**(2)**

The complex numbers $z$, $z^2$ and $\frac{1}{z}$ are represented by the points $A$, $B$ and $C$ respectively on an Argand diagram. The real number 1 is represented by the point $D$, and $O$ is the origin.

($d$) Show the points $A$, $B$, $C$ and $D$ on an Argand diagram.

**(2)**

($e$) Prove that $\triangle OAB$ is similar to $\triangle OCD$.

**(3)**

[P4 January 2003 Qn 6]

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**11.** (*a*) Using that 3 is the real root of the cubic equation $x^3 - 27 = 0$, show that the complex roots of the cubic satisfy the quadratic equation $x^2 + 3x + 9 = 0$.

**(2)**

(*b*) Hence, or otherwise, find the three cube roots of 27, giving your answers in the form $a + \mathrm{i}b$, where $a, b \in \mathbb{R}$.

**(3)**

(*c*) Show these roots on an Argand diagram.

**(2)**

[#P4 June 2003 Qn 3]

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**12.**

$$f(x) = 3^x - x - 6.$$

(*a*) Show that $f(x) = 0$ has a root $\alpha$ between $x = 1$ and $x = 2$.

**(2)**

(*b*) Starting with the interval (1, 2), use interval bisection three times to find an interval of width 0.125 which contains $\alpha$.

**(2)**

[*P4 June 2003 Qn 4]

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**13.**

$$z = \frac{a + 3\mathrm{i}}{2 + a\mathrm{i}}, \qquad a \in \mathbb{R}.$$

(*a*) Given that $a = 4$, find $|z|$.

**(3)**

(*b*) Show that there is only one value of $a$ for which $\arg z = \frac{\pi}{4}$, and find this value.

**(6)**

[P4 June 2003 Qn 5]

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**14.**

$$f(n) = (2n + 1)7^n - 1.$$

Prove by induction that, for all positive integers $n$, $f(n)$ is divisible by 4.

**(6)**

[P6 June 2003 Qn 2]

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**15.** Given that $z = 2 - 2\text{i}$ and $w = -\sqrt{3} + \text{i}$,

(*a*) find the modulus and argument of $wz^2$.

**(6)**

(*b*) Show on an Argand diagram the points $A$, $B$ and $C$ which represent $z$, $w$ and $wz^2$ respectively, and determine the size of angle $BOC$.

**(4)**

[P4 January 2004 Qn 3]

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**16.** (*a*) Show that $$\sum_{r=1}^{n}(r+1)(r+5) = \frac{1}{6}n(n+7)(2n+7).$$

**(4)**

(*b*) Hence calculate the value of $$\sum_{r=10}^{40}(r+1)(r+5).$$

**(2)**

[P4 June 2004 Qn 1]

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**17.**
$$\text{f}(x) = 2^x + x - 4.$$

The equation $\text{f}(x) = 0$ has a root $\alpha$ in the interval [1, 2].

Use linear interpolation on the values at the end points of this interval to find an approximation to $\alpha$.

**(2)**

[*P4 June 2004 Qn 2]

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**18.** The complex number $z = a + \text{i}b$, where $a$ and $b$ are real numbers, satisfies the equation

$$z^2 + 16 - 30\text{i} = 0.$$

(*a*) Show that $ab = 15$.

**(2)**

(*b*) Write down a second equation in $a$ and $b$ and hence find the roots of

$$z^2 + 16 - 30\text{i} = 0.$$

**(4)**

[P4 June 2004 Qn 3]

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**19.** Given that $z = 1 + \sqrt{3}\text{i}$ and that $\dfrac{w}{z} = 2 + 2\text{i}$, find

(*a*) $w$ in the form $a + \text{i}b$, where $a, b \in \mathbb{R}$,
**(3)**

(*b*) the argument of $w$,
**(2)**

(*c*) the exact value for the modulus of $w$.
**(2)**

On an Argand diagram, the point $A$ represents $z$ and the point $B$ represents $w$.

(*d*) Draw the Argand diagram, showing the points $A$ and $B$.
**(2)**

(*e*) Find the distance $AB$, giving your answer as a simplified surd.
**(2)**

[P4 June 2004 Qn 5]

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**20.** Show that the normal to the rectangular hyperbola $xy = c^2$, at the point $P\left(ct, \dfrac{c}{t}\right)$, $t \neq 0$ has equation

$$y = t^2x + \frac{c}{t} - ct^3.$$

**(5)**

[*P5 June 2004 Qn 8]

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**21.** Given that $z = -2\sqrt{2} + 2\sqrt{2}\,\mathrm{i}$ and $w = 1 - \mathrm{i}\sqrt{3}$, find

($a$) $\left|\dfrac{z}{w}\right|$,

**(3)**

($b$) $\arg\left(\dfrac{z}{w}\right)$.

**(3)**

($c$) On an Argand diagram, plot points $A$, $B$, $C$ and $D$ representing the complex numbers $z$, $w$, $\left(\dfrac{z}{w}\right)$ and 4, respectively.

**(3)**

($d$) Show that $\angle AOC = \angle DOB$.

**(2)**

($e$) Find the area of triangle $AOC$.

**(2)**

[#FP1/P4 January 2005 Qn 8]

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**22.** Given that $-2$ is a root of the equation $z^3 + 6z + 20 = 0$,

($a$) find the other two roots of the equation,

**(3)**

($b$) show, on a single Argand diagram, the three points representing the roots of the equation,

**(1)**

($c$) prove that these three points are the vertices of a right-angled triangle.

**(2)**

[#FP1/P4 June 2005 Qn 2]

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**23.**

$$\mathrm{f}(x) = 1 - \mathrm{e}^x + 3\sin 2x$$

The equation $\mathrm{f}(x) = 0$ has a root $\alpha$ in the interval $1.0 < x < 1.4$.

Starting with the interval (1.0, 1.4), use interval bisection three times to find the value of $\alpha$ to one decimal place.

**(3)**

[*FP1/P4 June 2005 Qn 4]

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**24.** $$z = -4 + 6\text{i}.$$

($a$) Calculate arg $z$, giving your answer in radians to 3 decimal places.

**(2)**

The complex number $w$ is given by $w = \dfrac{A}{2-\text{i}}$, where $A$ is a positive constant. Given that $|w| = \sqrt{20}$,

($b$) find $w$ in the form $a + \text{i}b$, where $a$ and $b$ are constants,

**(4)**

($c$) calculate arg $\dfrac{w}{z}$.

**(3)**

[FP1/P4 June 2005 Qn 5]

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**25.** The point $P(ap^2, 2ap)$ lies on the parabola $M$ with equation $y^2 = 4ax$, where $a$ is a positive constant.

($a$) Show that an equation of the tangent to $M$ at $P$ is

$$py = x + ap^2.$$

**(3)**

The point $Q(16ap^2, 8ap)$ also lies on $M$.

($b$) Write down an equation of the tangent to $M$ at $Q$.

**(2)**

[*FP2/P5 June 2005 Qn 5]

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**26.** ($a$) Express $\dfrac{6x+10}{x+3}$ in the form $p + \dfrac{q}{x+3}$, where $p$ and $q$ are integers to be found.

**(1)**

The sequence of real numbers $u_1, u_2, u_3, \ldots$ is such that $u_1 = 5.2$ and $u_{n+1} = \dfrac{6u_n + 10}{u_n + 3}$.

($b$) Prove by induction that $u_n > 5$, for $n \in \mathbb{Z}^+$.

**(4)**

[FP3/P6 June 2005 Qn 1]

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**27.** Prove that $\sum_{r=1}^{n}(r-1)(r+2) = \frac{1}{3}(n-1)n(n+4)$.

**(6)**

[FP1/P4 January 2006 Qn 1]

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**28.** Given that $\frac{z+2\text{i}}{z-\lambda\text{i}} = \text{i}$, where $\lambda$ is a positive, real constant,

(*a*) show that $z = \left(\frac{\lambda}{2}+1\right) + \text{i}\left(\frac{\lambda}{2}-1\right)$.

**(5)**

Given also that arg $z$ = arctan $\frac{1}{2}$, calculate

(*b*) the value of $\lambda$,

**(3)**

(*c*) the value of $|z|^2$.

**(2)**

[FP1/P4 January 2006 Qn 3]

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**29.** The temperature $\theta$ °C of a room $t$ hours after a heating system has been turned on is given by

$$\theta = t + 26 - 20\text{e}^{-0.5t}, \quad t \geq 0.$$

The heating system switches off when $\theta = 20$. The time $t = \alpha$, when the heating system switches off, is the solution of the equation $\theta - 20 = 0$, where $\alpha$ lies in the interval [1.8, 2].

(a) Using the end points of the interval [1.8, 2], find, by linear interpolation, an approximation to $\alpha$. Give your answer to 2 decimal places.

**(4)**

(*b*) Use your answer to part (*a*) to estimate, giving your answer to the nearest minute, the time for which the heating system was on.

**(1)**

[#*FP1/P4 January 2006 Qn 5]

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**30.** The parabola $C$ has equation $y^2 = 4ax$, where $a$ is a constant.

(*a*) Show that an equation for the normal to $C$ at the point $P(ap^2, 2ap)$ is

$$y + px = 2ap + ap^3.$$

**(4)**

The normals to $C$ at the points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$, $p \neq q$, meet at the point $R$.

(*b*) Find, in terms of $a$, $p$ and $q$, the coordinates of $R$.

**(5)**

[*FP2/P5 January 2006 Qn 9]

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**31.** A transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is represented by the matrix

$$\mathbf{A} = \begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix}, \text{ where } k \text{ is a constant.}$$

Find the image under $T$ of the line with equation $y = 2x + 1$.

**(2)**

[*FP3/P6 January 2006 Qn 3]

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**32.** Prove by induction that, for $n \in \mathbb{Z}^+$, $\sum_{r=1}^{n} r2^r = 2\{1 + (n-1)2^n\}$.

**(5)**

[*FP3/P6 January 2006 Qn 5]

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**33.** The complex numbers $z$ and $w$ satisfy the simultaneous equations

$$2z + \mathrm{i}w = -1,$$

$$z - w = 3 + 3\mathrm{i}.$$

(*a*) Use algebra to find $z$, giving your answers in the form $a + \mathrm{i}b$, where $a$ and $b$ are real.

**(4)**

(*b*) Calculate arg $z$, giving your answer in radians to 2 decimal places.

**(2)**

[FP1 June 2006 Qn 1]

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**34.** $$f(x) = 0.25x - 2 + 4\sin\sqrt{x}.$$

(*a*) Show that the equation f($x$) = 0 has a root $\alpha$ between $x = 0.24$ and $x = 0.28$.

**(2)**

(*b*) Starting with the interval [0.24, 0.28], use interval bisection three times to find an interval of width 0.005 which contains $\alpha$.

**(3)**

[*FP1 June 2006 Qn 6]

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**35.** (*a*) Find the roots of the equation

$$z^2 + 2z + 17 = 0,$$

giving your answers in the form $a$ + i$b$, where $a$ and $b$ are integers.

**(3)**

(*b*) Show these roots on an Argand diagram.

**(1)**

[FP1 January 2007 Qn 1]

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**36.** The complex numbers $z_1$ and $z_2$ are given by

$$z_1 = 5 + 3\text{i},$$
$$z_1 = 1 + p\text{i},$$

where $p$ is an integer.

(*a*) Find $\frac{z_2}{z_1}$, in the form $a$ + i$b$, where $a$ and $b$ are expressed in terms of $p$.

**(3)**

Given that $\arg\left(\frac{z_2}{z_1}\right) = \frac{\pi}{4}$,

(*b*) find the value of $p$.

**(2)**

[FP1 January 2007 Qn 3]

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**37.** $$f(x) = x^3 + 8x - 19.$$

(*a*) Show that the equation $f(x) = 0$ has only one real root.
**(3)**

(*b*) Show that the real root of $f(x) = 0$ lies between 1 and 2.
**(2)**

(*c*) Obtain an approximation to the real root of $f(x) = 0$ by performing two applications of the Newton-Raphson procedure to $f(x)$, using $x = 2$ as the first approximation. Give your answer to 3 decimal places.
**(4)**

(*d*) By considering the change of sign of $f(x)$ over an appropriate interval, show that your answer to part (*c*) is accurate to 3 decimal places.
**(2)**

[FP1 June 2007 Qn 4]

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**38.** $$z = \sqrt{3} - \mathrm{i}.$$

$z^*$ is the complex conjugate of $z$.

(*a*) Show that $\dfrac{z}{z^*} = \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}\mathrm{i}$.
**(3)**

(*b*) Find the value of $\left|\dfrac{z}{z^*}\right|$.
**(2)**

(*c*) Verify, for $z = \sqrt{3} - \mathrm{i}$, that $\arg \dfrac{z}{z^*} = \arg z - \arg z^*$.
**(4)**

(*d*) Display $z$, $z^*$ and $\dfrac{z}{z^*}$ on a single Argand diagram.
**(2)**

(*e*) Find a quadratic equation with roots $z$ and $z^*$ in the form $ax^2 + bx + c = 0$, where $a$, $b$ and $c$ are real constants to be found.
**(2)**

[FP1 June 2007 Qn 6]

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**39.** The points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$, $p \neq q$, lie on the parabola $C$ with equation $y^2 = 4ax$, where $a$ is a constant.

(*a*) Show that an equation for the chord $PQ$ is $(p + q)\, y = 2(x + apq)$ .

**(3)**

The normals to $C$ at $P$ and $Q$ meet at the point $R$.

(*b*) Show that the coordinates of $R$ are $(\, a(p^2 + q^2 + pq + 2), -apq(p + q)\,)$.

**(7)**

[*FP2 June 2007 Qn 8]

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**40.** Prove by induction that, for $n \in \mathbb{Z}^+$, $\sum_{r=1}^{n}(2r-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$.

**(5)**

[FP3 June 2007 Qn 5]

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**41.** Given that

$$f(n) = 3^{4n} + 2^{4n+2},$$

(*a*) show that, for $k \in \mathbb{Z}^+$, $f(k + 1) - f(k)$ is divisible by 15,

**(4)**

(*b*) prove that, for $n \in \mathbb{Z}^+$, $f(n)$ is divisible by 5,

**(3)**

[*FP3 June 2007 Qn 6]

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**42.** Given that $x = -\frac{1}{2}$ is the real solution of the equation

$$2x^3 - 11x^2 + 14x + 10 = 0,$$

find the two complex solutions of this equation.

**(6)**

[FP1 January 2008 Qn 2]

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**43.** $$\mathrm{f}(x) = 3x^2 + x - \tan\left(\frac{x}{2}\right) - 2, \qquad -\pi < x < \pi.$$

The equation $\mathrm{f}(x) = 0$ has a root $\alpha$ in the interval [0.7, 0.8].

Use linear interpolation, on the values at the end points of this interval, to obtain an approximation to $\alpha$. Give your answer to 3 decimal places.

**(4)**

[*FP1 January 2008 Qn 4]

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**44.** $$z = -2 + \mathrm{i}.$$

(*a*) Express in the form $a + \mathrm{i}b$

(i) $\dfrac{1}{z}$

(ii) $z^2$.

**(4)**

(*b*) Show that $|z^2 - z| = 5\sqrt{2}$.

**(2)**

(*c*) Find arg $(z^2 - z)$.

**(2)**

(*d*) Display $z$ and $z^2 - z$ on a single Argand diagram.

**(2)**

[FP1 January 2008 Qn 6]

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**45.** (*a*) Write down the value of the real root of the equation

$$x^3 - 64 = 0.$$

**(1)**

(*b*) Find the complex roots of $x^3 - 64 = 0$, giving your answers in the form $a + \mathrm{i}b$, where $a$ and $b$ are real.

**(4)**

(*c*) Show the three roots of $x^3 - 64 = 0$ on an Argand diagram.

**(2)**

[FP1 June 2008 Qn 1]

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**46.** The complex number $z$ is defined by

$$z = \frac{a + 2\mathrm{i}}{a - \mathrm{i}}, \qquad a \in \mathbb{R}, \quad a > 0\,.$$

Given that the real part of $z$ is $\frac{1}{2}$, find

(*a*) the value of $a$,

**(4)**

(*b*) the argument of $z$, giving your answer in radians to 2 decimal places.

**(3)**

[FP1 June 2008 Qn 3]

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**47.**

$$\mathbf{A} = \begin{pmatrix} k & -2 \\ 1-k & k \end{pmatrix}, \text{ where } k \text{ is constant.}$$

A transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is represented by the matrix **A**.

(*a*) Find the value of $k$ for which the line $y = 2x$ is mapped onto itself under $T$.

**(3)**

(*b*) Show that **A** is non-singular for all values of $k$.

**(3)**

(*c*) Find $\mathbf{A}^{-1}$ in terms of $k$.

**(2)**

A point $P$ is mapped onto a point $Q$ under $T$.

The point $Q$ has position vector $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ relative to an origin $O$.

Given that $k = 3$,

(d) find the position vector of $P$.

**(3)**

[FP3 June 2008 Qn 5]

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