Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number | Candidate Number

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Monday 14 May 2018 – Afternoon
Time: 1 hour 30 minutes

Paper Reference
6667/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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Pearson

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1. $$f(z) = 2z^3 - 4z^2 + 15z - 13$$

Given that $f(z) \equiv (z - 1)(2z^2 + az + b)$, where a and b are real constants,

(a) find the value of a and the value of b.

(2)

(b) Hence use algebra to find the three roots of the equation $f(z) = 0$

(4)

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Question 1 continued

Q1

(Total 6 marks)

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2. $$f(x) = \frac{3}{2}x^2 + \frac{4}{3x} + 2x - 5, \qquad x < 0$$

The equation $f(x) = 0$ has a single root α.

(a) Show that α lies in the interval $[-3, -2.5]$

(2)

(b) Taking -3 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places.

(5)

(c) Use linear interpolation once on the interval $[-3, -2.5]$ to find another approximation to α, giving your answer to 3 decimal places.

(3)

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Question 2 continued

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Question 2 continued

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Question 2 continued

(Total 10 marks)

Q2

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3. (i) Given that

$$\mathbf{A} = \begin{pmatrix} -2 & 3 \\ 1 & 1 \end{pmatrix}, \quad \mathbf{AB} = \begin{pmatrix} -1 & 5 & 12 \\ 3 & -5 & -1 \end{pmatrix}$$

(a) find $\mathbf{A}^{-1}$

(2)

(b) Hence, or otherwise, find the matrix **B**, giving your answer in its simplest form.

(3)

(ii) Given that

$$\mathbf{C} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(a) describe fully the single geometrical transformation represented by the matrix **C**.

(2)

(b) Hence find the matrix $\mathbf{C}^{39}$

(2)

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Question 3 continued

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Question 3 continued

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Question 3 continued

(Total 9 marks)

Q3

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4. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n}\left(r^2 - r - 8\right) = \frac{1}{3}n(n-a)(n+a)$$

where a is a positive integer to be determined.

(4)

(b) Hence, or otherwise, state the positive value of n that satisfies

$$\sum_{r=1}^{n}\left(r^2 - r - 8\right) = 0$$

(1)

Given that

$$\sum_{r=3}^{17}\left(kr^3 + r^2 - r - 8\right) = 6710 \qquad \text{where } k \text{ is a constant}$$

(c) find the exact value of k.

(4)

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Question 4 continued

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Question 4 continued

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Question 4 continued

(Total 9 marks)

Q4

P 5 1 5 6 7 A 0 1 5 3 6

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5. The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant.

Given that $P\left(ct, \frac{c}{t}\right)$, $t \neq 0$, is a general point on H,

(a) use calculus to show that the equation of the tangent to H at P can be written as

$$t^2y + x = 2ct$$

(4)

The points A and B lie on H.

The tangent to H at A and the tangent to H at B meet at the point $\left(-\frac{8c}{5}, \frac{3c}{5}\right)$.

Given that the x coordinate of A is positive,

(b) find, in terms of c, the coordinates of A and the coordinates of B.

(5)

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Question 5 continued

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Question 5 continued

Q5

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6. $$\mathbf{M} = \begin{pmatrix} 8 & -1 \\ -4 & 2 \end{pmatrix}$$

(a) Find the value of det **M**

(1)

The triangle T has vertices at the points (4, 1), (6, k) and (12, 1), where k is a constant.

The triangle T is transformed onto the triangle T' by the transformation represented by the matrix **M**.

Given that the area of triangle T' is 216 square units,

(b) find the possible values of k.

(5)

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Question 6 continued

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Question 6 continued

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Question 6 continued

(Total 6 marks)

Q6

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7. The parabola C has equation $y^2 = 4ax$, where a is a positive constant. The point S is the focus of C.

The straight line l passes through the point S and meets the directrix of C at the point D.

Given that the y coordinate of D is $\dfrac{24a}{5}$,

(a) show that an equation of the line l is

$$12x + 5y = 12a$$

(2)

The point $P(ak^2, 2ak)$, where k is a positive constant, lies on the parabola C.

Given that the line segment SP is perpendicular to l,

(b) find, in terms of a, the coordinates of the point P.

(6)

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 8 marks)

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8. Prove by induction that

$$f(n) = 2^{n+2} + 3^{2n+1}$$

is divisible by 7 for all positive integers n.

(6)

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Question 8 continued

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Question 8 continued

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Question 8 continued

(Total 6 marks)

Q8

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9. (i) Given that

$$\frac{3w+7}{5} = \frac{p-4\mathrm{i}}{3-\mathrm{i}} \qquad \text{where } p \text{ is a real constant}$$

(a) express w in the form $a + b\mathrm{i}$, where a and b are real constants. Give your answer in its simplest form in terms of p.

(5)

Given that $\arg w = -\frac{\pi}{2}$

(b) find the value of p.

(1)

(ii) Given that

$$(z + 1 - 2\mathrm{i})^* = 4\mathrm{i}z$$

find z, giving your answer in the form $z = x + \mathrm{i}y$, where x and y are real constants.

(6)

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Question 9 continued

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Question 9 continued

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Question 9 continued

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Question 9 continued

Q9

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

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