Edexcel Maths FP1

Past Paper Pack

2009-2014

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 7 / 0 1	Signature	

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Friday 30 January 2009 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Orange)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Examiner's use only

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Question Number	Leave Blank
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Instructions to Candidates

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Check that you have the correct question paper.
Answer ALL the questions. Write your answers in the spaces provided in this question paper.
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Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 10 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. $$f(x) = 2x^3 - 8x^2 + 7x - 3$$

Given that $x = 3$ is a solution of the equation $f(x) = 0$, solve $f(x) = 0$ completely.

(5)

Leave blank

2. (a) Show, using the formulae for $\sum r$ and $\sum r^2$, that

$$\sum_{r=1}^{n}(6r^2+4r-1)=n(n+2)(2n+1)$$

(5)

(b) Hence, or otherwise, find the value of $\sum_{r=11}^{20}(6r^2+4r-1)$.

(2)

Leave blank

3. The rectangular hyperbola, H, has parametric equations $x = 5t$, $y = \frac{5}{t}$, $t \neq 0$.

(a) Write the cartesian equation of H in the form $xy = c^2$.

(1)

Points A and B on the hyperbola have parameters $t = 1$ and $t = 5$ respectively.

(b) Find the coordinates of the mid-point of AB.

(3)

Leave blank

4. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$$

(5)

Leave blank

5. $$f(x) = 3\sqrt{x} + \frac{18}{\sqrt{x}} - 20$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [1.1, 1.2].

(2)

(b) Find $f'(x)$.

(3)

(c) Using $x_0 = 1.1$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 significant figures.

(4)

Leave blank

6. A series of positive integers $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 6 \text{ and } u_{n+1} = 6u_n - 5, \quad \text{for } n \geqslant 1.$$

Prove by induction that $u_n = 5 \times 6^{n-1} + 1$, for $n \geqslant 1$.

(5)

Leave blank

7. Given that $\mathbf{X} = \begin{pmatrix} 2 & a \\ -1 & -1 \end{pmatrix}$, where a is a constant, and $a \neq 2$,

(a) find $\mathbf{X}^{-1}$ in terms of a.

(3)

Given that $\mathbf{X} + \mathbf{X}^{-1} = \mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix,

(b) find the value of a.

(3)

Leave blank

8. A parabola has equation $y^2 = 4ax$, $a > 0$. The point Q $(aq^2, 2aq)$ lies on the parabola.

(a) Show that an equation of the tangent to the parabola at Q is

$$yq = x + aq^2.$$

(4)

This tangent meets the y-axis at the point R.

(b) Find an equation of the line l which passes through R and is perpendicular to the tangent at Q.

(3)

(c) Show that l passes through the focus of the parabola.

(1)

(d) Find the coordinates of the point where l meets the directrix of the parabola.

(2)

Leave blank

9. Given that $z_1 = 3 + 2\text{i}$ and $z_2 = \dfrac{12 - 5\text{i}}{z_1}$,

(a) find z_2 in the form $a + \text{i}b$, where a and b are real.
(2)

(b) Show on an Argand diagram the point P representing z_1 and the point Q representing z_2.
(2)

(c) Given that O is the origin, show that $\angle POQ = \dfrac{\pi}{2}$.
(2)

The circle passing through the points O, P and Q has centre C. Find

(d) the complex number represented by C,
(2)

(e) the exact value of the radius of the circle.
(2)

Leave blank

10. $\mathbf{A} = \begin{pmatrix} 3\sqrt{2} & 0 \\ 0 & 3\sqrt{2} \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

(a) Describe fully the transformations described by each of the matrices **A**, **B** and **C**.
(4)

It is given that the matrix **D** = **CA**, and that the matrix **E** = **DB**.

(b) Find **D**.
(2)

(c) Show that $\mathbf{E} = \begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}$.
(1)

The triangle *ORS* has vertices at the points with coordinates (0, 0), (–15, 15) and (4, 21). This triangle is transformed onto the triangle *OR′S′* by the transformation described by **E**.

(d) Find the coordinates of the vertices of triangle *OR′S′*.
(4)

(e) Find the area of triangle *OR′S′* and deduce the area of triangle *ORS*.
(3)

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Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Wednesday 17 June 2009 – Morning

Time: 1 hour 30 minutes

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1. The complex numbers z_1 and z_2 are given by

$$z_1 = 2 - \mathrm{i} \quad \text{and} \quad z_2 = -8 + 9\mathrm{i}$$

(a) Show z_1 and z_2 on a single Argand diagram.
(1)

Find, showing your working,

(b) the value of $|z_1|$,
(2)

(c) the value of arg z_1, giving your answer in radians to 2 decimal places,
(2)

(d) $\frac{z_2}{z_1}$ in the form $a + b\mathrm{i}$, where a and b are real.
(3)

Leave blank

2. (a) Using the formulae for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$, show that

$$\sum_{r=1}^{n} r(r+1)(r+3) = \frac{1}{12}n(n+1)(n+2)(3n+k),$$

where k is a constant to be found.

(7)

(b) Hence evaluate $\sum_{r=21}^{40} r(r+1)(r+3)$.

(2)

Leave blank

3. $$f(x) = (x^2 + 4)(x^2 + 8x + 25)$$

(a) Find the four roots of $f(x) = 0$.

(5)

(b) Find the sum of these four roots.

(2)

Leave blank

4. Given that α is the only real root of the equation

$$x^3 - x^2 - 6 = 0$$

(a) show that $2.2 < \alpha < 2.3$

(2)

(b) Taking 2.2 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x) = x^3 - x^2 - 6$ to obtain a second approximation to α, giving your answer to 3 decimal places.

(5)

(c) Use linear interpolation once on the interval [2.2, 2.3] to find another approximation to α, giving your answer to 3 decimal places.

(3)

Leave blank

5. $\mathbf{R} = \begin{pmatrix} a & 2 \\ a & b \end{pmatrix}$, where a and b are constants and $a > 0$.

(a) Find $\mathbf{R}^2$ in terms of a and b.

(3)

Given that $\mathbf{R}^2$ represents an enlargement with centre (0, 0) and scale factor 15,

(b) find the value of a and the value of b.

(5)

Leave blank

6. The parabola C has equation $y^2 = 16x$.

(a) Verify that the point $P(4t^2, 8t)$ is a general point on C.

(1)

(b) Write down the coordinates of the focus S of C.

(1)

(c) Show that the normal to C at P has equation

$$y + tx = 8t + 4t^3$$

(5)

The normal to C at P meets the x-axis at the point N.

(d) Find the area of triangle PSN in terms of t, giving your answer in its simplest form.

(4)

Leave blank

7. $$\mathbf{A} = \begin{pmatrix} a & -2 \\ -1 & 4 \end{pmatrix}, \text{ where } a \text{ is a constant.}$$

(a) Find the value of a for which the matrix **A** is singular.

(2)

$$\mathbf{B} = \begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}$$

(b) Find $\mathbf{B}^{-1}$.

(3)

The transformation represented by **B** maps the point P onto the point Q.

Given that Q has coordinates $(k - 6, 3k + 12)$, where k is a constant,

(c) show that P lies on the line with equation $y = x + 3$.

(3)

Leave blank

8. Prove by induction that, for $n \in \mathbb{Z}^+$,

(a) $f(n) = 5^n + 8n + 3$ is divisible by 4,

(7)

(b) $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^n = \begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix}$

(7)

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 7 / 0 1	Signature	

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Monday 1 February 2010 – Afternoon

Time: 1 hour 30 minutes

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Team Leader's use only

Question Number	Leave Blank
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Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

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1. The complex numbers z_1 and z_2 are given by

$$z_1 = 2 + 8\text{i} \quad \text{and} \quad z_2 = 1 - \text{i}$$

Find, showing your working,

(a) $\frac{z_1}{z_2}$ in the form $a + b\text{i}$, where a and b are real,

(3)

(b) the value of $\left|\frac{z_1}{z_2}\right|$,

(2)

(c) the value of $\arg \frac{z_1}{z_2}$, giving your answer in radians to 2 decimal places.

(2)

Leave blank

2.
$$f(x) = 3x^2 - \frac{11}{x^2}$$

(a) Write down, to 3 decimal places, the value of f(1.3) and the value of f(1.4).
(1)

The equation $f(x) = 0$ has a root α between 1.3 and 1.4

(b) Starting with the interval [1.3, 1.4], use interval bisection to find an interval of width 0.025 which contains α.
(3)

(c) Taking 1.4 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to α, giving your answer to 3 decimal places.
(5)

Leave blank

3. A sequence of numbers is defined by

$$u_1 = 2,$$
$$u_{n+1} = 5u_n - 4, \qquad n \geqslant 1.$$

Prove by induction that, for $n \in \mathbb{Z}^+$, $u_n = 5^{n-1} + 1$.

(4)

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4.

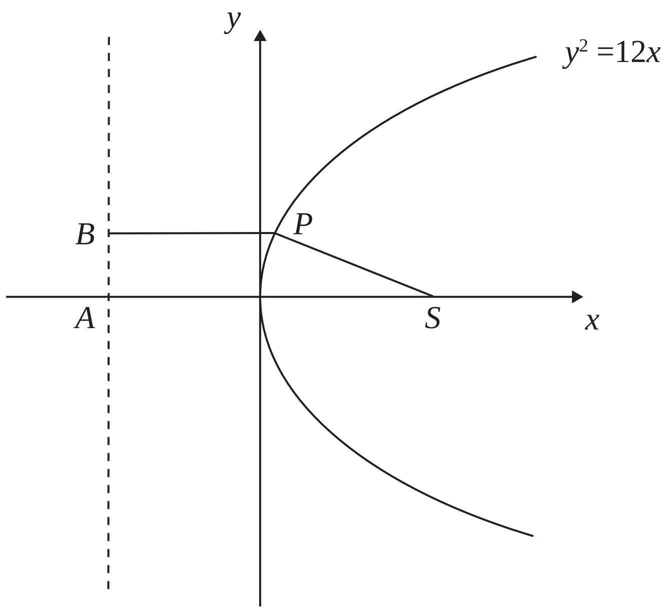

Figure 1

Figure 1 shows a sketch of part of the parabola with equation $y^2 = 12x$.

The point P on the parabola has x-coordinate $\frac{1}{3}$.

The point S is the focus of the parabola.

(a) Write down the coordinates of S.

(1)

The points A and B lie on the directrix of the parabola.
The point A is on the x-axis and the y-coordinate of B is positive.

Given that $ABPS$ is a trapezium,

(b) calculate the perimeter of $ABPS$.

(5)

Leave blank

5. $$\mathbf{A} = \begin{pmatrix} a & -5 \\ 2 & a+4 \end{pmatrix}, \text{ where } a \text{ is real.}$$

(a) Find det **A** in terms of a.

(2)

(b) Show that the matrix **A** is non-singular for all values of a.

(3)

Given that $a = 0$,

(c) find $\mathbf{A}^{-1}$.

(3)

Leave blank

6. Given that 2 and 5 + 2i are roots of the equation

$$x^3 - 12x^2 + cx + d = 0, \qquad c, d \in \mathbb{R},$$

(a) write down the other complex root of the equation.

(1)

(b) Find the value of c and the value of d.

(5)

(c) Show the three roots of this equation on a single Argand diagram.

(2)

7. The rectangular hyperbola H has equation $xy = c^2$, where c is a constant.

The point $P\left(ct, \frac{c}{t}\right)$ is a general point on H.

(a) Show that the tangent to H at P has equation

$$t^2 y + x = 2ct$$

(4)

The tangents to H at the points A and B meet at the point $(15c, -c)$.

(b) Find, in terms of c, the coordinates of A and B.

(5)

Leave blank

8. (a) Prove by induction that, for any positive integer n,

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

(5)

(b) Using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$, show that

$$\sum_{r=1}^{n}(r^3 + 3r + 2) = \frac{1}{4}n(n+2)(n^2+7)$$

(5)

(c) Hence evaluate $\sum_{r=15}^{25}(r^3 + 3r + 2)$

(2)

Leave blank

9.

$$\mathbf{M} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(a) Describe fully the geometrical transformation represented by the matrix **M**.

(2)

The transformation represented by **M** maps the point *A* with coordinates (p, q) onto the point *B* with coordinates $(3\sqrt{2}, 4\sqrt{2})$.

(b) Find the value of *p* and the value of *q*.

(4)

(c) Find, in its simplest surd form, the length *OA*, where *O* is the origin.

(2)

(d) Find $\mathbf{M}^2$.

(2)

The point *B* is mapped onto the point *C* by the transformation represented by $\mathbf{M}^2$.

(e) Find the coordinates of *C*.

(2)

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Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Tuesday 22 June 2010 – Afternoon

Time: 1 hour 30 minutes

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Question Number	Leave Blank
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Mathematical Formulae (Pink)	Nil

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1. $$z = 2 - 3\text{i}$$

(a) Show that $z^2 = -5 - 12\text{i}$.

(2)

Find, showing your working,

(b) the value of $|z^2|$,

(2)

(c) the value of $\arg(z^2)$, giving your answer in radians to 2 decimal places.

(2)

(d) Show z and z^2 on a single Argand diagram.

(1)

Leave blank

2. $$\mathbf{M} = \begin{pmatrix} 2a & 3 \\ 6 & a \end{pmatrix}, \text{ where } a \text{ is a real constant.}$$

(a) Given that $a = 2$, find $\mathbf{M}^{-1}$.

(3)

(b) Find the values of a for which $\mathbf{M}$ is singular.

(2)

Leave blank

3. $$f(x) = x^3 - \frac{7}{x} + 2, \quad x > 0$$

(a) Show that $f(x) = 0$ has a root α between 1.4 and 1.5

(2)

(b) Starting with the interval $[1.4, 1.5]$, use interval bisection twice to find an interval of width 0.025 that contains α.

(3)

(c) Taking 1.45 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x) = x^3 - \frac{7}{x} + 2$ to obtain a second approximation to α, giving your answer to 3 decimal places.

(5)

4. $$f(x) = x^3 + x^2 + 44x + 150$$

Given that $f(x) = (x+3)(x^2 + ax + b)$, where a and b are real constants,

(a) find the value of a and the value of b.

(2)

(b) Find the three roots of $f(x) = 0$.

(4)

(c) Find the sum of the three roots of $f(x) = 0$.

(1)

Leave blank

5. The parabola C has equation $y^2 = 20x$.

(a) Verify that the point $P(5t^2, 10t)$ is a general point on C.

(1)

The point A on C has parameter $t = 4$.
The line l passes through A and also passes through the focus of C.

(b) Find the gradient of l.

(4)

Leave blank

6. Write down the 2 × 2 matrix that represents

(a) an enlargement with centre $(0, 0)$ and scale factor 8,

(1)

(b) a reflection in the *x*-axis.

(1)

Hence, or otherwise,

(c) find the matrix **T** that represents an enlargement with centre $(0, 0)$ and scale factor 8, followed by a reflection in the *x*-axis.

(2)

$$\mathbf{A} = \begin{pmatrix} 6 & 1 \\ 4 & 2 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} k & 1 \\ c & -6 \end{pmatrix}, \text{ where } k \text{ and } c \text{ are constants.}$$

(d) Find **AB**.

(3)

Given that **AB** represents the same transformation as **T**,

(e) find the value of *k* and the value of *c*.

(2)

Leave blank

7.

$$f(n) = 2^n + 6^n$$

(a) Show that $f(k+1) = 6f(k) - 4(2^k)$.

(3)

(b) Hence, or otherwise, prove by induction that, for $n \in \mathbb{Z}^+$, $f(n)$ is divisible by 8.

(4)

Leave blank

8. The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant.

The point A on H has x-coordinate $3c$.

(a) Write down the y-coordinate of A.

(1)

(b) Show that an equation of the normal to H at A is

$$3y = 27x - 80c$$

(5)

The normal to H at A meets H again at the point B.

(c) Find, in terms of c, the coordinates of B.

(5)

Leave blank

9. (a) Prove by induction that

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

(6)

Using the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$,

(b) show that

$$\sum_{r=1}^{n}(r+2)(r+3) = \frac{1}{3}n(n^2+an+b),$$

where a and b are integers to be found.

(5)

(c) Hence show that

$$\sum_{r=n+1}^{2n}(r+2)(r+3) = \frac{1}{3}n(7n^2+27n+26)$$

(3)

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Monday 31 January 2011 – Afternoon

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1. $$z = 5 - 3\text{i}, \quad w = 2 + 2\text{i}$$

Express in the form $a + b\text{i}$, where a and b are real constants,

(a) z^2,

(2)

(b) $\dfrac{z}{w}$.

(3)

Q1

(Total 5 marks)

Leave blank

2.

$$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$$

(a) Find **AB**.

(3)

Given that

$$\mathbf{C} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

(b) describe fully the geometrical transformation represented by **C**,

(2)

(c) write down $\mathbf{C}^{100}$.

(1)

Q2

(Total 6 marks)

Leave blank

3.

$$f(x) = 5x^2 - 4x^{\frac{3}{2}} - 6, \quad x \geqslant 0$$

The root α of the equation $f(x) = 0$ lies in the interval $[1.6, 1.8]$.

(a) Use linear interpolation once on the interval $[1.6, 1.8]$ to find an approximation to α. Give your answer to 3 decimal places.

(4)

(b) Differentiate $f(x)$ to find $f'(x)$.

(2)

(c) Taking 1.7 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places.

(4)

Leave blank

4. Given that 2 – 4i is a root of the equation

$$z^2 + pz + q = 0,$$

where p and q are real constants,

(a) write down the other root of the equation,

(1)

(b) find the value of p and the value of q.

(3)

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5. (a) Use the results for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$, to prove that

$$\sum_{r=1}^{n} r(r+1)(r+5) = \frac{1}{4}n(n+1)(n+2)(n+7)$$

for all positive integers n.

(5)

(b) Hence, or otherwise, find the value of

$$\sum_{r=20}^{50} r(r+1)(r+5)$$

(2)

Leave blank

6.

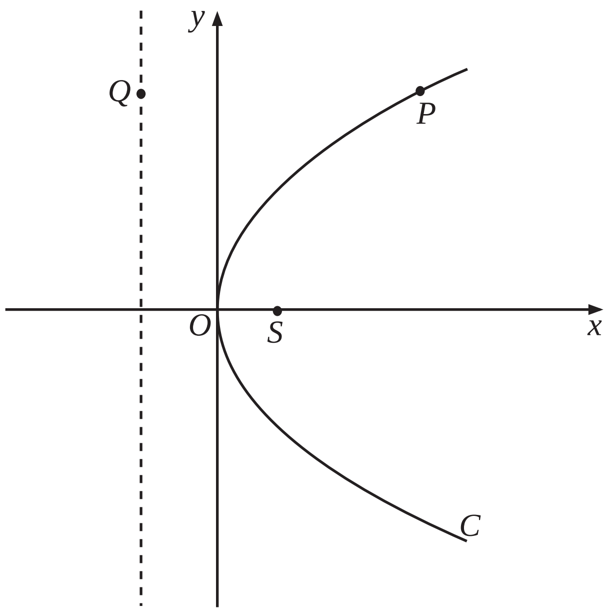

Figure 1

Figure 1 shows a sketch of the parabola C with equation $y^2 = 36x$.
The point S is the focus of C.

(a) Find the coordinates of S.

(1)

(b) Write down the equation of the directrix of C.

(1)

Figure 1 shows the point P which lies on C, where $y > 0$, and the point Q which lies on the directrix of C. The line segment QP is parallel to the x-axis.

Given that the distance PS is 25,

(c) write down the distance QP,

(1)

(d) find the coordinates of P,

(3)

(e) find the area of the trapezium $OSPQ$.

(2)

Leave blank

7. $$z = -24 - 7\text{i}$$

(a) Show z on an Argand diagram.
(1)

(b) Calculate arg z, giving your answer in radians to 2 decimal places.
(2)

It is given that

$$w = a + b\text{i}, \quad a \in \mathbb{R},\ b \in \mathbb{R}$$

Given also that $|w| = 4$ and $\arg w = \dfrac{5\pi}{6}$,

(c) find the values of a and b,
(3)

(d) find the value of $|zw|$.
(3)

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8.

$$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$$

(a) Find det **A**.

(1)

(b) Find $\mathbf{A}^{-1}$.

(2)

The triangle R is transformed to the triangle S by the matrix **A**.
Given that the area of triangle S is 72 square units,

(c) find the area of triangle R.

(2)

The triangle S has vertices at the points $(0, 4)$, $(8, 16)$ and $(12, 4)$.

(d) Find the coordinates of the vertices of R.

(4)

Leave blank

9. A sequence of numbers $u_1, u_2, u_3, u_4, \ldots$ is defined by

$$u_{n+1} = 4u_n + 2, \quad u_1 = 2$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = \frac{2}{3}\left(4^n - 1\right)$$

(5)

Leave blank

10. The point $P\left(6t, \frac{6}{t}\right), t \neq 0$, lies on the rectangular hyperbola H with equation $xy = 36$.

(a) Show that an equation for the tangent to H at P is

$$y = -\frac{1}{t^2}x + \frac{12}{t}$$

(5)

The tangent to H at the point A and the tangent to H at the point B meet at the point $(-9, 12)$.

(b) Find the coordinates of A and B.

(7)

Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	7	/	0	1	Signature	

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Wednesday 22 June 2011 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
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Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
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Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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Leave blank

1. $$f(x) = 3^x + 3x - 7$$

(a) Show that the equation $f(x) = 0$ has a root α between $x = 1$ and $x = 2$.

(2)

(b) Starting with the interval $[1, 2]$, use interval bisection twice to find an interval of width 0.25 which contains α.

(3)

Leave blank

2. $$z_1 = -2 + \mathrm{i}$$

(a) Find the modulus of z_1.

(1)

(b) Find, in radians, the argument of z_1, giving your answer to 2 decimal places.

(2)

The solutions to the quadratic equation

$$z^2 - 10z + 28 = 0$$

are z_2 and z_3.

(c) Find z_2 and z_3, giving your answers in the form $p \pm \mathrm{i}\sqrt{q}$, where p and q are integers.

(3)

(d) Show, on an Argand diagram, the points representing your complex numbers z_1, z_2 and z_3.

(2)

Leave blank

3. (a) Given that

$$\mathbf{A} = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}$$

(i) find $\mathbf{A}^2$,

(ii) describe fully the geometrical transformation represented by $\mathbf{A}^2$.

(4)

(b) Given that

$$\mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

describe fully the geometrical transformation represented by **B**.

(2)

(c) Given that

$$\mathbf{C} = \begin{pmatrix} k+1 & 12 \\ k & 9 \end{pmatrix}$$

where k is a constant, find the value of k for which the matrix **C** is singular.

(3)

Leave blank

4. $$f(x) = x^2 + \frac{5}{2x} - 3x - 1, \quad x \neq 0$$

(a) Use differentiation to find $f'(x)$.

(2)

The root α of the equation $f(x) = 0$ lies in the interval $[0.7, 0.9]$.

(b) Taking 0.8 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places.

(4)

Leave blank

5. $$\mathbf{A} = \begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}, \text{ where } a \text{ and } b \text{ are constants.}$$

Given that the matrix **A** maps the point with coordinates $(4, 6)$ onto the point with coordinates $(2, -8)$,

(a) find the value of a and the value of b.

(4)

A quadrilateral R has area 30 square units.
It is transformed into another quadrilateral S by the matrix **A**.
Using your values of a and b,

(b) find the area of quadrilateral S.

(4)

Leave blank

6. Given that $z = x + \mathrm{i}y$, find the value of x and the value of y such that

$$z + 3\mathrm{i}\,z^* = -1 + 13\mathrm{i}$$

where z^* is the complex conjugate of z.

(7)

Leave blank

7. (a) Use the results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} (2r-1)^2 = \frac{1}{3}n(2n+1)(2n-1)$$

for all positive integers n.

(6)

(b) Hence show that

$$\sum_{r=n+1}^{3n} (2r-1)^2 = \frac{2}{3}n\left(an^2+b\right)$$

where a and b are integers to be found.

(4)

Leave blank

8. The parabola C has equation $y^2 = 48x$.

The point $P\left(12t^2, 24t\right)$ is a general point on C.

(a) Find the equation of the directrix of C.

(2)

(b) Show that the equation of the tangent to C at $P\left(12t^2, 24t\right)$ is

$$x - ty + 12t^2 = 0$$

(4)

The tangent to C at the point $(3, 12)$ meets the directrix of C at the point X.

(c) Find the coordinates of X.

(4)

Leave blank

9. Prove by induction, that for $n \in \mathbb{Z}^+$,

(a) $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^n = \begin{pmatrix} 3^n & 0 \\ 3(3^n - 1) & 1 \end{pmatrix}$,

(6)

(b) $\mathrm{f}(n) = 7^{2n-1} + 5$ is divisible by 12.

(6)

Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	7	/	0	1	Signature	

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Monday 30 January 2012 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
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Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

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Instructions to Candidates

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Information for Candidates

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Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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PEARSON

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1. Given that $z_1 = 1 - \text{i}$,

(a) find $\arg(z_1)$.

(2)

Given also that $z_2 = 3 + 4\text{i}$, find, in the form $a + \text{i}b$, $a, b \in \mathbb{R}$,

(b) $z_1 z_2$,

(2)

(c) $\dfrac{z_2}{z_1}$.

(3)

In part (b) and part (c) you must show all your working clearly.

Leave blank

2. (a) Show that $f(x) = x^4 + x - 1$ has a real root α in the interval $[0.5, 1.0]$.

(2)

(b) Starting with the interval $[0.5, 1.0]$, use interval bisection twice to find an interval of width 0.125 which contains α.

(3)

(c) Taking 0.75 as a first approximation, apply the Newton Raphson process twice to $f(x)$ to obtain an approximate value of α. Give your answer to 3 decimal places.

(5)

Leave blank

3. A parabola C has cartesian equation $y^2 = 16x$. The point $P(4t^2, 8t)$ is a general point on C.

(a) Write down the coordinates of the focus F and the equation of the directrix of C.
(3)

(b) Show that the equation of the normal to C at P is $y + tx = 8t + 4t^3$.
(5)

Leave blank

4. A right angled triangle T has vertices $A(1, 1)$, $B(2, 1)$ and $C(2, 4)$. When T is transformed by the matrix $\mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, the image is T'.

(a) Find the coordinates of the vertices of T'.

(2)

(b) Describe fully the transformation represented by **P**.

(2)

The matrices $\mathbf{Q} = \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ represent two transformations. When T is transformed by the matrix **QR**, the image is T''.

(c) Find **QR**.

(2)

(d) Find the determinant of **QR**.

(2)

(e) Using your answer to part (d), find the area of T''.

(3)

Leave blank

5. The roots of the equation

$$z^3 - 8z^2 + 22z - 20 = 0$$

are z_1, z_2 and z_3.

(a) Given that $z_1 = 3 + \mathrm{i}$, find z_2 and z_3.

(4)

(b) Show, on a single Argand diagram, the points representing z_1, z_2 and z_3.

(2)

Leave blank

6. (a) Prove by induction

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

(5)

(b) Using the result in part (a), show that

$$\sum_{r=1}^{n} (r^3 - 2) = \frac{1}{4}n(n^3 + 2n^2 + n - 8)$$

(3)

(c) Calculate the exact value of $\sum_{r=20}^{50} (r^3 - 2)$.

(3)

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7. A sequence can be described by the recurrence formula

$$u_{n+1} = 2u_n + 1, \qquad n \geqslant 1, \quad u_1 = 1$$

(a) Find u_2 and u_3.

(2)

(b) Prove by induction that $u_n = 2^n - 1$

(5)

Leave blank

8. $$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$$

(a) Show that **A** is non-singular.
(2)

(b) Find **B** such that $\mathbf{BA}^2 = \mathbf{A}$.
(4)

Leave blank

9. The rectangular hyperbola H has cartesian equation $xy = 9$

The points $P\left(3p, \frac{3}{p}\right)$ and $Q\left(3q, \frac{3}{q}\right)$ lie on H, where $p \neq \pm q$.

(a) Show that the equation of the tangent at P is $x + p^2 y = 6p$.

(4)

(b) Write down the equation of the tangent at Q.

(1)

The tangent at the point P and the tangent at the point Q intersect at R.

(c) Find, as single fractions in their simplest form, the coordinates of R in terms of p and q.

(4)

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 7 / 0 1	Signature	

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Friday 1 June 2012 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
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Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

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Information for Candidates

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Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 10 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
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Turn over

PEARSON

Leave blank

1. $$f(x) = 2x^3 - 6x^2 - 7x - 4$$

(a) Show that $f(4) = 0$

(1)

(b) Use algebra to solve $f(x) = 0$ completely.

(4)

Leave blank

2. (a) Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 3 \\ 4 & 5 & 5 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix}$$

find **AB**.

(2)

(b) Given that

$$\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 8 & 6 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 5 & 2k \\ 4 & k \end{pmatrix}, \text{ where } k \text{ is a constant}$$

and

$$\mathbf{E} = \mathbf{C} + \mathbf{D}$$

find the value of k for which **E** has no inverse.

(4)

Leave blank

3. $$f(x) = x^2 + \frac{3}{4\sqrt{x}} - 3x - 7, \quad x > 0$$

A root α of the equation $f(x) = 0$ lies in the interval $[3, 5]$.

Taking 4 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 2 decimal places.

(6)

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4. (a) Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=1}^{n}\left(r^3 + 6r - 3\right) = \frac{1}{4}n^2(n^2 + 2n + 13)$$

for all positive integers n.

(5)

(b) Hence find the exact value of

$$\sum_{r=16}^{30}\left(r^3 + 6r - 3\right)$$

(2)

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5.

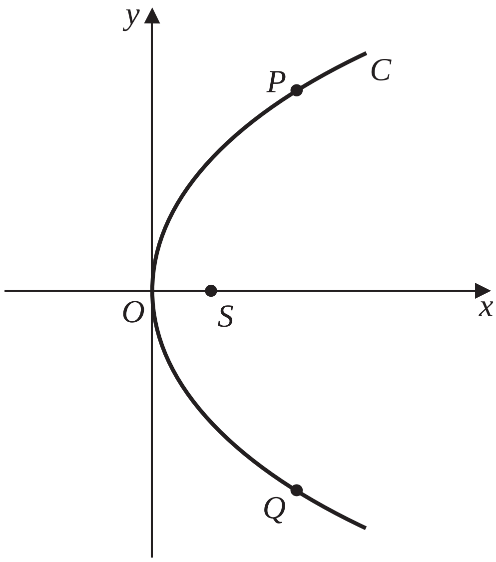

Figure 1

Figure 1 shows a sketch of the parabola C with equation $y^2 = 8x$.
The point P lies on C, where $y > 0$, and the point Q lies on C, where $y < 0$
The line segment PQ is parallel to the y-axis.

Given that the distance PQ is 12,

(a) write down the y-coordinate of P,

(1)

(b) find the x-coordinate of P.

(2)

Figure 1 shows the point S which is the focus of C.
The line l passes through the point P and the point S.

(c) Find an equation for l in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Leave blank

6. $$f(x) = \tan\left(\frac{x}{2}\right) + 3x - 6, \quad -\pi < x < \pi$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[1, 2]$.

(2)

(b) Use linear interpolation once on the interval $[1, 2]$ to find an approximation to α. Give your answer to 2 decimal places.

(3)

Leave blank

7. $$z = 2 - \mathrm{i}\sqrt{3}$$

(a) Calculate arg z, giving your answer in radians to 2 decimal places.
(2)

Use algebra to express

(b) $z + z^2$ in the form $a + b\mathrm{i}\sqrt{3}$, where a and b are integers,
(3)

(c) $\dfrac{z+7}{z-1}$ in the form $c + d\mathrm{i}\sqrt{3}$, where c and d are integers.
(4)

Given that

$$w = \lambda - 3\mathrm{i}$$

where λ is a real constant, and $\arg(4 - 5\mathrm{i} + 3w) = -\dfrac{\pi}{2}$,

(d) find the value of λ.
(2)

Leave blank

8. The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant.

The point $P\left(ct, \frac{c}{t}\right)$, $t \neq 0$, is a general point on H.

(a) Show that an equation for the tangent to H at P is

$$x + t^2 y = 2ct$$

(4)

The tangent to H at the point P meets the x-axis at the point A and the y-axis at the point B.

Given that the area of the triangle OAB, where O is the origin, is 36,

(b) find the exact value of c, expressing your answer in the form $k\sqrt{2}$, where k is an integer.

(4)

Leave blank

9. $$\mathbf{M} = \begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}$$

(a) Find det **M**.
(1)

The transformation represented by **M** maps the point $S(2a - 7, a - 1)$, where a is a constant, onto the point $S'(25, -14)$.

(b) Find the value of a.
(3)

The point R has coordinates (6, 0).

Given that O is the origin,

(c) find the area of triangle ORS.
(2)

Triangle ORS is mapped onto triangle $OR'S'$ by the transformation represented by **M**.

(d) Find the area of triangle $OR'S'$.
(2)

Given that

$$\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

(e) describe fully the single geometrical transformation represented by **A**.
(2)

The transformation represented by **A** followed by the transformation represented by **B** is equivalent to the transformation represented by **M**.

(f) Find **B**.
(4)

Leave blank

10. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 2^{2n-1} + 3^{2n-1} \text{ is divisible by 5.}$$

(6)

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 7 / 0 1	Signature	

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Monday 28 January 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
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Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

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Information for Candidates

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There are 9 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

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PEARSON

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1. Show, using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, that

$$\sum_{r=1}^{n} 3(2r-1)^2 = n(2n+1)(2n-1), \text{ for all positive integers } n.$$

(5)

Leave blank

2.

$$z = \frac{50}{3 + 4\mathrm{i}}$$

Find, in the form $a + \mathrm{i}\,b$ where $a, b \in \mathbb{R}$,

(a) z,

(2)

(b) z^2.

(2)

Find

(c) $|z|$,

(2)

(d) $\arg z^2$, giving your answer in degrees to 1 decimal place.

(2)

Leave blank

3.

$$f(x) = 2x^{\frac{1}{2}} + x^{-\frac{1}{2}} - 5, \qquad x > 0$$

(a) Find $f'(x)$.

(2)

The equation $f(x) = 0$ has a root α in the interval [4.5, 5.5].

(b) Using $x_0 = 5$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 significant figures.

(4)

Leave blank

4. The transformation U, represented by the 2×2 matrix **P**, is a rotation through 90° anticlockwise about the origin.

(a) Write down the matrix **P**.

(1)

The transformation V, represented by the 2×2 matrix **Q**, is a reflection in the line $y = -x$.

(b) Write down the matrix **Q**.

(1)

Given that U followed by V is transformation T, which is represented by the matrix **R**,

(c) express **R** in terms of **P** and **Q**,

(1)

(d) find the matrix **R**,

(2)

(e) give a full geometrical description of T as a single transformation.

(2)

Leave blank

5.

$$f(x) = (4x^2 + 9)(x^2 - 6x + 34)$$

(a) Find the four roots of $f(x) = 0$

Give your answers in the form $x = p + iq$, where p and q are real.

(5)

(b) Show these four roots on a single Argand diagram.

(2)

Leave blank

6. $$\mathbf{X} = \begin{pmatrix} 1 & a \\ 3 & 2 \end{pmatrix}, \text{ where } a \text{ is a constant.}$$

(a) Find the value of a for which the matrix **X** is singular.
(2)

$$\mathbf{Y} = \begin{pmatrix} 1 & -1 \\ 3 & 2 \end{pmatrix}$$

(b) Find $\mathbf{Y}^{-1}$.
(2)

The transformation represented by **Y** maps the point A onto the point B.

Given that B has coordinates $(1 - \lambda, 7\lambda - 2)$, where λ is a constant,

(c) find, in terms of λ, the coordinates of point A.
(4)

Leave blank

7. The rectangular hyperbola, H, has cartesian equation $xy = 25$

The point $P\left(5p, \frac{5}{p}\right)$, and the point $Q\left(5q, \frac{5}{q}\right)$, where $p, q \neq 0$, $p \neq q$, are points on the rectangular hyperbola H.

(a) Show that the equation of the tangent at point P is

$$p^2y + x = 10p$$

(4)

(b) Write down the equation of the tangent at point Q.

(1)

The tangents at P and Q meet at the point N.

Given $p + q \neq 0$,

(c) show that point N has coordinates $\left(\frac{10pq}{p+q}, \frac{10}{p+q}\right)$.

(4)

The line joining N to the origin is perpendicular to the line PQ.

(d) Find the value of p^2q^2.

(5)

Leave blank

8. (a) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} r(r+3) = \tfrac{1}{3}n(n+1)(n+5)$$

(6)

(b) A sequence of positive integers is defined by

$$u_1 = 1,$$
$$u_{n+1} = u_n + n(3n+1), \qquad n \in \mathbb{Z}^+$$

Prove by induction that

$$u_n = n^2(n-1)+1, \qquad n \in \mathbb{Z}^+$$

(5)

Leave blank

9.

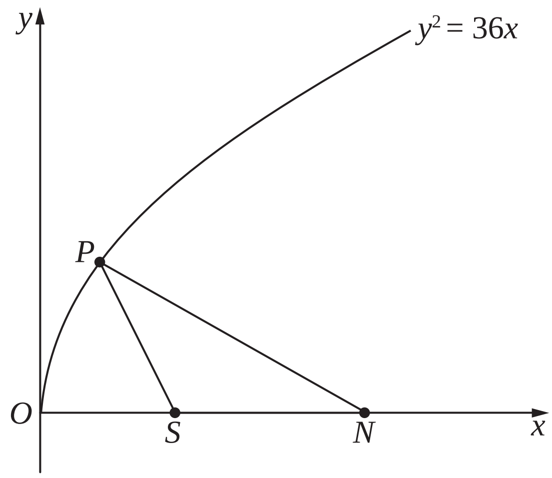

Figure 1

Figure 1 shows a sketch of part of the parabola with equation $y^2 = 36x$.

The point P (4, 12) lies on the parabola.

(a) Find an equation for the normal to the parabola at P.

(5)

This normal meets the x-axis at the point N and S is the focus of the parabola, as shown in Figure 1.

(b) Find the area of triangle PSN.

(4)

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 7 / 0 1	Signature	

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Monday 10 June 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
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Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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P 4 3 1 3 8 A 0 1 3 2

Turn over

PEARSON

Leave blank

1.

$$\mathbf{M} = \begin{pmatrix} x & x-2 \\ 3x-6 & 4x-11 \end{pmatrix}$$

Given that the matrix **M** is singular, find the possible values of x.

(4)

Leave blank

2. $$f(x) = \cos(x^2) - x + 3, \qquad 0 < x < \pi$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [2.5, 3].

(2)

(b) Use linear interpolation once on the interval [2.5, 3] to find an approximation for α, giving your answer to 2 decimal places.

(3)

Leave blank

3. Given that $x = \frac{1}{2}$ is a root of the equation

$$2x^3 - 9x^2 + kx - 13 = 0, \qquad k \in \mathbb{R}$$

find

(a) the value of k,

(3)

(b) the other 2 roots of the equation.

(4)

Leave blank

4. The rectangular hyperbola H has Cartesian equation $xy = 4$

The point $P\left(2t, \frac{2}{t}\right)$ lies on H, where $t \neq 0$

(a) Show that an equation of the normal to H at the point P is

$$ty - t^3x = 2 - 2t^4$$

(5)

The normal to H at the point where $t = -\frac{1}{2}$ meets H again at the point Q.

(b) Find the coordinates of the point Q.

(4)

Leave blank

5. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} (r+2)(r+3) = \frac{1}{3}n(n^2 + 9n + 26)$$

for all positive integers n.

(6)

(b) Hence show that

$$\sum_{r=n+1}^{3n} (r+2)(r+3) = \frac{2}{3}n(an^2 + bn + c)$$

where a, b and c are integers to be found.

(4)

Leave blank

6. A parabola C has equation $y^2 = 4ax$, $a > 0$

The points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ lie on C, where $p \neq 0$, $q \neq 0$, $p \neq q$.

(a) Show that an equation of the tangent to the parabola at P is

$$py - x = ap^2$$

(4)

(b) Write down the equation of the tangent at Q.

(1)

The tangent at P meets the tangent at Q at the point R.

(c) Find, in terms of p and q, the coordinates of R, giving your answers in their simplest form.

(4)

Given that R lies on the directrix of C,

(d) find the value of pq.

(2)

Leave blank

7. $z_1 = 2 + 3\text{i}, \quad z_2 = 3 + 2\text{i}, \quad z_3 = a + b\text{i}, \quad a, b \in \mathbb{R}$

(a) Find the exact value of $|z_1 + z_2|$.

(2)

Given that $w = \dfrac{z_1 z_3}{z_2}$,

(b) find w in terms of a and b, giving your answer in the form $x + \text{i}y$, $\quad x, y \in \mathbb{R}$

(4)

Given also that $w = \dfrac{17}{13} - \dfrac{7}{13}\text{i}$,

(c) find the value of a and the value of b,

(3)

(d) find arg w, giving your answer in radians to 3 decimal places.

(2)

Leave blank

8.

$$\mathbf{A} = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}$$

and **I** is the 2 × 2 identity matrix.

(a) Prove that

$$\mathbf{A}^2 = 7\mathbf{A} + 2\mathbf{I}$$

(2)

(b) Hence show that

$$\mathbf{A}^{-1} = \frac{1}{2}(\mathbf{A} - 7\mathbf{I})$$

(2)

The transformation represented by **A** maps the point P onto the point Q.

Given that Q has coordinates $(2k + 8, -2k - 5)$, where k is a constant,

(c) find, in terms of k, the coordinates of P.

(4)

Leave blank

9. (a) A sequence of numbers is defined by

$$u_1 = 8$$
$$u_{n+1} = 4u_n - 9n, \quad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = 4^n + 3n + 1$$

(5)

(b) Prove by induction that, for $m \in \mathbb{Z}^+$,

$$\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^m = \begin{pmatrix} 2m+1 & -4m \\ m & 1-2m \end{pmatrix}$$

(5)

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 7 / 0 1 R	Signature	

Paper Reference(s)

6667/01R

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Monday 10 June 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
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Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 10 questions in this question paper. The total mark for this paper is 75.
There are 36 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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Turn over

PEARSON

Leave blank

1. The complex numbers z and w are given by

$$z = 8 + 3\text{i}, \quad w = -2\text{i}$$

Express in the form $a + b\text{i}$, where a and b are real constants,

(a) $z - w$,

(1)

(b) zw.

(2)

Leave blank

2. (i)

$$\mathbf{A} = \begin{pmatrix} 2k+1 & k \\ -3 & -5 \end{pmatrix}, \quad \text{where } k \text{ is a constant}$$

Given that

$$\mathbf{B} = \mathbf{A} + 3\mathbf{I}$$

where **I** is the 2 × 2 identity matrix, find

(a) **B** in terms of k,

(2)

(b) the value of k for which **B** is singular.

(2)

(ii) Given that

$$\mathbf{C} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}, \quad \mathbf{D} = (2 \ \ -1 \ \ 5)$$

and

$$\mathbf{E} = \mathbf{CD}$$

find **E**.

(2)

Leave blank

3. $$f(x) = \frac{1}{2}x^4 - x^3 + x - 3$$

(a) Show that the equation $f(x) = 0$ has a root α between $x = 2$ and $x = 2.5$

(2)

(b) Starting with the interval $[2, 2.5]$ use interval bisection twice to find an interval of width 0.125 which contains α.

(3)

The equation $f(x) = 0$ has a root β in the interval $[-2, -1]$.

(c) Taking -1.5 as a first approximation to β, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to β.
Give your answer to 2 decimal places.

(5)

Leave blank

4. $$f(x) = (4x^2 + 9)(x^2 - 2x + 5)$$

(a) Find the four roots of $f(x) = 0$

(4)

(b) Show the four roots of $f(x) = 0$ on a single Argand diagram.

(2)

Leave blank

5.

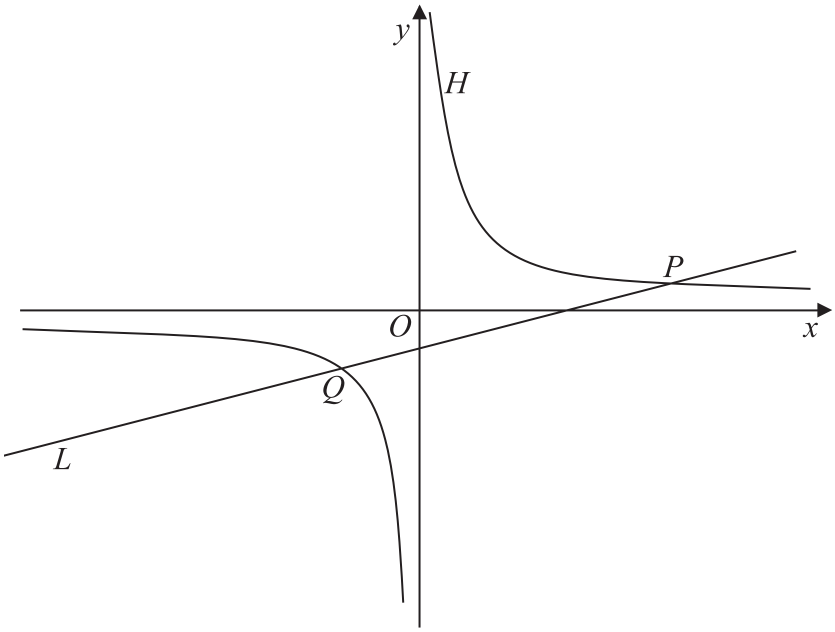

Figure 1

Figure 1 shows a rectangular hyperbola H with parametric equations

$$x = 3t, \quad y = \frac{3}{t}, \quad t \neq 0$$

The line L with equation $6y = 4x - 15$ intersects H at the point P and at the point Q as shown in Figure 1.

(a) Show that L intersects H where $4t^2 - 5t - 6 = 0$

(3)

(b) Hence, or otherwise, find the coordinates of points P and Q.

(5)

P 4 2 8 2 8 A 0 1 4 3 6

Leave blank

6.

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$$

The transformation represented by **B** followed by the transformation represented by **A** is equivalent to the transformation represented by **P**.

(a) Find the matrix **P**.

(2)

Triangle T is transformed to the triangle T' by the transformation represented by **P**.

Given that the area of triangle T' is 24 square units,

(b) find the area of triangle T.

(3)

Triangle T' is transformed to the original triangle T by the matrix represented by **Q**.

(c) Find the matrix **Q**.

(2)

P 4 2 8 2 8 A 0 1 8 3 6

Leave blank

7. The parabola C has equation $y^2 = 4ax$, where a is a positive constant.

The point $P(at^2, 2at)$ is a general point on C.

(a) Show that the equation of the tangent to C at $P(at^2, 2at)$ is

$$ty = x + at^2$$

(4)

The tangent to C at P meets the y-axis at a point Q.

(b) Find the coordinates of Q.

(1)

Given that the point S is the focus of C,

(c) show that PQ is perpendicular to SQ.

(3)

Leave blank

8. (a) Prove by induction, that for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} r(2r-1) = \frac{1}{6}n(n+1)(4n-1)$$

(6)

(b) Hence, show that

$$\sum_{r=n+1}^{3n} r(2r-1) = \frac{1}{3}n(an^2 + bn + c)$$

where a, b and c are integers to be found.

(4)

Leave blank

9. The complex number w is given by

$$w = 10 - 5\text{i}$$

(a) Find $|w|$.

(1)

(b) Find arg w, giving your answer in radians to 2 decimal places.

(2)

The complex numbers z and w satisfy the equation

$$(2 + \text{i})(z + 3\text{i}) = w$$

(c) Use algebra to find z, giving your answer in the form $a + b\text{i}$, where a and b are real numbers.

(4)

Given that

$$\arg(\lambda + 9\text{i} + w) = \frac{\pi}{4}$$

where λ is a real constant,

(d) find the value of λ.

(2)

Leave blank

10. (i) Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to evaluate

$$\sum_{r=1}^{24} (r^3 - 4r)$$

(2)

(ii) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=0}^{n} (r^2 - 2r + 2n + 1) = \frac{1}{6}(n+1)(n+a)(bn+c)$$

for all integers $n \geqslant 0$, where a, b and c are constant integers to be found.

(6)

P 4 2 8 2 8 A 0 3 2 3 6

Write your name here

Surname	Other names

Pearson Edexcel International Advanced Level

Centre Number	Candidate Number

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Wednesday 29 January 2014 – Morning **Time: 1 hour 30 minutes**	Paper Reference **6667A/01**

You must have: Mathematical Formulae and Statistical Tables (Pink)	Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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P 4 3 0 1 9 A 0 1 3 2

PEARSON

Leave blank

1. $f(x) = 2x - 5\cos x$, where x is in radians.

(a) Show that the equation $f(x) = 0$ has a root α in the interval [1, 1.4].

(2)

(b) Starting with the interval [1, 1.4], use interval bisection twice to find an interval of width 0.1 which contains α.

(3)

Leave blank

Question 1 continued

Q1

(Total 5 marks)

Leave blank

2.

(i) $$\mathbf{A} = \begin{pmatrix} -4 & 10 \\ -3 & k \end{pmatrix}, \quad \text{where } k \text{ is a constant.}$$

The triangle T is transformed to the triangle T' by the transformation represented by **A**.

Given that the area of triangle T' is twice the area of triangle T, find the possible values of k.

(4)

(ii) Given that

$$\mathbf{B} = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 5 & 1 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 2 & 8 \\ 0 & 2 \\ 1 & -2 \end{pmatrix}$$

find **BC**.

(3)

Leave blank

Question 2 continued

Q2

(Total 7 marks)

Leave blank

3. A rectangular hyperbola has parametric equations

$$x = 2t, \quad y = \frac{2}{t}, \qquad t \neq 0$$

Points P and Q on this hyperbola have parameters $t = \frac{1}{2}$ and $t = 4$ respectively.

The line L, which passes through the origin O, is perpendicular to the chord PQ.

Find an equation for L.

(4)

Leave blank

Question 3 continued

Q3

(Total 4 marks)

Leave blank

4.

$$f(x) = 2x^{\frac{1}{2}} - \frac{6}{x^2} - 3, \quad x > 0$$

A root β of the equation $f(x) = 0$ lies in the interval [3, 4].

Taking 3.5 as a first approximation to β, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to β. Give your answer to 3 decimal places.

(5)

Leave blank

Question 4 continued

Q4

(Total 5 marks)

Leave blank

5. $$z = 5 + \text{i}\sqrt{3}, \qquad w = \sqrt{3} - \text{i}$$

(a) Find the value of $|w|$.

(1)

Find in the form $a + \text{i}b$, where a and b are real constants,

(b) zw, showing clearly how you obtained your answer,

(2)

(c) $\dfrac{z}{w}$, showing clearly how you obtained your answer.

(3)

Given that

$$\arg(z + \lambda) = \frac{\pi}{3}, \quad \text{where } \lambda \text{ is a real constant,}$$

(d) find the value of λ.

(2)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 8 marks)

Leave blank

6. (a) Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to show that for all positive integers n,

$$\sum_{r=1}^{n} r(r+1)(r-1) = \frac{1}{4}n(n+1)(n-1)(n+a)$$

where a is an integer to be determined.

(4)

(b) Hence find the value of n, where $n > 1$, that satisfies

$$\sum_{r=1}^{n} r(r+1)(r-1) = 10\sum_{r=1}^{n} r^2$$

(5)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 9 marks)

Leave blank

7. $$\mathbf{P} = \begin{pmatrix} 3a & -2a \\ -b & 2b \end{pmatrix}, \quad \mathbf{M} = \begin{pmatrix} -6a & 7a \\ 2b & -b \end{pmatrix}$$

where a and b are non-zero constants.

(a) Find $\mathbf{P}^{-1}$, leaving your answer in terms of a and b.
(3)

Given that

$$\mathbf{M} = \mathbf{PQ}$$

(b) find the matrix $\mathbf{Q}$, giving your answer in its simplest form.
(3)

Leave blank

Question 7 continued

Q7

(Total 6 marks)

8. The parabola C has equation $y^2 = 4ax$, where a is a positive constant.

The point $P(ap^2, 2ap)$ lies on the parabola C.

(a) Show that an equation of the normal to C at P is

$$y + px = ap^3 + 2ap$$

(5)

The normal to C at the point P meets the x-axis at the point $(6a, 0)$ and meets the directrix of C at the point D. Given that $p > 0$,

(b) find, in terms of a, the coordinates of D.

(4)

Given also that the directrix of C cuts the x-axis at the point X,

(c) find, in terms of a, the area of the triangle XPD, giving your answer in its simplest form.

(3)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 12 marks)

Leave blank

9. Given that $z = x + \mathrm{i}y$, where $x \in \mathbb{R}$, $y \in \mathbb{R}$, find the value of x and the value of y such that

$$(3 - \mathrm{i})z^* + 2\mathrm{i}z = 9 - \mathrm{i}$$

where z^* is the complex conjugate of z.

(8)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 8 marks)

Leave blank

10. (i) A sequence of numbers $u_1, u_2, u_3, \ldots$, is defined by

$$u_{n+1} = 5u_n + 3, \quad u_1 = 3$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = \frac{3}{4}(5^n - 1)$$

(5)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 5(5^n) - 4n - 5 \text{ is divisible by } 16.$$

(6)

P 4 3 0 1 9 A 0 2 8 3 2

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Question 10 continued

Leave blank

Question 10 continued

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Question 10 continued

Leave blank

Question 10 continued

Q10

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 7 / 0 1	Signature	

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Tuesday 10 June 2014 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
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Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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Turn over

PEARSON

Leave blank

1. The complex numbers z_1 and z_2 are given by

$$z_1 = p + 2\text{i} \text{ and } z_2 = 1 - 2\text{i}$$

where p is an integer.

(a) Find $\frac{z_1}{z_2}$ in the form $a + b\text{i}$ where a and b are real. Give your answer in its simplest form in terms of p.

(4)

Given that $\left|\frac{z_1}{z_2}\right| = 13$,

(b) find the possible values of p.

(4)

Leave blank

2.

$$f(x) = x^3 - \frac{5}{2x^{\frac{3}{2}}} + 2x - 3, \quad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [1.1, 1.5].

(2)

(b) Find $f'(x)$.

(2)

(c) Using $x_0 = 1.1$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 decimal places.

(3)

Leave blank

3. Given that 2 and 1 – 5i are roots of the equation

$$x^3 + px^2 + 30x + q = 0, \qquad p, q \in \mathbb{R}$$

(a) write down the third root of the equation. **(1)**

(b) Find the value of p and the value of q. **(5)**

(c) Show the three roots of this equation on a single Argand diagram. **(2)**

Leave blank

4. (i) Given that

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix},$$

(a) find **AB**.

(b) Explain why $\mathbf{AB} \neq \mathbf{BA}$.

(4)

(ii) Given that

$$\mathbf{C} = \begin{pmatrix} 2k & -2 \\ 3 & k \end{pmatrix}, \text{ where } k \text{ is a real number}$$

find $\mathbf{C}^{-1}$, giving your answer in terms of k.

(3)

Leave blank

5. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} (2r-1)^2 = \frac{1}{3}n(4n^2-1)$$

(6)

(b) Hence show that

$$\sum_{r=2n+1}^{4n} (2r-1)^2 = an(bn^2-1)$$

where a and b are constants to be found.

(3)

Leave blank

Question 5 continued

6. The rectangular hyperbola H has cartesian equation $xy = c^2$.

The point $P\left(ct, \frac{c}{t}\right)$, $t > 0$, is a general point on H.

(a) Show that an equation of the tangent to H at the point P is

$$t^2y + x = 2ct$$

(4)

An equation of the normal to H at the point P is $t^3x - ty = ct^4 - c$

Given that the normal to H at P meets the x-axis at the point A and the tangent to H at P meets the x-axis at the point B,

(b) find, in terms of c and t, the coordinates of A and the coordinates of B.

(2)

Given that $c = 4$,

(c) find, in terms of t, the area of the triangle APB. Give your answer in its simplest form.

(3)

Leave blank

Question 6 continued

Leave blank

7. (i) In each of the following cases, find a 2×2 matrix that represents

(a) a reflection in the line $y = -x$,

(b) a rotation of 135° anticlockwise about (0, 0),

(c) a reflection in the line $y = -x$ followed by a rotation of 135° anticlockwise about (0, 0).

(4)

(ii) The triangle T has vertices at the points $(1, k)$, $(3, 0)$ and $(11, 0)$, where k is a constant.

Triangle T is transformed onto the triangle T' by the matrix

$$\begin{pmatrix} 6 & -2 \\ 1 & 2 \end{pmatrix}$$

Given that the area of triangle T' is 364 square units, find the value of k.

(6)

Leave blank

Question 7 continued

Leave blank

8. The points $P(4k^2, 8k)$ and $Q(k^2, 4k)$, where k is a constant, lie on the parabola C with equation $y^2 = 16x$.

The straight line l_1 passes through the points P and Q.

(a) Show that an equation of the line l_1 is given by

$$3ky - 4x = 8k^2$$

(4)

The line l_2 is perpendicular to the line l_1 and passes through the focus of the parabola C. The line l_2 meets the directrix of C at the point R.

(b) Find, in terms of k, the y coordinate of the point R.

(7)

Leave blank

Question 8 continued

Leave blank

9. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 8^n - 2^n$$

is divisible by 6

(6)

Leave blank

Question 9 continued

Q9

(Total 6 marks)

TOTAL FOR PAPER: 75 MARKS

END

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 7 / 0 1 R	Signature	

Paper Reference(s)

6667/01R

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Tuesday 10 June 2014 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
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7	
8	
9	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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PEARSON

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1. The roots of the equation

$$2z^3 - 3z^2 + 8z + 5 = 0$$

are z_1, z_2 and z_3

Given that $z_1 = 1 + 2i$, find z_2 and z_3

(5)

Leave blank

2.

$$f(x) = 3\cos 2x + x - 2, \quad -\pi \leqslant x < \pi$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [2, 3].

(2)

(b) Use linear interpolation once on the interval [2, 3] to find an approximation to α.

Give your answer to 3 decimal places.

(3)

(c) The equation $f(x) = 0$ has another root β in the interval [–1, 0]. Starting with this interval, use interval bisection to find an interval of width 0.25 which contains β.

(4)

Leave blank

3. (i)

$$\mathbf{A} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(a) Describe fully the single transformation represented by the matrix **A**.

(2)

The matrix **B** represents an enlargement, scale factor –2, with centre the origin.

(b) Write down the matrix **B**.

(1)

(ii)

$$\mathbf{M} = \begin{pmatrix} 3 & k \\ -2 & 3 \end{pmatrix}, \quad \text{where } k \text{ is a positive constant.}$$

Triangle T has an area of 16 square units.

Triangle T is transformed onto the triangle T' by the transformation represented by the matrix **M**.

Given that the area of the triangle T' is 224 square units, find the value of k.

(3)

Leave blank

4. The complex number z is given by

$$z = \frac{p + 2\text{i}}{3 + p\text{i}}$$

where p is an integer.

(a) Express z in the form $a + b\text{i}$ where a and b are real. Give your answer in its simplest form in terms of p.

(4)

(b) Given that $\arg(z) = \theta$, where $\tan\theta = 1$ find the possible values of p.

(5)

Leave blank

Question 4 continued

Leave blank

5. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$ to show that

$$\sum_{r=1}^{n} r(r^2 - 3) = \frac{1}{4}n(n+1)(n+3)(n-2)$$

(5)

(b) Calculate the value of $\sum_{r=10}^{50} r(r^2 - 3)$

(3)

Leave blank

6.

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}$$

Given that $\mathbf{M} = (\mathbf{A} + \mathbf{B})(2\mathbf{A} - \mathbf{B})$,

(a) calculate the matrix **M**,

(6)

(b) find the matrix **C** such that **MC** = **A**.

(4)

Leave blank

7. The parabola C has cartesian equation $y^2 = 4ax$, $a > 0$

The points $P(ap^2, 2ap)$ and $P'(ap^2, -2ap)$ lie on C.

(a) Show that an equation of the normal to C at the point P is

$$y + px = 2ap + ap^3$$

(5)

(b) Write down an equation of the normal to C at the point P'.

(1)

The normal to C at P meets the normal to C at P' at the point Q.

(c) Find, in terms of a and p, the coordinates of Q.

(2)

Given that S is the focus of the parabola,

(d) find the area of the quadrilateral $SPQP'$.

(3)

Leave blank

Question 7 continued

Leave blank

8. The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant.

The point $P\left(ct, \frac{c}{t}\right)$, $t \neq 0$, is a general point on H.

An equation for the tangent to H at P is given by

$$y = -\frac{1}{t^2}x + \frac{2c}{t}$$

The points A and B lie on H.

The tangent to H at A and the tangent to H at B meet at the point $\left(-\frac{6}{7}c, \frac{12}{7}c\right)$.

Find, in terms of c, the coordinates of A and the coordinates of B.

(5)

Leave blank

Question 8 continued

Leave blank

9. (a) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} (r+1)2^{r-1} = n2^n$$

(5)

(b) A sequence of numbers is defined by

$$u_1 = 0, \qquad u_2 = 32,$$

$$u_{n+2} = 6u_{n+1} - 8u_n \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = 4^{n+1} - 2^{n+3}$$

(7)

Leave blank

Question 9 continued

Q9

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

Further Pure Mathematics FP1

Candidates sitting FP1 may also require those formulae listed under Core Mathematics C1 and C2.

Summations

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Numerical solution of equations

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Conics

	Parabola	**Rectangular Hyperbola**
Standard Form	$y^2 = 4ax$	$xy = c^2$
Parametric Form	$(at^2, 2at)$	$\left(ct, \dfrac{c}{t}\right)$
Foci	$(a, 0)$	Not required
Directrices	$x = -a$	Not required

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

In FP1, θ will be a multiple of 45°.

Core Mathematics C1

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$u_n = a + (n - 1)d$

$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$a^2 = b^2 + c^2 - 2bc \cos A$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times \ldots \times r}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + ... + y_{n-1})\}$, where $h = \frac{b-a}{n}$