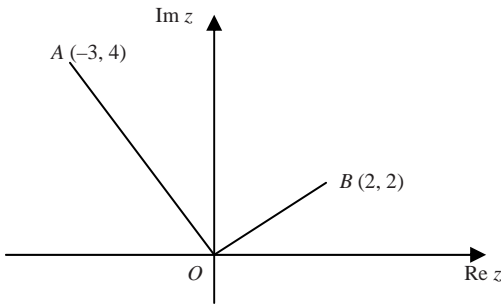


## 6667/01: Further Pure Mathematics FP1

Question number	Scheme	Marks
1.	<p>(a) <math>f'(x) = 3x^2 - 6x + 5</math></p> <p>(b) <math>f(1.4) = -0.136</math>  <math>f'(1.4) = 2.48</math>  <math>x_0 = 1.4, x_1 = 1.4 - \frac{-0.136}{2.48}</math>  <math>= 1.455</math> (3 dpl)</p>	<p>M1A1 (2)</p> <p>B1</p> <p>B1ft</p> <p>M1</p> <p>A1 (4)</p> <p><b>(6 marks)</b></p>
2.	<p>(a) <math>\begin{pmatrix} a &amp; 4 \\ -1 &amp; 1 \end{pmatrix} \begin{pmatrix} 0 &amp; 1 &amp; 1 \\ 2 &amp; 0 &amp; 2 \end{pmatrix} = \begin{pmatrix} 8 &amp; a &amp; a+8 \\ 2 &amp; -1 &amp; 1 \end{pmatrix}</math></p> <p>(b) <math>\det \mathbf{A} = a - (-4) = a + 4</math></p> <p>(c) Area of <math>R = 2</math>  Area of <math>R' = 18</math>  Area scale factor is <math>9 = a + 4</math>  <math>\therefore a = 5</math></p>	<p>M1 A1 A1 (3)</p> <p>B1 (1)</p> <p>B1</p> <p>M1</p> <p>A1 (3)</p> <p><b>(7 marks)</b></p>
3.	<p>(a) <math>\mathbf{R}^2 = \begin{pmatrix} 0 &amp; 1 \\ -1 &amp; 0 \end{pmatrix}</math></p> <p>(b) Rotation of <math>90^\circ</math>, clockwise (about (0,0))</p> <p>(c) Rotation of <math>45^\circ</math> clockwise</p>	<p>M1 A1 (2)</p> <p>B1, B1 (2)</p> <p>B1ft (1)</p> <p><b>(5 marks)</b></p>
4.	<p>End points: (4, -8) and (5, 2)</p> <p><math>\frac{\alpha - 4}{8} = \frac{5 - \alpha}{2}</math> (or equiv.)</p> <p><math>\alpha = 4.8</math></p>	<p>B1</p> <p>M1</p> <p>A1 (3)</p> <p><b>(3 marks)</b></p>

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5. (a)	$\sum_{r=1}^n (r^2 - r - 1) = \sum_{r=1}^n r^2 - \sum_{r=1}^n r - \sum_{r=1}^n 1$ $\sum_{r=1}^n 1 = n$ $\sum_{r=1}^n (r^2 - r - 1) = \frac{n}{6}(n+1)(2n+1) - \frac{1}{2}n(n+1) - n$ $= \frac{n}{6}(2n^2 - 8)$ $= \frac{1}{3}(n-2)n(n+2) \quad (*)$	M1 B1 M1 M1 A1 A1 (6)
(b)	$\sum_{r=10}^{40} (r^2 - r - 1) = \sum_{r=1}^{40} (r^2 - r - 1) - \sum_{r=1}^9 (r^2 - r - 1)$ $= \frac{1}{3} \times 3 \times 40 \times 42 - \frac{1}{3} \times 7 \times 9 \times 11 = \frac{1449}{21049}$	M1 M1 A1 (3)
<b>(9 marks)</b>		
6. (a)	$ z  = \sqrt{3^2 + 4^2} = 5$	M1 A1 (2)
(b)	$\arg z = \pi - \arctan \frac{4}{3} = 2.21$	M1 A1 (2)
(c)	$w = \frac{-14 + 2i}{-3 + 4i} = \frac{(-14 + 2i)(-3 - 4i)}{(-3 + 4i)(-3 - 4i)}$ $= \frac{(42 + 8) + i(-6 + 56)}{9 + 16}$ $= \frac{50 + 50i}{25} = 2 + 2i$	M1 A1 A1 A1 (4)
(d)		B1 B1 (2)
<b>(10 marks)</b>		

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7. (a)	$a = 4$	B1 (1)
(b)	$y = 2a^{\frac{1}{2}}x^{\frac{1}{2}} \Rightarrow y' = a^{\frac{1}{2}}x^{-\frac{1}{2}}$ $y =$ and attempt $y'$ $y' = \frac{1}{t}$ sub $x = 4t^2$ Tangent is $y - 8t = \frac{1}{t}(x - 4t^2)$ $yt = x + 4t^2$ (*)	M1 M1 M1
(c)	$x = -4$ $15t = -4 + 4t^2$ Substitute $(-4, 15)$ $4t^2 - 15t - 4 = 0$ $(4t + 1)(t - 4) = 0$ Attempt to solve $t = 4$ or $-\frac{1}{4}$ $A = (64, 32)$ $B = (\frac{1}{4}, -2)$ M for attempt A or B	B1ft M1 M1 A1 M1 A1 A1 (7)
		<b>(12 marks)</b>
8. (a)	$1 + 2i$	B1 (1)
(b)	$(x - 1 + 2i)(x - 1 - 2i)$ are factors of $f(x)$ so $x^2 - 2x + 5$ is a factor of $f(x)$ $f(x) = (x^2 - 2x + 5)(2x - 1)$ Third root is $\frac{1}{2}$	M1 M1 A1 M1 A1ft A1 (6)
(c)	$p = 10 + 2$ $= 12$	M1 A1 (2)
		<b>(9 marks)</b>

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<p>9. (a)</p> <p>(b)</p>	$\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}^1 = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}, \text{ for } n = 1, \begin{pmatrix} n+1 & n \\ -n & 1-n \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$ <p style="text-align: center;"><math>\therefore</math> true for <math>n = 1</math></p> <p>Assume true for <math>n = k</math>,</p> $\begin{pmatrix} k+1 & k \\ -k & 1-k \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 2k+2-k & k+1-0 \\ -2k-1+k & -k+0 \end{pmatrix}$ $= \begin{pmatrix} (k+1)+1 & k+1 \\ -(k+1) & 1-(k+1) \end{pmatrix}$ <p style="text-align: center;"><math>\therefore</math> true for <math>n = k + 1</math> if true for <math>n = k</math></p> <p style="text-align: center;"><math>\therefore</math> true for <math>n \in \mathbb{Z}^+</math> by induction</p> <p><math>f(1) = 4 + 6 - 1 = 9 = 3 \times 3</math></p> <p style="text-align: center;"><math>\therefore</math> true for <math>n = 1</math></p> <p>Assume true for <math>n = k</math>, <math>f(k) = 4^k + 6k - 1</math> is divisible by 3</p> $f(k+1) = 4^{k+1} + 6(k+1) - 1$ $= 4 \times 4^k + 6(k+1) - 1$ <p><math>f(k+1) - f(k) = 3 \times 4^k + 6</math></p> <p style="text-align: center;"><math>\therefore f(k+1) = 3(4^k + 2) - f(k)</math> which is divisible by 3</p> <p style="text-align: center;"><math>\therefore</math> true for <math>n = k + 1</math> if true for <math>n = k</math></p> <p style="text-align: center;"><math>\therefore</math> true for <math>n \in \mathbb{Z}^+</math> by induction</p>	<p>B1</p> <p>M1 A2/1/0</p> <p>M1 A1</p> <p>A1 (7)</p> <p>B1</p> <p>M1 A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1 (7)</p> <p><b>(14 marks)</b></p>