Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	7	/	0	1	Signature	

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Friday 30 January 2009 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
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6	
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9	
10	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Orange)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions. Write your answers in the spaces provided in this question paper.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 10 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. $$f(x) = 2x^3 - 8x^2 + 7x - 3$$

Given that $x = 3$ is a solution of the equation $f(x) = 0$, solve $f(x) = 0$ completely.

(5)

Leave blank

2. (a) Show, using the formulae for $\sum r$ and $\sum r^2$, that

$$\sum_{r=1}^{n}(6r^2+4r-1)=n(n+2)(2n+1)$$

(5)

(b) Hence, or otherwise, find the value of $\sum_{r=11}^{20}(6r^2+4r-1)$.

(2)

PMT

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3. The rectangular hyperbola, H, has parametric equations $x = 5t$, $y = \dfrac{5}{t}$, $t \neq 0$.

(a) Write the cartesian equation of H in the form $xy = c^2$.

(1)

Points A and B on the hyperbola have parameters $t = 1$ and $t = 5$ respectively.

(b) Find the coordinates of the mid-point of AB.

(3)

PMT

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4. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$$

(5)

Leave blank

5.

$$f(x) = 3\sqrt{x} + \frac{18}{\sqrt{x}} - 20$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [1.1, 1.2].

(2)

(b) Find $f'(x)$.

(3)

(c) Using $x_0 = 1.1$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 significant figures.

(4)

Leave blank

6. A series of positive integers $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 6 \text{ and } u_{n+1} = 6u_n - 5, \quad \text{for } n \geqslant 1.$$

Prove by induction that $u_n = 5 \times 6^{n-1} + 1$, for $n \geqslant 1$.

(5)

PMT

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7. Given that $\mathbf{X} = \begin{pmatrix} 2 & a \\ -1 & -1 \end{pmatrix}$, where a is a constant, and $a \neq 2$,

(a) find $\mathbf{X}^{-1}$ in terms of a.

(3)

Given that $\mathbf{X} + \mathbf{X}^{-1} = \mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix,

(b) find the value of a.

(3)

Leave blank

8. A parabola has equation $y^2 = 4ax$, $a > 0$. The point Q $(aq^2, 2aq)$ lies on the parabola.

(a) Show that an equation of the tangent to the parabola at Q is

$$yq = x + aq^2.$$

(4)

This tangent meets the y-axis at the point R.

(b) Find an equation of the line l which passes through R and is perpendicular to the tangent at Q.

(3)

(c) Show that l passes through the focus of the parabola.

(1)

(d) Find the coordinates of the point where l meets the directrix of the parabola.

(2)

Leave blank

9. Given that $z_1 = 3 + 2\text{i}$ and $z_2 = \dfrac{12 - 5\text{i}}{z_1}$,

(a) find z_2 in the form $a + \text{i}b$, where a and b are real.
(2)

(b) Show on an Argand diagram the point P representing z_1 and the point Q representing z_2.
(2)

(c) Given that O is the origin, show that $\angle POQ = \dfrac{\pi}{2}$.
(2)

The circle passing through the points O, P and Q has centre C. Find

(d) the complex number represented by C,
(2)

(e) the exact value of the radius of the circle.
(2)

Leave blank

10. $\mathbf{A} = \begin{pmatrix} 3\sqrt{2} & 0 \\ 0 & 3\sqrt{2} \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

(a) Describe fully the transformations described by each of the matrices **A**, **B** and **C**.
(4)

It is given that the matrix **D** = **CA**, and that the matrix **E** = **DB**.

(b) Find **D**.
(2)

(c) Show that $\mathbf{E} = \begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}$.
(1)

The triangle *ORS* has vertices at the points with coordinates (0, 0), (–15, 15) and (4, 21). This triangle is transformed onto the triangle *OR′S′* by the transformation described by **E**.

(d) Find the coordinates of the vertices of triangle *OR′S′*.
(4)

(e) Find the area of triangle *OR′S′* and deduce the area of triangle *ORS*.
(3)