PMT

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced Level

Friday 30 January 2009 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Orange)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP1), the paper reference (6667), your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 10 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

N34694A

1. $$\mathrm{f}(x) = 2x^3 - 8x^2 + 7x - 3.$$

Given that $x = 3$ is a solution of the equation $\mathrm{f}(x) = 0$, solve $\mathrm{f}(x) = 0$ completely.

(5)

---

2. (*a*) Show, using the formulae for $\sum r$ and $\sum r^2$, that

$$\sum_{r=1}^{n} (6r^2 + 4r - 1) = n(n+2)(2n+1).$$

(5)

(*b*) Hence, or otherwise, find the value of $\sum_{r=11}^{20} (6r^2 + 4r - 1)$.

(2)

---

3. The rectangular hyperbola, H, has parametric equations $x = 5t$, $y = \frac{5}{t}$, $t \neq 0$.

(*a*) Write the cartesian equation of H in the form $xy = c^2$.

(1)

Points A and B on the hyperbola have parameters $t = 1$ and $t = 5$ respectively.

(*b*) Find the coordinates of the mid-point of AB.

(3)

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4. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}.$$

(5)

---

5. $$\mathrm{f}(x) = 3\sqrt{x} + \frac{18}{\sqrt{x}} - 20.$$

(*a*) Show that the equation $\mathrm{f}(x) = 0$ has a root α in the interval $[1.1, 1.2]$.

(2)

(*b*) Find $\mathrm{f}'(x)$.

(3)

(*c*) Using $x_0 = 1.1$ as a first approximation to α, apply the Newton-Raphson procedure once to $\mathrm{f}(x)$ to find a second approximation to α, giving your answer to 3 significant figures.

(4)

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6. A series of positive integers $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 6 \text{ and } u_{n+1} = 6u_n - 5, \text{ for } n \geq 1.$$

Prove by induction that $u_n = 5 \times 6^{n-1} + 1$, for $n \geq 1$.

(5)

---

7. Given that $\mathbf{X} = \begin{pmatrix} 2 & a \\ -1 & -1 \end{pmatrix}$, where a is a constant, and $a \neq 2$,

(*a*) find $\mathbf{X}^{-1}$ in terms of a.

(3)

Given that $\mathbf{X} + \mathbf{X}^{-1} = \mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix,

(*b*) find the value of a.

(3)

---

8. A parabola has equation $y^2 = 4ax$, $a > 0$. The point Q $(aq^2, 2aq)$ lies on the parabola.

(*a*) Show that an equation of the tangent to the parabola at Q is

$$yq = x + aq^2.$$

(4)

This tangent meets the y-axis at the point R.

(*b*) Find an equation of the line l which passes through R and is perpendicular to the tangent at Q.

(3)

(*c*) Show that l passes through the focus of the parabola.

(1)

(*d*) Find the coordinates of the point where l meets the directrix of the parabola.

(2)

---

9. Given that $z_1 = 3 + 2\text{i}$ and $z_2 = \dfrac{12 - 5\text{i}}{z_1}$,

(*a*) find z_2 in the form $a + \text{i}b$, where a and b are real.

(2)

(*b*) Show, on an Argand diagram, the point P representing z_1 and the point Q representing z_2.

(2)

(*c*) Given that O is the origin, show that $\angle POQ = \dfrac{\pi}{2}$.

(2)

The circle passing through the points O, P and Q has centre C. Find

(*d*) the complex number represented by C,

(2)

(*e*) the exact value of the radius of the circle.

(2)

10.

$$\mathbf{A} = \begin{pmatrix} 3\sqrt{2} & 0 \\ 0 & 3\sqrt{2} \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

(*a*) Describe fully the transformations described by each of the matrices **A**, **B** and **C**.

(4)

It is given that the matrix **D** = **CA**, and that the matrix **E** = **DB**.

(*b*) Find **D**.

(2)

(*c*) Show that $\mathbf{E} = \begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}$.

(1)

The triangle ORS has vertices at the points with coordinates (0, 0), (–15, 15) and (4, 21). This triangle is transformed onto the triangle $OR'S'$ by the transformation described by **E**.

(*d*) Find the coordinates of the vertices of triangle $OR'S'$.

(4)

(*e*) Find the area of triangle $OR'S'$ and deduce the area of triangle ORS.

(3)

TOTAL FOR PAPER: 75 MARKS

END

PMT

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced Subsidiary

Wednesday 17 June 2009 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Orange)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

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There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

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M35146A

1. The complex numbers z_1 and z_2 are given by

$$z_1 = 2 - \mathrm{i} \text{ and } z_2 = -8 + 9\mathrm{i}$$

(*a*) Show z_1 and z_2 on a single Argand diagram. **(1)**

Find, showing your working,

(*b*) the value of $|z_1|$, **(2)**

(*c*) the value of arg z_1, giving your answer in radians to 2 decimal places, **(2)**

(*d*) $\dfrac{z_2}{z_1}$ in the form $a + b\mathrm{i}$, where a and b are real. **(3)**

---

2. (*a*) Using the formulae for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$, show that

$$\sum_{r=1}^{n} r(r+1)(r+3) = \frac{1}{12}n(n+1)(n+2)(3n+k),$$

where k is a constant to be found. **(7)**

(*b*) Hence evaluate $\sum_{r=21}^{40} r(r+1)(r+3)$. **(2)**

---

3. $$\mathrm{f}(x) = (x^2 + 4)(x^2 + 8x + 25)$$

(*a*) Find the four roots of $\mathrm{f}(x) = 0$. **(5)**

(*b*) Find the sum of these four roots. **(2)**

---

4. Given that α is the only real root of the equation

$$x^3 - x^2 - 6 = 0,$$

(*a*) show that $2.2 < \alpha < 2.3$

(2)

(*b*) Taking 2.2 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x) = x^3 - x^2 - 6$ to obtain a second approximation to α, giving your answer to 3 decimal places.

(5)

(*c*) Use linear interpolation once on the interval [2.2, 2.3] to find another approximation to α, giving your answer to 3 decimal places.

(3)

---

5. $\mathbf{R} = \begin{pmatrix} a & 2 \\ a & b \end{pmatrix}$, where a and b are constants and $a > 0$.

(*a*) Find $\mathbf{R}^2$ in terms of a and b.

(3)

Given that $\mathbf{R}^2$ represents an enlargement with centre (0, 0) and scale factor 15,

(*b*) find the value of a and the value of b.

(5)

---

6. The parabola C has equation $y^2 = 16x$.

(*a*) Verify that the point $P(4t^2, 8t)$ is a general point on C.

(1)

(*b*) Write down the coordinates of the focus S of C.

(1)

(*c*) Show that the normal to C at P has equation

$$y + tx = 8t + 4t^3 .$$

(5)

The normal to C at P meets the x-axis at the point N.

(*d*) Find the area of triangle PSN in terms of t, giving your answer in its simplest form.

(4)

---

7. $\mathbf{A} = \begin{pmatrix} a & -2 \\ -1 & 4 \end{pmatrix}$, where a is a constant.

(*a*) Find the value of a for which the matrix $\mathbf{A}$ is singular.

(2)

$$\mathbf{B} = \begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}$$

(*b*) Find $\mathbf{B}^{-1}$.

(3)

The transformation represented by $\mathbf{B}$ maps the point P onto the point Q.

Given that Q has coordinates $(k - 6, 3k + 12)$, where k is a constant,

(*c*) show that P lies on the line with equation $y = x + 3$.

(3)

---

8. Prove by induction that, for $n \in \mathbb{Z}^+$,

(*a*) $f(n) = 5^n + 8n + 3$ is divisible by 4,

(7)

(*b*) $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^n = \begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix}$.

(7)

TOTAL FOR PAPER: 75 MARKS

END

PMT

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Monday 1 February 2010 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Orange)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

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Information for Candidates

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There are 9 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

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Answers without working may not gain full credit.

N35143A

1. The complex numbers z_1 and z_2 are given by

$$z_1 = 2 + 8\text{i} \quad \text{and} \quad z_2 = 1 - \text{i}$$

Find, showing your working,

(*a*) $\dfrac{z_1}{z_2}$ in the form $a + b\text{i}$, where a and b are real,

(3)

(*b*) the value of $\left|\dfrac{z_1}{z_2}\right|$,

(2)

(*c*) the value of $\arg \dfrac{z_1}{z_2}$, giving your answer in radians to 2 decimal places.

(2)

---

2. $$\text{f}(x) = 3x^2 - \frac{11}{x^2}.$$

(*a*) Write down, to 3 decimal places, the value of f(1.3) and the value of f(1.4).

(1)

The equation $\text{f}(x) = 0$ has a root α between 1.3 and 1.4

(*b*) Starting with the interval [1.3, 1.4], use interval bisection to find an interval of width 0.025 which contains α.

(3)

(*c*) Taking 1.4 as a first approximation to α, apply the Newton-Raphson procedure once to $\text{f}(x)$ to obtain a second approximation to α, giving your answer to 3 decimal places.

(5)

---

3. A sequence of numbers is defined by

$$u_1 = 2,$$

$$u_{n+1} = 5u_n - 4, \qquad n \geq 1.$$

Prove by induction that, for $n \in \mathbb{Z}^+$, $u_n = 5^{n-1} + 1$.

(4)

4.

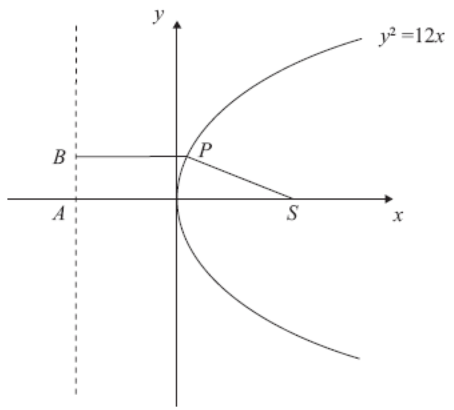

Figure 1

Figure 1 shows a sketch of part of the parabola with equation $y^2 = 12x$.

The point P on the parabola has x-coordinate $\frac{1}{3}$.

The point S is the focus of the parabola.

(a) Write down the coordinates of S.

(1)

The points A and B lie on the directrix of the parabola.
The point A is on the x-axis and the y-coordinate of B is positive.

Given that $ABPS$ is a trapezium,

(b) calculate the perimeter of $ABPS$.

(5)

5. $\mathbf{A} = \begin{pmatrix} a & -5 \\ 2 & a+4 \end{pmatrix}$, where a is real.

(a) Find det **A** in terms of a.

(2)

(b) Show that the matrix **A** is non-singular for all values of a.

(3)

Given that $a = 0$,

(c) find $\mathbf{A}^{-1}$.

(3)

6. Given that 2 and 5 + 2i are roots of the equation

$$x^3 - 12x^2 + cx + d = 0, \qquad c, d \in \mathbb{R},$$

(a) write down the other complex root of the equation.

(1)

(b) Find the value of c and the value of d.

(5)

(c) Show the three roots of this equation on a single Argand diagram.

(2)

7. The rectangular hyperbola H has equation $xy = c^2$, where c is a constant.

The point $P\left(ct, \frac{c}{t}\right)$ is a general point on H.

(a) Show that the tangent to H at P has equation

$$t^2y + x = 2ct.$$

(4)

The tangents to H at the points A and B meet at the point $(15c, -c)$.

(b) Find, in terms of c, the coordinates of A and B.

(5)

8. (*a*) Prove by induction that, for any positive integer n,

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2.$$

(5)

(*b*) Using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$, show that

$$\sum_{r=1}^{n}(r^3+3r+2) = \frac{1}{4}n(n+2)(n^2+7).$$

(5)

(*c*) Hence evaluate $\sum_{r=15}^{25}(r^3+3r+2)$.

(2)

9.

$$\mathbf{M} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

(*a*) Describe fully the geometrical transformation represented by the matrix **M**.

(2)

The transformation represented by **M** maps the point A with coordinates (p, q) onto the point B with coordinates $(3\sqrt{2}, 4\sqrt{2})$.

(*b*) Find the value of p and the value of q.

(4)

(*c*) Find, in its simplest surd form, the length OA, where O is the origin.

(2)

(*d*) Find $\mathbf{M}^2$.

(2)

The point B is mapped onto the point C by the transformation represented by $\mathbf{M}^2$.

(*e*) Find the coordinates of C.

(2)

TOTAL FOR PAPER: 75 MARKS

END

PMT

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced Subsidiary

Tuesday 22 June 2010 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

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H35387A

PMT

1. $$z = 2 - 3\text{i}$$

(*a*) Show that $z^2 = -5 - 12\text{i}$.
(2)

Find, showing your working,

(*b*) the value of $|z^2|$,
(2)

(*c*) the value of $\arg(z^2)$, giving your answer in radians to 2 decimal places.
(2)

(*d*) Show z and z^2 on a single Argand diagram.
(1)

---

2. $$\mathbf{M} = \begin{pmatrix} 2a & 3 \\ 6 & a \end{pmatrix}, \text{ where a is a real constant.}$$

(*a*) Given that $a = 2$, find $\mathbf{M}^{-1}$.
(3)

(*b*) Find the values of a for which $\mathbf{M}$ is singular.
(2)

---

3. $$\text{f}(x) = x^3 - \frac{7}{x} + 2, \; x > 0.$$

(*a*) Show that $\text{f}(x) = 0$ has a root α between 1.4 and 1.5.
(2)

(*b*) Starting with the interval [1.4, 1.5], use interval bisection twice to find an interval of width 0.025 that contains α.
(3)

(*c*) Taking 1.45 as a first approximation to α, apply the Newton-Raphson procedure once to $\text{f}(x) = x^3 - \frac{7}{x} + 2$, $x > 0$ to obtain a second approximation to α, giving your answer to 3 decimal places.
(5)

---

4. $$\text{f}(x) = x^3 + x^2 + 44x + 150.$$

Given that $\text{f}(x) = (x + 3)(x^2 + ax + b)$, where a and b are real constants,

(*a*) find the value of a and the value of b.
(2)

(*b*) Find the three roots of $\text{f}(x) = 0$.
(4)

(*c*) Find the sum of the three roots of $\text{f}(x) = 0$.
(1)

---

5. The parabola C has equation $y^2 = 20x$.

(*a*) Verify that the point $P(5t^2, 10t)$ is a general point on C.
(1)

The point A on C has parameter $t = 4$.
The line l passes through A and also passes through the focus of C.

(*b*) Find the gradient of l.
(4)

---

6. Write down the 2×2 matrix that represents

(*a*) an enlargement with centre (0, 0) and scale factor 8,
(1)

(*b*) a reflection in the x-axis.
(1)

Hence, or otherwise,

(*c*) find the matrix $\mathbf{T}$ that represents an enlargement with centre (0, 0) and scale factor 8, followed by a reflection in the x-axis.
(2)

$$\mathbf{A} = \begin{pmatrix} 6 & 1 \\ 4 & 2 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} k & 1 \\ c & -6 \end{pmatrix}, \text{ where } k \text{ and } c \text{ are constants.}$$

(*d*) Find $\mathbf{AB}$.
(3)

Given that $\mathbf{AB}$ represents the same transformation as $\mathbf{T}$,

(*e*) find the value of k and the value of c.
(2)

---

7. $$\mathrm{f}(n) = 2^n + 6^n.$$

(*a*) Show that $\mathrm{f}(k+1) = 6\mathrm{f}(k) - 4(2^k)$.

(3)

(*b*) Hence, or otherwise, prove by induction that, for $n \in \mathbb{Z}^+$, $\mathrm{f}(n)$ is divisible by 8.

(4)

---

8. The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant.

The point A on H has x-coordinate $3c$.

(*a*) Write down the y-coordinate of A.

(1)

(*b*) Show that an equation of the normal to H at A is

$$3y = 27x - 80c.$$

(5)

The normal to H at A meets H again at the point B.

(*c*) Find, in terms of c, the coordinates of B.

(5)

---

9. (*a*) Prove by induction that

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1).$$

(6)

Using the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$,

(*b*) show that

$$\sum_{r=1}^{n} (r+2)(r+3) = \frac{1}{3}n(n^2 + an + b),$$

where a and b are integers to be found.

(5)

(*c*) Hence show that

$$\sum_{r=n+1}^{2n} (r+2)(r+3) = \frac{1}{3}n(7n^2 + 27n + 26).$$

(3)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Monday 31 January 2011 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

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N35406A

1. $$z = 5 - 3\text{i}, \quad w = 2 + 2\text{i}$$

Express in the form $a + b\text{i}$, where a and b are real constants,

(*a*) z^2,

(2)

(*b*) $\dfrac{z}{w}$.

(3)

---

2. $$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$$

(*a*) Find **AB**.

(3)

Given that

$$\mathbf{C} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

(*b*) describe fully the geometrical transformation represented by **C**,

(2)

(*c*) write down $\mathbf{C}^{100}$.

(1)

---

3. $$\text{f}(x) = 5x^2 - 4x^{\frac{3}{2}} - 6, \quad x \geq 0.$$

The root α of the equation f $(x) = 0$ lies in the interval [1.6,1.8].

(*a*) Use linear interpolation once on the interval [1.6, 1.8] to find an approximation to α. Give your answer to 3 decimal places.

(4)

(*b*) Differentiate f(x) to find f′(x).

(2)

(*c*) Taking 1.7 as a first approximation to α, apply the Newton-Raphson process once to f(x) to obtain a second approximation to α. Give your answer to 3 decimal places.

(4)

---

4. Given that $2 - 4\text{i}$ is a root of the equation

$$z^2 + pz + q = 0,$$

where p and q are real constants,

(*a*) write down the other root of the equation,

(1)

(*b*) find the value of p and the value of q.

(3)

---

5. (*a*) Use the results for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$, to prove that

$$\sum_{r=1}^{n} r(r+1)(r+5) = \frac{1}{4}n(n+1)(n+2)(n+7)$$

for all positive integers n.

(5)

(*b*) Hence, or otherwise, find the value of

$$\sum_{r=20}^{50} r(r+1)(r+5).$$

(2)

6.

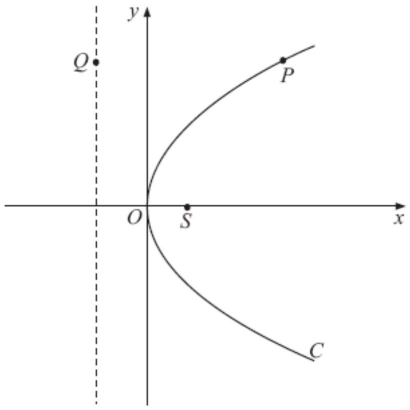

Figure 1

Figure 1 shows a sketch of the parabola C with equation $y^2 = 36x$.
The point S is the focus of C.

(*a*) Find the coordinates of S.
(1)

(*b*) Write down the equation of the directrix of C.
(1)

Figure 1 shows the point P which lies on C, where $y > 0$, and the point Q which lies on the directrix of C. The line segment QP is parallel to the x-axis.

Given that the distance PS is 25,

(*c*) write down the distance QP,
(1)

(*d*) find the coordinates of P,
(3)

(*e*) find the area of the trapezium $OSPQ$.
(2)

7.

$$z = -24 - 7\text{i}$$

(*a*) Show z on an Argand diagram.
(1)

(*b*) Calculate arg z, giving your answer in radians to 2 decimal places.
(2)

It is given that

$$w = a + b\text{i}, \quad a \in \mathbb{R}, \quad b \in \mathbb{R}.$$

Given also that $|w| = 4$ and $\arg w = \dfrac{5\pi}{6}$,

(*c*) find the values of a and b,
(3)

(*d*) find the value of $|zw|$.
(3)

8.

$$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$$

(*a*) Find det **A**.
(1)

(*b*) Find $\mathbf{A}^{-1}$.
(2)

The triangle R is transformed to the triangle S by the matrix **A**.
Given that the area of triangle S is 72 square units,

(*c*) find the area of triangle R.
(2)

The triangle S has vertices at the points (0, 4), (8, 16) and (12, 4).

(*d*) Find the coordinates of the vertices of R.
(4)

---

9. A sequence of numbers $u_1, u_2, u_3, u_4, \ldots$, is defined by

$$u_{n+1} = 4u_n + 2, \quad u_1 = 2.$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = \frac{2}{3}(4^n - 1).$$

(5)

---

10. The point $P\left(6t, \frac{6}{t}\right)$, $t \neq 0$, lies on the rectangular hyperbola H with equation $xy = 36$.

(*a*) Show that an equation for the tangent to H at P is

$$y = -\frac{1}{t^2}x + \frac{12}{t}.$$

(5)

The tangent to H at the point A and the tangent to H at the point B meet at the point $(-9, 12)$.

(*b*) Find the coordinates of A and B.

(7)

TOTAL FOR PAPER: 75 MARKS

END

PMT

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/ Advanced Subsidiary

Wednesday 22 June 2011 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics FP1), the paper reference (6667), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P38168A

PMT

1. $$f(x) = 3^x + 3x - 7$$

(*a*) Show that the equation $f(x) = 0$ has a root α between $x = 1$ and $x = 2$.

(2)

(*b*) Starting with the interval [1, 2], use interval bisection twice to find an interval of width 0.25 which contains α.

(3)

---

2. $$z_1 = -2 + i$$

(*a*) Find the modulus of z_1.

(1)

(*b*) Find, in radians, the argument of z_1, giving your answer to 2 decimal places.

(2)

The solutions to the quadratic equation

$$z^2 - 10z + 28 = 0$$

are z_2 and z_3.

(*c*) Find z_2 and z_3, giving your answers in the form $p \pm i\sqrt{q}$, where p and q are integers.

(3)

(*d*) Show, on an Argand diagram, the points representing your complex numbers z_1, z_2 and z_3.

(2))

---

3. (*a*) Given that

$$\mathbf{A} = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix},$$

(i) find $\mathbf{A}^2$,

(ii) describe fully the geometrical transformation represented by $\mathbf{A}^2$.

(4)

(*b*) Given that

$$\mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix},$$

describe fully the geometrical transformation represented by **B**.

(2)

(*c*) Given that

$$\mathbf{C} = \begin{pmatrix} k+1 & 12 \\ k & 9 \end{pmatrix},$$

where k is a constant, find the value of k for which the matrix **C** is singular.

(3)

---

4. $$f(x) = x^2 + \frac{5}{2x} - 3x - 1, \quad x \neq 0.$$

(*a*) Use differentiation to find $f'(x)$.

(2)

The root α of the equation $f(x) = 0$ lies in the interval [0.7, 0.9].

(*b*) Taking 0.8 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places.

(4)

---

5. $$\mathbf{A} = \begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}, \text{ where } a \text{ and } b \text{ are constants.}$$

Given that the matrix **A** maps the point with coordinates (4, 6) onto the point with coordinates (2, −8),

(*a*) find the value of a and the value of b.

(4)

A quadrilateral R has area 30 square units.
It is transformed into another quadrilateral S by the matrix **A**.
Using your values of a and b,

(*b*) find the area of quadrilateral S.

(4)

---

6. Given that $z = x + \text{i}y$, find the value of x and the value of y such that

$$z + 3\text{i}z^* = -1 + 13\text{i}$$

where z^* is the complex conjugate of z.

(7)

---

7. (*a*) Use the results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n}(2r-1)^2 = \frac{1}{3}n(2n+1)(2n-1)$$

for all positive integers n.

(6)

(*b*) Hence show that

$$\sum_{r=n+1}^{3n}(2r-1)^2 = \frac{2}{3}n(an^2+b)$$

where a and b are integers to be found.

(4)

---

8. The parabola C has equation $y^2 = 48x$.

The point $P(12t^2, 24t)$ is a general point on C.

(*a*) Find the equation of the directrix of C.

(2)

(*b*) Show that the equation of the tangent to C at $P(12t^2, 24t)$ is

$$x - ty + 12t^2 = 0.$$

(4)

The tangent to C at the point (3, 12) meets the directrix of C at the point X.

(*c*) Find the coordinates of X.

(4)

---

9. Prove by induction, that for $n \in \mathbb{Z}^+$,

(*a*) $$\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^n = \begin{pmatrix} 3^n & 0 \\ 3(3^n - 1) & 1 \end{pmatrix}$$

(6)

(*b*) $\text{f}(n) = 7^{2n-1} + 5$ is divisible by 12.

(6)

TOTAL FOR PAPER: 75 MARKS

END

PMT

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced Subsidiary

Monday 30 January 2012 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP1), the paper reference (6667), your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P40086A

1. Given that $z_1 = 1 - \text{i}$,

(*a*) find $\arg(z_1)$.

(2)

Given also that $z_2 = 3 + 4\text{i}$, find, in the form $a + \text{i}b$, $a, b \in \mathbb{R}$,

(*b*) $z_1 z_2$,

(2)

(*c*) $\dfrac{z_2}{z_1}$.

(3)

In part (*b*) and part (*c*) you must show all your working clearly.

---

2. (*a*) Show that $\text{f}(x) = x^4 + x - 1$ has a real root α in the interval [0.5, 1.0].

(2)

(*b*) Starting with the interval [0.5, 1.0], use interval bisection twice to find an interval of width 0.125 which contains α.

(3)

(*c*) Taking 0.75 as a first approximation, apply the Newton Raphson process twice to $\text{f}(x)$ to obtain an approximate value of α. Give your answer to 3 decimal places.

(5)

---

3. A parabola C has cartesian equation $y^2 = 16x$. The point $P(4t^2, 8t)$ is a general point on C.

(*a*) Write down the coordinates of the focus F and the equation of the directrix of C.

(3)

(*b*) Show that the equation of the normal to C at P is $y + tx = 8t + 4t^3$.

(5)

PMT

4. A right angled triangle T has vertices $A(1, 1)$, $B(2, 1)$ and $C(2, 4)$. When T is transformed by the matrix $\mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, the image is T'.

(*a*) Find the coordinates of the vertices of T'.
(2)

(*b*) Describe fully the transformation represented by $\mathbf{P}$.
(2)

The matrices $\mathbf{Q} = \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ represent two transformations. When T is transformed by the matrix $\mathbf{QR}$, the image is T''.

(*c*) Find $\mathbf{QR}$.
(2)

(*d*) Find the determinant of $\mathbf{QR}$.
(2)

(*e*) Using your answer to part (*d*), find the area of T''.
(3)

---

5. The roots of the equation

$$z^3 - 8z^2 + 22z - 20 = 0$$

are z_1, z_2 and z_3 .

(*a*) Given that $z_1 = 3 + \mathrm{i}$, find z_2 and z_3.
(4)

(*b*) Show, on a single Argand diagram, the points representing z_1, z_2 and z_3.
(2)

---

6. (*a*) Prove by induction

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2.$$

(5)

(*b*) Using the result in part (*a*), show that

$$\sum_{r=1}^{n} (r^3 - 2) = \frac{1}{4}n(n^3 + 2n^2 + n - 8).$$

(3)

(*c*) Calculate the exact value of $\sum_{r=20}^{50} (r^3 - 2)$.
(3)

---

7. A sequence can be described by the recurrence formula

$$u_{n+1} = 2u_n + 1, \qquad n \geq 1, \quad u_1 = 1.$$

(*a*) Find u_2 and u_3.
(2)

(*b*) Prove by induction that $u_n = 2^n - 1$.
(5)

8.
$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}.$$

(*a*) Show that $\mathbf{A}$ is non-singular.
(2)

(*b*) Find $\mathbf{B}$ such that $\mathbf{BA}^2 = \mathbf{A}$.
(4)

---

9. The rectangular hyperbola H has cartesian equation $xy = 9$.

The points $P\left(3p, \frac{3}{p}\right)$ and $Q\left(3q, \frac{3}{q}\right)$ lie on H, where $p \neq \pm q$.

(*a*) Show that the equation of the tangent at P is $x + p^2y = 6p$.
(4)

(*b*) Write down the equation of the tangent at Q.
(1)

The tangent at the point P and the tangent at the point Q intersect at R.

(*c*) Find, as single fractions in their simplest form, the coordinates of R in terms of p and q.
(4)

TOTAL FOR PAPER: 75 MARKS

END

PMT

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced Subsidiary

Friday 1 June 2012 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics FP1), the paper reference (6667), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 10 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P40688A

1. $$\mathrm{f}(x) = 2x^3 - 6x^2 - 7x - 4.$$

(*a*) Show that $\mathrm{f}(4) = 0$.

(1)

(*b*) Use algebra to solve $\mathrm{f}(x) = 0$ completely.

(4)

---

2. (*a*) Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 3 \\ 4 & 5 & 5 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix},$$

find **AB**.

(2)

(*b*) Given that

$$\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 8 & 6 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} 5 & 2k \\ 4 & k \end{pmatrix}, \text{ where } k \text{ is a constant}$$

and

$$\mathbf{E} = \mathbf{C} + \mathbf{D},$$

find the value of k for which **E** has no inverse.

(4)

---

3. $$\mathrm{f}(x) = x^2 + \frac{3}{4\sqrt{x}} - 3x - 7, \quad x > 0.$$

A root α of the equation $\mathrm{f}(x) = 0$ lies in the interval [3, 5].

Taking 4 as a first approximation to α, apply the Newton-Raphson process once to $\mathrm{f}(x)$ to obtain a second approximation to α. Give your answer to 2 decimal places.

(6)

PMT

4. (a) Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=1}^{n}(r^3 + 6r - 3) = \frac{1}{4}n^2(n + 2n + 13)$$

for all positive integers n.

(5)

(b) Hence find the exact value of

$$\sum_{r=16}^{30}(r^3 + 6r - 3).$$

(2)

5.

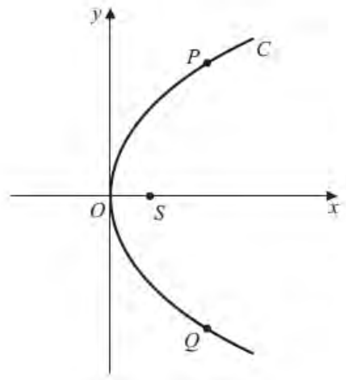

Figure 1

Figure 1 shows a sketch of the parabola C with equation $y^2 = 8x$.
The point P lies on C, where $y > 0$, and the point Q lies on C, where $y < 0$.
The line segment PQ is parallel to the y-axis.

Given that the distance PQ is 12,

(a) write down the y-coordinate of P,

(1)

(b) find the x-coordinate of P.

(2)

Figure 1 shows the point S which is the focus of C.

The line l passes through the point P and the point S.

(c) Find an equation for l in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

PMT

6. $$f(x) = \tan\left(\frac{x}{2}\right) + 3x - 6, \quad -\pi < x < \pi.$$

(*a*) Show that the equation $f(x) = 0$ has a root α in the interval $[1, 2]$.

(2)

(*b*) Use linear interpolation once on the interval $[1, 2]$ to find an approximation to α.
Give your answer to 2 decimal places.

(3)

---

7. $$z = 2 - i\sqrt{3}.$$

(*a*) Calculate $\arg z$, giving your answer in radians to 2 decimal places.

(2)

Use algebra to express

(*b*) $z + z^2$ in the form $a + bi\sqrt{3}$, where a and b are integers,

(3)

(*c*) $\dfrac{z+7}{z-1}$ in the form $c + di\sqrt{3}$, where c and d are integers.

(4)

Given that

$$w = \lambda - 3i,$$

where λ is a real constant, and $\arg(4 - 5i + 3w) = -\dfrac{\pi}{2}$,

(*d*) find the value of λ.

(2)

---

8. The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant.

The point $P\left(ct, \dfrac{c}{t}\right)$, $t \neq 0$, is a general point on H.

(*a*) Show that an equation for the tangent to H at P is

$$x + t^2 y = 2ct.$$

(4)

The tangent to H at the point P meets the x-axis at the point A and the y-axis at the point B.

Given that the area of the triangle OAB, where O is the origin, is 36,

(*b*) find the exact value of c, expressing your answer in the form $k\sqrt{2}$, where k is an integer.

(4)

PMT

9. $$\mathbf{M} = \begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}.$$

(*a*) Find det **M**.

(1)

The transformation represented by **M** maps the point $S(2a - 7, a - 1)$, where a is a constant, onto the point $S'(25, -14)$.

(*b*) Find the value of a.

(3)

The point R has coordinates $(6, 0)$.

Given that O is the origin,

(*c*) find the area of triangle ORS.

(2)

Triangle ORS is mapped onto triangle $OR'S'$ by the transformation represented by **M**.

(*d*) Find the area of triangle $OR'S'$.

(2)

Given that

$$\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

(*e*) describe fully the single geometrical transformation represented by **A**.

(2)

The transformation represented by **A** followed by the transformation represented by **B** is equivalent to the transformation represented by **M**.

(*f*) Find **B**.

(4)

---

10. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 2^{2n-1} + 3^{2n-1}$$

is divisible by 5.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced Subsidiary/Advanced Level

Monday 28 January 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP1), the paper reference (6667), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P41485A

1. Show, using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, that

$$\sum_{r=1}^{n} 3(2r-1)^2 = n(2n+1)(2n-1), \text{ for all positive integers } n.$$

(5)

---

2. $$z = \frac{50}{3+4\mathrm{i}}.$$

Find, in the form a + ib where $a, b \in \mathbb{R}$,

(a) z,

(2)

(b) z^2.

(2)

Find

(c) $|z|$,

(2)

(d) arg z^2, giving your answer in degrees to 1 decimal place.

(2)

---

3. $$f(x) = 2x^{\frac{1}{2}} + x^{-\frac{1}{2}} - 5, \qquad x > 0.$$

(a) Find $f'(x)$.

(2)

The equation $f(x) = 0$ has a root α in the interval [4.5, 5.5].

(b) Using $x_0 = 5$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 significant figures.

(4)

---

4. The transformation U, represented by the 2 × 2 matrix **P**, is a rotation through 90° anticlockwise about the origin.

(a) Write down the matrix **P**.

(1)

The transformation V, represented by the 2 × 2 matrix **Q**, is a reflection in the line $y = -x$.

(b) Write down the matrix **Q**.

(1)

Given that U followed by V is transformation T, which is represented by the matrix **R**,

(c) express **R** in terms of **P** and **Q**,

(1)

(d) find the matrix **R**,

(2)

(e) give a full geometrical description of T as a single transformation.

(2)

---

5. $$f(x) = (4x^2 + 9)(x^2 - 6x + 34).$$

(a) Find the four roots of $f(x) = 0$.

Give your answers in the form $x = p + \mathrm{i}q$, where p and q are real.

(5)

(b) Show these four roots on a single Argand diagram.

(2)

PMT

6. $$\mathbf{X} = \begin{pmatrix} 1 & a \\ 3 & 2 \end{pmatrix}, \text{ where } a \text{ is a constant.}$$

(*a*) Find the value of a for which the matrix $\mathbf{X}$ is singular.
(2)

$$\mathbf{Y} = \begin{pmatrix} 1 & -1 \\ 3 & 2 \end{pmatrix}.$$

(*b*) Find $\mathbf{Y}^{-1}$.
(2)

The transformation represented by $\mathbf{Y}$ maps the point A onto the point B.

Given that B has coordinates $(1 - \lambda, 7\lambda - 2)$, where λ is a constant,

(*c*) find, in terms of λ, the coordinates of point A.
(4)

---

7. The rectangular hyperbola, H, has cartesian equation $xy = 25$.

The point $P\left(5p, \frac{5}{p}\right)$ and the point $Q\left(5q, \frac{5}{q}\right)$, where $p, q \neq 0$, $p \neq q$, are points on the rectangular hyperbola H.

(*a*) Show that the equation of the tangent at point P is

$$p^2y + x = 10p.$$
(4)

(*b*) Write down the equation of the tangent at point Q.
(1)

The tangents at P and Q meet at the point N.

Given $p + q \neq 0$,

(*c*) show that point N has coordinates $\left(\frac{10pq}{p+q}, \frac{10}{p+q}\right)$.
(4)

The line joining N to the origin is perpendicular to the line PQ.

(*d*) Find the value of p^2q^2.
(5)

8. (*a*) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} r(r+3) = \frac{1}{3}n(n+1)(n+5).$$
(6)

(*b*) A sequence of positive integers is defined by

$$u_1 = 1,$$

$$u_{n+1} = u_n + n(3n+1), \quad n \in \mathbb{Z}^+.$$

Prove by induction that

$$u_n = n^2(n-1) + 1, \quad n \in \mathbb{Z}^+.$$
(5)

9.

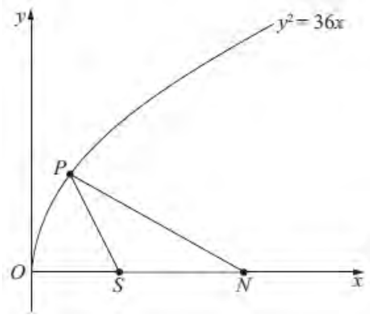

Figure 1

Figure 1 shows a sketch of part of the parabola with equation $y^2 = 36x$.

The point $P(4, 12)$ lies on the parabola.

(*a*) Find an equation for the normal to the parabola at P.
(5)

This normal meets the x-axis at the point N and S is the focus of the parabola, as shown in Figure 1.

(*b*) Find the area of triangle PSN.
(4)

TOTAL FOR PAPER: 75 MARKS

END

PMT

Paper Reference(s)
6667/01R

Edexcel GCE

Further Pure Mathematics FP1 (R)

Advanced/Advanced Subsidiary

Monday 10 June 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

This paper is strictly for students outside the UK.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 10 questions in this question paper. The total mark for this paper is 75.
There are 36 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P42828A

1. The complex numbers z and w are given by

$$z = 8 + 3\text{i}, \qquad w = -2\text{i}$$

Express in the form $a + b\text{i}$, where a and b are real constants,

(*a*) $z - w$,

(1)

(*b*) zw.

(2)

---

2. (i) $$\mathbf{A} = \begin{pmatrix} 2k+1 & k \\ -3 & -5 \end{pmatrix}, \text{ where } k \text{ is a constant}$$

Given that

$$\mathbf{B} = \mathbf{A} + 3\mathbf{I}$$

where $\mathbf{I}$ is the 2×2 identity matrix, find

(*a*) $\mathbf{B}$ in terms of k,

(2)

(*b*) the value of k for which $\mathbf{B}$ is singular.

(2)

(ii) Given that

$$\mathbf{C} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}, \qquad \mathbf{D} = \begin{pmatrix} 2 & -1 & 5 \end{pmatrix}$$

and

$$\mathbf{E} = \mathbf{CD}$$

find $\mathbf{E}$.

(2)

---

3. $$f(x) = \frac{1}{2}x^4 - x^3 + x - 3$$

(a) Show that the equation $f(x) = 0$ has a root α between $x = 2$ and $x = 2.5$.

(2)

(b) Starting with the interval [2, 2.5] use interval bisection twice to find an interval of width 0.125 which contains α.

(3)

The equation $f(x) = 0$ has a root β in the interval $[-2, -1]$.

(c) Taking -1.5 as a first approximation to β, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to β.
Give your answer to 2 decimal places.

(5)

---

4. $$f(x) = (4x^2 + 9)(x^2 - 2x + 5)$$

(a) Find the four roots of $f(x) = 0$.

(4)

(b) Show the four roots of $f(x) = 0$ on a single Argand diagram.

(2)

5.

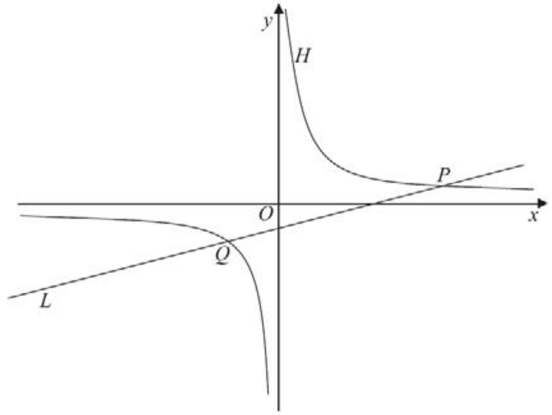

Figure 1

Figure 1 shows a rectangular hyperbola H with parametric equations

$$x = 3t, \quad y = \frac{3}{t}, \quad t \neq 0$$

The line L with equation $6y = 4x - 15$ intersects H at the point P and at the point Q as shown in Figure 1.

(a) Show that L intersects H where $4t^2 - 5t - 6 = 0$.

(3)

(b) Hence, or otherwise, find the coordinates of points P and Q.

(5)

PMT

6. $$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$$

The transformation represented by **B** followed by the transformation represented by **A** is equivalent to the transformation represented by **P**.

(*a*) Find the matrix **P**.

(2)

Triangle T is transformed to the triangle T' by the transformation represented by **P**.

Given that the area of triangle T' is 24 square units,

(*b*) find the area of triangle T.

(3)

Triangle T' is transformed to the original triangle T by the matrix represented by **Q**.

(*c*) Find the matrix **Q**.

(2)

---

7. The parabola C has equation $y^2 = 4ax$, where a is a positive constant.

The point $P(at^2, 2at)$ is a general point on C.

(*a*) Show that the equation of the tangent to C at $P(at^2, 2at)$ is

$$ty = x + at^2$$

(4)

The tangent to C at P meets the y-axis at a point Q.

(*b*) Find the coordinates of Q.

(1)

Given that the point S is the focus of C,

(*c*) show that PQ is perpendicular to SQ.

(3)

---

8. (*a*) Prove by induction, that for $n \in \square^+$,

$$\sum_{r=1}^{n} r(2r-1) = \frac{1}{6}n(n+1)(4n-1)$$

(6)

(*b*) Hence, show that

$$\sum_{r=n+1}^{3n} r(2r-1) = \frac{1}{3}n(an^2 + bn + c)$$

where a, b and c are integers to be found.

(4)

PMT

9. The complex number w is given by

$$w = 10 - 5\mathrm{i}$$

(a) Find $|w|$.

(1)

(b) Find arg w, giving your answer in radians to 2 decimal places

(2)

The complex numbers z and w satisfy the equation

$$(2 + \mathrm{i})(z + 3\mathrm{i}) = w$$

(c) Use algebra to find z, giving your answer in the form $a + b\mathrm{i}$, where a and b are real numbers.

(4)

Given that

$$\arg(\lambda + 9\mathrm{i} + w) = \frac{\pi}{4}$$

where λ is a real constant,

(d) find the value of λ.

(2)

10. (i) Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to evaluate

$$\sum_{r=1}^{24} (r^3 - 4r)$$

(2)

(ii) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=0}^{n} (r^2 - 2r + 2n + 1) = \frac{1}{6}(n+1)(n+a)(bn+c)$$

for all integers $n \geq 0$, where a, b and c are constant integers to be found.

(6)

TOTAL FOR PAPER: 75 MARKS

END

PMT

Paper Reference(s)

6667/01
Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Monday 10 June 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43138A

1.

$$\mathbf{M} = \begin{pmatrix} x & x-2 \\ 3x-6 & 4x-11 \end{pmatrix}$$

Given that the matrix **M** is singular, find the possible values of x.

(4)

2.

$$f(x) = \cos(x^2) - x + 3, \qquad 0 < x < \pi$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [2.5, 3].

(2)

(b) Use linear interpolation once on the interval [2.5, 3] to find an approximation for α, giving your answer to 2 decimal places.

(3)

---

3. Given that $x = \frac{1}{2}$ is a root of the equation

$$2x^3 - 9x^2 + kx - 13 = 0, \qquad k \in \mathbb{R}$$

find

(a) the value of k,

(3)

(b) the other 2 roots of the equation.

(4)

---

4. The rectangular hyperbola H has Cartesian equation $xy = 4$.

The point $P\left(2t, \frac{2}{t}\right)$ lies on H, where $t \neq 0$.

(a) Show that an equation of the normal to H at the point P is

$$ty - t^3x = 2 - 2t^4$$

(5)

The normal to H at the point where $t = -\frac{1}{2}$ meets H again at the point Q.

(b) Find the coordinates of the point Q.

(4)

PMT

5. (*a*) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n}(r+2)(r+3) = \frac{1}{3}n(n^2+9n+26)$$

for all positive integers n.

(6)

(*b*) Hence show that

$$\sum_{r=n+1}^{3n}(r+2)(r+3) = \frac{2}{3}n(an^2+bn+c)$$

where a, b and c are integers to be found.

(4)

---

6. A parabola C has equation $y^2 = 4ax, \quad a > 0$

The points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ lie on C, where $p \neq 0$, $q \neq 0$, $p \neq q$.

(*a*) Show that an equation of the tangent to the parabola at P is

$$py - x = ap^2$$

(4)

(*b*) Write down the equation of the tangent at Q.

(1)

The tangent at P meets the tangent at Q at the point R.

(*c*) Find, in terms of p and q, the coordinates of R, giving your answers in their simplest form.

(4)

Given that R lies on the directrix of C,

(*d*) find the value of pq.

(2)

---

7. $$z_1 = 2 + 3\text{i}, \qquad z_2 = 3 + 2\text{i}, \qquad z_3 = a + b\text{i}, \qquad a, b \in \mathbb{R}$$

(*a*) Find the exact value of $|z_1 + z_2|$.

(2)

Given that $w = \dfrac{z_1 z_3}{z_2}$,

(*b*) find w in terms of a and b, giving your answer in the form $x + \text{i}y, \quad x, y \in \mathbb{R}$.

(4)

Given also that $w = \dfrac{17}{13} - \dfrac{7}{13}\text{i}$,

(*c*) find the value of a and the value of b,

(3)

(*d*) find arg w, giving your answer in radians to 3 decimal places.

(2)

8.

$$\mathbf{A} = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}$$

and $\mathbf{I}$ is the 2×2 identity matrix.

(*a*) Prove that

$$\mathbf{A}^2 = 7\mathbf{A} + 2\mathbf{I}$$

(2)

(*b*) Hence show that

$$\mathbf{A}^{-1} = \frac{1}{2}(\mathbf{A} - 7\mathbf{I})$$

(2)

The transformation represented by $\mathbf{A}$ maps the point P onto the point Q.

Given that Q has coordinates $(2k + 8, -2k - 5)$, where k is a constant,

(*c*) find, in terms of k, the coordinates of P.

(4)

PMT

9. (*a*) A sequence of numbers is defined by

$$u_1 = 8$$

$$u_{n+1} = 4u_n - 9n, n \geq 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = 4^n + 3n + 1$$

(5)

(*b*) Prove by induction that, for $m \in \mathbb{Z}^+$,

$$\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^m = \begin{pmatrix} 2m+1 & -4m \\ m & 1-2m \end{pmatrix}$$

(5)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

WFM01/01

Pearson Edexcel International Advanced Level

Further Pure Mathematics F1

Advanced/Advanced Subsidiary

Wednesday 29 January 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Blue)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

P44967A

1. $$\mathrm{f}(x) = 6\sqrt{x} - x^2 - \frac{1}{2x}, \qquad x > 0$$

(*a*) Show that the equation $\mathrm{f}(x) = 0$ has a root α in the interval $[3, 4]$.
(2)

(*b*) Taking 3 as a first approximation to α, apply the Newton-Raphson process once to $\mathrm{f}(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places.
(5)

(*c*) Use linear interpolation once on the interval $[3, 4]$ to find another approximation to α. Give your answer to 3 decimal places.
(3)

---

2. The quadratic equation

$$5x^2 - 4x + 2 = 0$$

has roots α and β.

(*a*) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$.
(2)

(*b*) Find the value of $\alpha^2 + \beta^2$.
(2)

(*c*) Find a quadratic equation which has roots

$$\frac{1}{\alpha^2} \text{ and } \frac{1}{\beta^2}$$

giving your answer in the form $px^2 + qx + r = 0$, where p, q and r are integers.
(4)

---

3. $$\mathbf{A} = \begin{pmatrix} 6 & 4 \\ 1 & 1 \end{pmatrix}$$

(*a*) Show that $\mathbf{A}$ is non-singular.
(2)

The triangle R is transformed to the triangle S by the matrix $\mathbf{A}$.

Given that the area of triangle R is 10 square units,

(*b*) find the area of triangle S.
(2)

Given that

$$\mathbf{B} = \mathbf{A}^4$$

and that the triangle R is transformed to the triangle T by the matrix $\mathbf{B}$,

(*c*) find, without evaluating $\mathbf{B}$, the area of triangle T.
(2)

---

4. $$\mathrm{f}(x) = x^4 + 3x^3 - 5x^2 - 19x - 60$$

(*a*) Given that $x = -4$ and $x = 3$ are roots of the equation $\mathrm{f}(x) = 0$, use algebra to solve $\mathrm{f}(x) = 0$ completely.
(7)

(*b*) Show the four roots of $\mathrm{f}(x) = 0$ on a single Argand diagram.
(2)

---

5. (*a*) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} (9r^2 - 4r) = \frac{1}{2}n(n+1)(6n-1)$$

for all positive integers n.
(4)

Given that

$$\sum_{r=1}^{12} (9r^2 - 4r + k(2^r)) = 6630$$

(*b*) find the exact value of the constant k.
(4)

PMT

6.

(i) $$\mathbf{B} = \begin{pmatrix} -1 & 2 \\ 3 & -4 \end{pmatrix}, \qquad \mathbf{Y} = \begin{pmatrix} 4 & -2 \\ 1 & 0 \end{pmatrix}$$

(a) Find $\mathbf{B}^{-1}$.

(2)

The transformation represented by **Y** is equivalent to the transformation represented by **B** followed by the transformation represented by the matrix **A**.

(b) Find **A**.

(2)

(ii) $$\mathbf{M} = \begin{pmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{3} \end{pmatrix}$$

The matrix **M** represents an enlargement scale factor k, centre (0, 0), where $k > 0$, followed by a rotation anti-clockwise through an angle θ about (0, 0).

(a) Find the value of k.

(2)

(b) Find the value of θ.

(2)

---

7. (i) Given that

$$\frac{2w-3}{10} = \frac{4+7\mathrm{i}}{4-3\mathrm{i}}$$

find w, giving your answer in the form $a + b\mathrm{i}$, where a and b are real constants.
You must show your working.

(4)

(ii) Given that

$$z = (2 + \lambda\mathrm{i})(5 + \mathrm{i})$$

where λ is a real constant, and that

$$\arg z = \frac{\pi}{4}$$

find the value of λ.

(4)

---

8. The parabola C has equation $y^2 = 4ax$, where a is a positive constant.

The point $P(ap^2, 2ap)$ lies on the parabola C.

(a) Show that an equation of the tangent to C at P is

$$py = x + ap^2$$

(4)

The tangent to C at the point P intersects the directrix of C at the point B and intersects the x-axis at the point D.

Given that the y-coordinate of B is $\frac{5}{6}a$ and $p > 0$,

(b) find, in terms of a, the x-coordinate of D.

(6)

Given that O is the origin,

(c) find, in terms of a, the area of the triangle OPD, giving your answer in its simplest form.

(2)

---

9. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\mathrm{f}(n) = 7^n - 2^n \text{ is divisible by } 5$$

(6)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

WFM01/01

Pearson Edexcel International Advanced Level

Further Pure Mathematics F1

Advanced/Advanced Subsidiary

Monday 23 June 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Blue)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

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- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
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- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

P44965A

1. Find the value of

$$\sum_{r=1}^{200}(r+1)(r-1)$$

(4)

---

2. Given that $-2 + 3\text{i}$ is a root of the equation

$$z^2 + pz + q = 0$$

where p and q are real constants,

(*a*) write down the other root of the equation.

(1)

(*b*) Find the value of p and the value of q.

(3)

3.

$$\mathbf{A} = \begin{pmatrix} 4 & -2 \\ a & -3 \end{pmatrix}$$

where a is a real constant and $a \neq 6$.

(*a*) Find $\mathbf{A}^{-1}$ in terms of a.

(3)

Given that $\mathbf{A} + 2\mathbf{A}^{-1} = \mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix,

(*b*) find the value of a.

(3)

4.

$$\text{f}(x) = x^{\frac{3}{2}} - 3x^{\frac{1}{2}} - 3, \qquad x > 0$$

Given that α is the only real root of the equation $\text{f}(x) = 0$,

(*a*) show that $4 < \alpha < 5$.

(2)

(*b*) Taking 4.5 as a first approximation to α, apply the Newton-Raphson procedure once to $\text{f}(x)$ to find a second approximation to α, giving your answer to 3 decimal places.

(5)

(*c*) Use linear interpolation once on the interval [4, 5] to find another approximation to α, giving your answer to 3 decimal places.

(3)

PMT

5. Given that $z_1 = -3 - 4\text{i}$ and $z_2 = 4 - 3\text{i}$

(*a*) show, on an Argand diagram, the point P representing z_1 and the point Q representing z_2. **(2)**

(*b*) Given that O is the origin, show that OP is perpendicular to OQ. **(2)**

(*c*) Show the point R on your diagram, where R represents $z_1 + z_2$. **(1)**

(*d*) Prove that $OPRQ$ is a square. **(2)**

---

6. It is given that α and β are roots of the equation $3x^2 + 5x - 1 = 0$.

(*a*) Find the exact value of $\alpha^3 + \beta^3$. **(3)**

(*b*) Find a quadratic equation which has roots $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$, giving your answer in the form $ax^2 + bx + c = 0$, where a, b and c are integers. **(5)**

7.

$$\mathbf{P} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

(*a*) Describe fully the single geometrical transformation U represented by the matrix $\mathbf{P}$. **(3)**

The transformation V, represented by the 2×2 matrix $\mathbf{Q}$, is a reflection in the x-axis.

(*b*) Write down the matrix $\mathbf{Q}$. **(1)**

Given that V followed by U is the transformation T, which is represented by the matrix $\mathbf{R}$,

(*c*) find the matrix $\mathbf{R}$. **(2)**

(*d*) Show that there is a real number k for which the transformation T maps the point $(1, k)$ onto itself. Give the exact value of k in its simplest form. **(5)**

---

8. The hyperbola H has cartesian equation $xy = 16$.
The parabola P has parametric equations $x = 8t^2$, $y = 16t$.

(*a*) Find, using algebra, the coordinates of the point A where H meets P. **(3)**

Another point B (8, 2) lies on the hyperbola H.

(*b*) Find the equation of the normal to H at the point (8, 2), giving your answer in the form $y = mx + c$, where m and c are constants. **(5)**

(*c*) Find the coordinates of the points where this normal at B meets the parabola P. **(6)**

---

9. (i) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} r(r+1)(r+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

(5)

(ii) Prove by induction that,

$4^n + 6n + 8$ is divisible by 18

for all positive integers n. **(6)**

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6667/01R

Edexcel GCE

Further Pure Mathematics FP1 (R)

Advanced/Advanced Subsidiary

Tuesday 10 June 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

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Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
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There are 9 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43152A

1. The roots of the equation

$$2z^3 - 3z^2 + 8z + 5 = 0$$

are z_1, z_2 and z_3.

Given that $z_1 = 1 + 2\text{i}$, find z_2 and z_3.

(5)

---

2. $$\text{f}(x) = 3\cos 2x + x - 2, \qquad -\pi \leq x < \pi$$

(*a*) Show that the equation $\text{f}(x) = 0$ has a root α in the interval [2, 3].

(2)

(*b*) Use linear interpolation once on the interval [2, 3] to find an approximation to α.

Give your answer to 3 decimal places.

(3)

(*c*) The equation $\text{f}(x) = 0$ has another root β in the interval [–1, 0]. Starting with this interval, use interval bisection to find an interval of width 0.25 which contains β.

(4)

---

3. (i)

$$\mathbf{A} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(*a*) Describe fully the single transformation represented by the matrix **A**.

(2)

The matrix **B** represents an enlargement, scale factor –2, with centre the origin.

(*b*) Write down the matrix **B**.

(1)

(ii)

$$\mathbf{M} = \begin{pmatrix} 3 & k \\ -2 & 3 \end{pmatrix}, \qquad \text{where } k \text{ is a positive constant.}$$

Triangle T has an area of 16 square units.

Triangle T is transformed onto the triangle T' by the transformation represented by the matrix **M**.

Given that the area of the triangle T' is 224 square units, find the value of k.

(3)

PMT

4. The complex number z is given by

$$z = \frac{p + 2\mathrm{i}}{3 + p\mathrm{i}}$$

where p is an integer.

(a) Express z in the form $a + b\mathrm{i}$ where a and b are real. Give your answer in its simplest form in terms of p.
(4)

(b) Given that $\arg(z) = \theta$, where $\tan\theta = 1$ find the possible values of p.
(5)

---

5. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$ to show that

$$\sum_{r=1}^{n} r\left(r^2 - 3\right) = \frac{1}{4}n(n+1)(n+3)(n-2)$$

(5)

(b) Calculate the value of $\sum_{r=10}^{50} r\left(r^2 - 3\right)$.
(3)

6.

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}$$

Given that $\mathbf{M} = (\mathbf{A} + \mathbf{B})(2\mathbf{A} - \mathbf{B})$,

(a) calculate the matrix $\mathbf{M}$,
(6)

(b) find the matrix $\mathbf{C}$ such that $\mathbf{MC} = \mathbf{A}$.
(4)

---

7. The parabola C has cartesian equation $y^2 = 4ax$, $a > 0$.

The points $P(ap^2, 2ap)$ and $P'(ap^2, -2ap)$ lie on C.

(a) Show that an equation of the normal to C at the point P is

$$y + px = 2ap + ap^3$$

(5)

(b) Write down an equation of the normal to C at the point P'.
(1)

The normal to C at P meets the normal to C at P' at the point Q.

(c) Find, in terms of a and p, the coordinates of Q.
(2)

Given that S is the focus of the parabola,

(d) find the area of the quadrilateral $SPQP'$.
(3)

---

8. The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant.

The point $P\left(ct, \frac{c}{t}\right)$, $t \neq 0$ is a general point on H.

An equation for the tangent to H at P is given by

$$y = \frac{1}{t^2}x + \frac{2c}{t}$$

The points A and B lie on H.

The tangent to H at A and the tangent to H at B meet at the point $\left(-\frac{6}{7}c, \frac{12}{7}c\right)$.

Find, in terms of c, the coordinates of A and the coordinates of B.
(5)

PMT

9. (*a*) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} (r+1)2^{r-1} = n2^n$$

(5)

(*b*) A sequence of numbers is defined by

$$u_1 = 0, \qquad u_2 = 32,$$

$$u_{n+2} = 6u_{n+1} - 8u_n \qquad n \geq 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = 4^{n+1} - 2^{n+3}$$

(7)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Tuesday 10 June 2014 – Morning

Time: 1 hour 30 minutes

<u>**Materials required for examination**</u>	<u>**Items included with question papers**</u>
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43153A

PMT

1. The complex numbers z_1 and z_2 are given by

$$z_1 = p + 2\text{i} \text{ and } z_2 = 1 - 2\text{i}$$

where p is an integer.

(*a*) Find $\dfrac{z_1}{z_2}$ in the form $a + b\text{i}$ where a and b are real. Give your answer in its simplest form in terms of p.

(4)

Given that $\left|\dfrac{z_1}{z_2}\right| = 13$,

(*b*) find the possible values of p.

(4)

2.

$$\text{f}(x) = x^3 - \frac{5}{2x^{\frac{3}{2}}} + 2x - 3, \qquad x > 0$$

(*a*) Show that the equation $\text{f}(x) = 0$ has a root α in the interval $[1.1, 1.5]$.

(2)

(*b*) Find $\text{f}'(x)$.

(2)

(*c*) Using $x_0 = 1.1$ as a first approximation to α, apply the Newton-Raphson procedure once to $\text{f}(x)$ to find a second approximation to α, giving your answer to 3 decimal places.

(3)

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3. Given that 2 and $1 - 5\text{i}$ are roots of the equation

$$x^3 + px^2 + 30x + q = 0, \qquad p, q \in \mathbb{R}$$

(*a*) write down the third root of the equation.

(1)

(*b*) Find the value of p and the value of q.

(5)

(*c*) Show the three roots of this equation on a single Argand diagram.

(2)

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4. (i) Given that

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix},$$

(*a*) find **AB**.

(*b*) Explain why $\mathbf{AB} \neq \mathbf{BA}$.

(4)

(ii) Given that

$$\mathbf{C} = \begin{pmatrix} 2k & -2 \\ 3 & k \end{pmatrix}, \text{ where } k \text{ is a real number}$$

find $\mathbf{C}^{-1}$, giving your answer in terms of k.

(3)

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5. (*a*) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} (2r-1)^2 = \frac{1}{3}n(4n^2 - 1)$$

(6)

(*b*) Hence show that

$$\sum_{r=2n+1}^{4n} (2r-1)^2 = an(bn^2 - 1)$$

where a and b are constants to be found.

(3)

PMT

6. The rectangular hyperbola H has cartesian equation $xy = c^2$.

The point $P\left(ct, \frac{c}{t}\right)$, $t > 0$, is a general point on H.

(*a*) Show that an equation of the tangent to H at the point P is

$$t^2y + x = 2ct$$

(4)

An equation of the normal to H at the point P is $t^3x - ty = ct^4 - c$.

Given that the normal to H at P meets the x-axis at the point A and the tangent to H at P meets the x-axis at the point B,

(*b*) find, in terms of c and t, the coordinates of A and the coordinates of B.

(2)

Given that $c = 4$,

(*c*) find, in terms of t, the area of the triangle APB. Give your answer in its simplest form.

(3)

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7. (i) In each of the following cases, find a 2 × 2 matrix that represents

(*a*) a reflection in the line $y = -x$,

(*b*) a rotation of 135° anticlockwise about (0, 0),

(*c*) a reflection in the line $y = -x$ followed by a rotation of 135° anticlockwise about (0, 0).

(4)

(ii) The triangle T has vertices at the points $(1, k)$, $(3, 0)$ and $(11, 0)$, where k is a constant. Triangle T is transformed onto the triangle T' by the matrix

$$\begin{pmatrix} 6 & -2 \\ 1 & 2 \end{pmatrix}$$

Given that the area of triangle T' is 364 square units, find the value of k.

(6)

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8. The points $P(4k^2, 8k)$ and $Q(k^2, 4k)$, where k is a constant, lie on the parabola C with equation $y^2 = 16x$.

The straight line l_1 passes through the points P and Q.

(*a*) Show that an equation of the line l_1 is given by

$$3ky - 4x = 8k^2$$

(4)

The line l_2 is perpendicular to the line l_1 and passes through the focus of the parabola C. The line l_2 meets the directrix of C at the point R.

(*b*) Find, in terms of k, the y coordinate of the point R.

(7)

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9. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 8^n - 2^n$$

is divisible by 6.

(6)

TOTAL FOR PAPER: 75 MARKS

END