Mark Scheme (Final)

Summer 2018

Pearson Edexcel GCE
In Further Pure Mathematics FP1 (6667/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Question Number	Scheme	Notes	Marks
1.	$\text{f}(z) = 2z^3 - 4z^2 + 15z - 13 \equiv (z-1)(2z^2 + az + b)$		
(a)	$a = -2,\ b = 13$	At least one of either $a = -2$ or $b = 13$ or seen as their coefficients.	B1
		Both $a = -2$ and $b = 13$ or seen as their coefficients.	B1
			[2]
(b)	$\{z =\}$ 1 is a root	1 is a root, seen anywhere.	B1
	$\left\{2z^2 - 2z + 13 = 0 \Rightarrow z^2 - z + \frac{13}{2} = 0\right\}$		
	Either • $z = \dfrac{2 \pm \sqrt{4 - 4(2)(13)}}{2(2)}$	Correct method for solving a 3-term quadratic equation. Do not allow M1 here for an attempt at factorising.	M1
	or • $\left(z - \frac{1}{2}\right)^2 - \frac{1}{4} + \frac{13}{2} = 0$ and $z = \ldots$		
	or • $(2z-1)^2 - 1 + +13 = 0$ and $z = \ldots$		
	So, $\{z =\}$ $\frac{1}{2} + \frac{5}{2}\text{i},\ \frac{1}{2} - \frac{5}{2}\text{i}$	At least one of either $\frac{1}{2} + \frac{5}{2}\text{i}$ or $\frac{1}{2} - \frac{5}{2}\text{i}$ or any equivalent form.	A1
		For conjugate of first complex root	A1ft
			[4]
			Total 6

Question Number	Scheme	Notes	Marks
2. (a)	$f(-3) = 2.05555555...$ $f(-2.5) = -1.15833333...$	Attempt **both** of $f(-3) = \text{awrt } 2.1 \text{ or trunc } 2 \text{ or } 2.0 \text{ or } \frac{37}{18}$ **and** $f(-2.5) = \text{awrt } -1.2 \text{ or trunc } -1.1 \text{ or } -\frac{139}{120}$	M1
	Sign change oe (and $f(x)$ is continuous) therefore a root α {exists in the interval $[-3, -2.5]$.}	Both $f(-3) = \text{awrt } 2.1$ and $f(-2.5) = \text{awrt } -1.2$, sign change and 'root' or 'α'. Any errors award A0.	A1
			[2]
(b)	$f'(x) = 3x - \frac{4}{3x^2} + 2$	$\frac{3}{2}x^2 \to \pm Ax$ or $\frac{4}{3x} \to \pm Bx^{-2}$ or $2x - 5 \to 2$ Calculus must be seen for this to be awarded.	M1
		At least two terms differentiated correctly	A1
		Correct derivative.	A1
	$\alpha = -3 - \left(\frac{"2.055..."}{"-7.148..."}\right)$	Correct application of Newton-Raphson **using their values from calculus**.	M1
	$= -2.71243523...$ or $-\frac{1047}{386}$ or $-2\frac{275}{386}$	Exact value or awrt -2.712	A1
			[5]
(c)	$\frac{-2.5-\alpha}{"1.158..."} = \frac{\alpha - -3}{"2.055..."}$ or $\frac{\alpha - -3}{"2.055..."} = \frac{-2.5 - -3}{"2.055..." + "1.158..."}$	A correct linear interpolation statement with correct signs. $\frac{-2.5+\alpha}{"1.158..."} = \frac{-\alpha - -3}{"2.055..."}$ provided α sign changed at the end. Do not award until α is seen.	M1
	$\alpha = -3 + \left(\frac{"2.055..."}{"2.055..." + "1.158..."}\right)(0.5)$ or $\alpha = -3 + \left(\frac{"2.055..."}{"3.213..."}\right)(0.5)$ or $\alpha = \left(\frac{(-2.5)("2.055...") - 3("1.158...")}{"2.055..." + "1.158..."}\right)$	Achieves a correct linear interpolation statement with correct signs for $\alpha = ...$ dependent on the previous method mark.	dM1
	$= -2.68020743...$ or $-\frac{3101}{1157}$ or $-2\frac{787}{1157}$		
	$= -2.680$ (3 dp)	-2.680: only penalise accuracy once in (b) and (c), but must be to at least 3sf.	A1 **cao**

ALT (c)	The gradient of the line between (-3, 2.055…) and (-2.5, -1.158…) is $\frac{2.055...--1.158...}{-3--2.5}=-6.427...$		
	Equation of the line joining the points $y-2.055...=-6.427...(x--3)$	Correct attempt to find the equation of a line between the two points.	M1
	At $y=0$, $0-2.055...=-6.427...(x--3)$	Subs $y=0$ in their line and achieves $x=...$	dM1
	$\Rightarrow x=-2.680$	-2.680: only penalise accuracy once in (b) and (c), but must be to at least 3sf.	A1 cao
			[3]
			Total 10

Question Number	Scheme	Notes	Marks
3. (i) (a)	$\mathbf{A}^{-1} = \frac{1}{-2-3}\begin{pmatrix} 1 & -3 \\ -1 & -2 \end{pmatrix}$	Either $\frac{1}{-2-3}$ or $-\frac{1}{5}$ or $\begin{pmatrix} 1 & -3 \\ -1 & -2 \end{pmatrix}$	M1
		Correct expression for $\mathbf{A}^{-1}$	A1
			[2]
(b)	$\{\mathbf{B} = \mathbf{A}^{-1}(\mathbf{AB})\}$		
	$\mathbf{B} = -\frac{1}{5}\begin{pmatrix} 1 & -3 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} -1 & 5 & 12 \\ 3 & -5 & -1 \end{pmatrix}$	Writing down their $\mathbf{A}^{-1}$ multiplied by $\mathbf{AB}$	M1
	$= \left\{-\frac{1}{5}\right\}\begin{pmatrix} -10 & 20 & 15 \\ -5 & 5 & -10 \end{pmatrix}$	At least one correct row or at least two correct columns of $\begin{pmatrix} \dots \\ \dots \end{pmatrix}$. (Ignore $-\frac{1}{5}$).	A1
	$= \begin{pmatrix} 2 & -4 & -3 \\ 1 & -1 & 2 \end{pmatrix}$	Correct simplified matrix for **B**	A1
			[3]
ALT (b)	Let $\mathbf{B} = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$		
	$-2a + 3d = -1$ $-2b + 3e = 5$ $a + d = 3$ $b + e = -5$ $-2c + 3f = 12$ $c + f = -1$	Writes down at least 2 correct sets of simultaneous equations	M1
	$\{a = 2, d = 1, b = -4, e = -1, c = -3, f = 2\}$		
	$\mathbf{B} = \begin{pmatrix} 2 & -4 & -3 \\ 1 & -1 & 2 \end{pmatrix}$	At least one correct row or at least two correct columns for the matrix **B**	A1
		Correct matrix for **B**	A1
			[3]
(ii) (a)	Rotation	Rotation only.	M1
	90° clockwise about the origin	**90º** $\left(\text{or } \frac{\pi}{2}\right)$ **clockwise** about the **origin** or **270º** $\left(\text{or } \frac{3\pi}{2}\right)$ (anti-clockwise) about the **origin.** **-90º** $\left(\text{or } -\frac{\pi}{2}\right)$ **(**anticlockwise**)** about the **origin.** Origin can be written as $(0, 0)$ or O.	A1
			[2]
(b)	$\{\mathbf{C}^{39}\} = \mathbf{C}^{-1}$ or $\mathbf{C}^{3} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	For stating $\mathbf{C}^{-1}$ or $\mathbf{C}^{3}$ or 'rotation of **270º** clockwise o.e. about the **origin.** Can be implied by correct matrix.	M1
		$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ Correct answer with no working award M1A1	A1
			[2]
			Total 9

Question Number	Scheme	Notes	Marks
4. (a)	$\sum_{r=1}^{n}\left(r^2 - r - 8\right)$		
	$= \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1) - 8n$	At least one of the first two terms is correct.	M1
		Correct expression	A1
	$= \frac{1}{6}n\left((2n+1)(n+1) - 3(n+1) - 48\right)$	An attempt to factorise out at least n.	M1
	$= \frac{1}{6}n\left(2n^2 + 3n + 1 - 3n - 3 - 48\right)$		
	$= \frac{1}{6}n\left(2n^2 - 50\right)$		
	$= \frac{2}{6}n\left(n^2 - 25\right)$		
	$= \frac{1}{3}n(n-5)(n+5)$	Achieves the correct answer.	A1
			[4]
(b)	$n = 5$	5. Give B0 for 2 or more possible values of n.	B1 cao
			[1]
(c)	$\left(\frac{k}{4}(17^2)(18^2) - \frac{k}{4}(3^2)(2^2)\right) + \left(\frac{1}{3}(17)(22)(12) - \frac{1}{3}(2)(-3)(7)\right)$	Applying ***at least one of*** $n=17$ **or** $n=2$ to **both** $\frac{1}{4}n^2(n+1)^2$ and their $\frac{1}{3}n(n-5)(n+5)$	M1
		Applying $n=17$ **and** $n=2$ **only** to **both** $\frac{1}{4}n^2(n+1)^2$ and their $\frac{1}{3}n(n-5)(n+5)$. Require differences only for both brackets.	M1
	$\{\sum = 6710 \Rightarrow\}\ 23409k - 9k + 1496 + 14 = 6710 \Rightarrow k = \frac{2}{9}$	Sets their sum to 6710 and solves to give $k = \ldots$	ddM1
		$k = \frac{2}{9}$ or $0.\dot{2}$	A1 **cso**
			[4]
			Total 9

Question Number	Scheme	Notes	Marks
5. (a)	$y = c^2x^{-1} \Rightarrow \dfrac{dy}{dx} = -c^2x^{-2}$ or (implicitly) $y + x\dfrac{dy}{dx} = 0$ or (chain rule) $\dfrac{dy}{dx} = -ct^{-2} \times \dfrac{1}{c}$	$\dfrac{dy}{dx} = \pm k x^{-2}$ or $y + x\dfrac{dy}{dx} = 0$ or $\dfrac{\text{their } \frac{dy}{dt}}{\text{their } \frac{dx}{dt}}$	M1
	When $x = ct$, $m_T = \dfrac{dy}{dx} = \dfrac{-c^2}{(ct)^2} = -\dfrac{1}{t^2}$ or at $P\left(ct, \dfrac{c}{t}\right)$, $m_T = \dfrac{dy}{dx} = -\dfrac{y}{x} = -\dfrac{ct^{-1}}{ct} = -\dfrac{1}{t^2}$	$\dfrac{dy}{dx} = -\dfrac{1}{t^2}$	A1
	T: $y - \dfrac{c}{t} = -\dfrac{1}{t^2}(x - ct)$	Applies $y - \dfrac{c}{t} = (\text{their } m_T)(x - ct)$ where their m_T has come from calculus	M1
	T: $t^2y - ct = -x + ct$	At least one line of working.	
	T: $t^2y + x = 2ct$ *	Correct solution.	A1 **cso** *
			[4]
(b)	$t^2\left(\dfrac{3c}{5}\right) + \left(-\dfrac{8c}{5}\right) = 2ct$	Substitutes $\left(-\dfrac{8c}{5}, \dfrac{3c}{5}\right)$ into tangent.	M1
	$3t^2 - 8 = 10t$	Correct 3TQ in terms of t Can include uncancelled c.	A1
	$\{3t^2 - 10t - 8 = 0 \Rightarrow\}\ (t-4)(3t+2) = 0 \Rightarrow t = ...$	Attempt to solve their 3TQ for t	M1
	$t = 4,\ -\dfrac{2}{3} \Rightarrow A\left(4c, \dfrac{c}{4}\right), B\left(-\dfrac{2}{3}c, -\dfrac{3c}{2}\right)$	Uses one of their values of t to find A or B	M1
		Correct coordinates. Condone A and B swapped or missing.	A1
			[5]
			Total 9
ALT 1 (b)	$y - \dfrac{3c}{5} = -\dfrac{1}{t^2}\left(x - -\dfrac{8c}{5}\right)$ $\Rightarrow \dfrac{c}{t} - \dfrac{3c}{5} = -\dfrac{1}{t^2}\left(ct + \dfrac{8c}{5}\right)$	Substitutes $\left(ct, \dfrac{c}{t}\right)$ into their $y - \dfrac{3c}{5} = -\dfrac{1}{t^2}\left(x - -\dfrac{8c}{5}\right)$	M1
	$3t^2 - 10t = 8$	Correct 3TQ in terms of t. Can include uncancelled c.	A1
	then apply the original mark scheme.		
ALT 2 (b)	$A\left(ct_1, \dfrac{c}{t_1}\right), B\left(ct_2, \dfrac{c}{t_2}\right)$ $t_1^2y + x = 2ct_1$ $t_2^2y + x = 2ct_2$	Substitutes A and B into the equation of the tangent, solves for x and y	M1
	$t_1 + t_2 = \dfrac{10}{3},\ t_1t_2 = -\dfrac{8}{3}$		
	$3t^2 - 8 = 10t$	Correct 3TQ in terms of t_1 or t_2 Can include uncancelled c.	A1
	then apply original scheme		

Question Number	Scheme		Marks
6. (a)	$\{\det \mathbf{M} = (8)(2) - (-1)(-4)\} \Rightarrow \det \mathbf{M} = 12$	12	B1
			[1]
(b)	Area $T = \dfrac{216}{12}\ \{= 18\}$	Area $T = \dfrac{216}{\text{their "det}\,\mathbf{M}\text{"}}$	M1
	$h = \pm(1-k)$	Uses $(k-1)$ or $(1-k)$ in their solution.	M1
	$\frac{1}{2}8(k-1) = 18$ or $\frac{1}{2}8(1-k) = 18$ or $(k-1) = \frac{18}{4}$ or $(1-k) = \frac{18}{4}$ or $\{\frac{1}{2}8h = 18\} \Rightarrow h = \frac{9}{2}, k = 1 \pm \frac{9}{2}$	**dependent on the two previous M marks** $\frac{1}{2}8(k-1)$ or $\frac{1}{2}8(1-k) = \dfrac{216}{\text{their "det}\,\mathbf{M}\text{"}}$ or $(k-1)$ or $(1-k) = \dfrac{216}{4(\text{their "det}\,\mathbf{M}\text{"})}$ or $h = \dfrac{216}{4(\text{their "det}\,\mathbf{M}\text{"})}, k = 1 \pm \dfrac{216}{4(\text{their "det}\,\mathbf{M}\text{"})}$	ddM1
	$\Rightarrow k = 5.5$ or $k = -3.5$	At least one of either $k = 5.5$ or $k = -3.5$	A1
		Both $k = 5.5$ and $k = -3.5$	A1
			[5]
ALT (b)	$\mathbf{T}' = \begin{pmatrix} 8 & -1 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} 4 & 6 & 12 \\ 1 & k & 1 \end{pmatrix}$		
	$\mathbf{T}' = \begin{pmatrix} 31 & 48-k & 95 \\ -14 & -24+2k & -46 \end{pmatrix}$ or 18 seen	At least 5 out of 6 elements are correct or 18 seen..	M1
	$\frac{1}{2}\begin{vmatrix} 31 & 48-k & 95 & 31 \\ -14 & -24+2k & -46 & -14 \end{vmatrix} = 216$ **or** $\frac{1}{2}\begin{vmatrix} 4 & 6 & 12 & 4 \\ 1 & k & 1 & 1 \end{vmatrix} = 18$	$\frac{1}{2}\lvert\text{their } \mathbf{T}'\rvert = 216$ or $\frac{1}{2}\begin{vmatrix} 4 & 6 & 12 & 4 \\ 1 & k & 1 & 1 \end{vmatrix} = 18$	M1
	$\frac{1}{2}\begin{vmatrix} -744 + 62k + 672 - 14k - 2208 + 46k \\ +2280 - 190k - 1330 + 1426 \end{vmatrix} = 216$ $\frac{1}{2}\lvert 4k - 6 + 6 - 12k + 12 - 4\rvert = 18$	**Dependent on the two previous M marks.** Full method of evaluating a determinant.	ddM1
	$\frac{1}{2}\lvert 96 - 96k\rvert = 216$ or $\frac{1}{2}\lvert 8 - 8k\rvert = 18$		
	So, $1 - k = 4.5$ or $k - 1 = 4.5$		
	$\Rightarrow k = -3.5$ or $k = 5.5$	At least one of either $k = -3.5$ or $k = 5.5$	A1
		Both $k = -3.5$ and $k = 5.5$	A1
			[5]
			Total 6

Question Number	Scheme	Notes	Marks
7.	$y^2 = 4ax,\ S(a,0),\ D\left(-a, \frac{24a}{5}\right),\ P(ak^2, 2ak)$		
(a)	$m_l = \dfrac{\frac{24a}{5} - 0}{-a - a}\ \left\{= \dfrac{\frac{24a}{5} - 0}{-2a} = -\dfrac{12}{5}\right\}$ $\dfrac{y - \frac{24a}{5}}{0 - \frac{24a}{5}} = \dfrac{x - -a}{a - -a}$ or $\dfrac{y - 0}{\frac{24a}{5} - 0} = \dfrac{x - a}{-a - a}$	Uses $S(a, 0)$ and $D\left(\text{their "}-a\text{"}, \frac{24a}{5}\right)$ to find an expression for the gradient of l or applies the formula $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$ Can be un-simplified or simplified.	M1
	$l:\ y - 0 = -\frac{12}{5}(x - a) \Rightarrow\ 5y = -12x + 12a$ $l:\ 12x + 5y = 12a$ (*)	Correct solution only leading to $12x + 5y = 12a$ No errors seen.	A1 *
			[2]
ALT (a)	$y = mx + c$ At S, $0 = ma + c$ At D, $\frac{24a}{5} = -ma + c$ $\Rightarrow c = \frac{12a}{5}, m = -\frac{12}{5}$	Uses $S(a, 0)$ and $D\left(\text{their "}-a\text{"}, \frac{24a}{5}\right)$ to find 2 simultaneous equations and solves to achieve $c = ..., m = ...$	M1
	$y = -\frac{12}{5}x + \frac{12a}{5} \Rightarrow 12x + 5y = 12a$*	Correct solution only leading to $12x + 5y = 12a$	A1*
			[2]
(b)	$m_{SP} = \dfrac{2ak}{ak^2 - a}\ \left\{= \dfrac{2k}{k^2 - 1}\right\}$	Attempts to find the gradient of SP	M1
	$m_l = -\left(\dfrac{ak^2 - a}{2ak}\right)$ or $m_{SP} = -\dfrac{1}{(-\frac{12}{5})}\ \left\{= \dfrac{5}{12}\right\}$	Some evidence of applying $m_1 m_2 = -1$	M1
	So $\left\{\dfrac{2k}{k^2 - 1} = \dfrac{5}{12} \Rightarrow\right\}\ 24k = 5k^2 - 5$	Correct 3TQ in terms of k in any form.	A1
	$\{5k^2 - 24k - 5 = 0 \Rightarrow\}\ (k - 5)(5k + 1) = 0 \Rightarrow k = ...$	Attempt to solve their 3TQ for k	M1
	$\{\text{As } k > 0, \text{ so } k = 5\} \Rightarrow (25a, 10a)$	Uses their k to find P	M1
		$(25a, 10a)$	A1
			[6]

ALT 1 (b)	SP: $y-0=\frac{5}{12}(x-a)$	$y-0=m_{SP}(x-a)$	M1
		$m_{SP}=-\frac{1}{(-\frac{12}{5})}\left\{=\frac{5}{12}\right\}$	M1
	$\left\{y^2=4ax \Rightarrow\right\}\left(\frac{5}{12}(x-a)\right)^2=4ax$	Can sub for x and achieve $\frac{12}{5}y+a$	
	$25(x^2-2ax+a^2)=576ax$		
	$25x^2-626ax+25a^2=0$	Correct 3TQ in terms of a and x or $5y^2-48ay-20a^2=0$	A1
	$(25x-a)(x-25a)=0 \Rightarrow x=...$	Attempt to solve their 3TQ for x	M1
	$x=\frac{a}{25} \Rightarrow y=\frac{5}{12}\left(\frac{a}{25}-a\right)\left\{=-\frac{2a}{5}\right\}$ $x=25a \Rightarrow y=\frac{5}{12}(25a-a)$ $\{=10a\}$	Uses their x to find y	M1
	$\{\text{As } k>0,\} \Rightarrow (25a, 10a)$	$(25a, 10a)$	A1
			[6]
ALT 2 (b)	$0=m_{SP}a+c$	Subs S into $y=m_{SP}x+c$ to find c	M1
	$m_{SP}=-\frac{1}{(-\frac{12}{5})}\left\{=\frac{5}{12}\right\}$	Some evidence of applying $m_1m_2=-1$	M1
	$y=\frac{5}{12}x-\frac{5}{12}a$		
	At P, $2ak=\frac{5}{12}ak^2-\frac{5}{12}a$	Correct 3TQ in terms of k	A1
	then as part (b)		
			Total 8

Question Number	Scheme	Notes	Marks
8.	$f(n) = 2^{n+2} + 3^{2n+1}$ divisible by 7		
	$f(1) = 2^3 + 3^3 = 35$ {which is divisible by 7}.	Shows $f(1) = 35$	B1
	$\{\therefore f(n)$ is divisible by 7 when $n = 1$ }		
	{Assume that for $n = k$,		
	$f(k) = 2^{k+2} + 3^{2k+1}$ is divisible by 7 for $k \in \mathbb{Z}^+$.}		
	$f(k+1) - f(k) = 2^{k+1+2} + 3^{2(k+1)+1} - (2^{k+2} + 3^{2k+1})$	Applies $f(k+1)$ with at least 1 power correct	M1
	$f(k+1) - f(k) = 2(2^{k+2}) + 9(3^{2k+1}) - (2^{k+2} + 3^{2k+1})$		
	$f(k+1) - f(k) = 2^{k+2} + 8(3^{2k+1})$		
	$= (2^{k+2} + 3^{2k+1}) + 7(3^{2k+1})$	$(2^{k+2} + 3^{2k+1})$ or $f(k)$; $7(3^{2k+1})$	A1; A1
	or $= 8(2^{k+2} + 3^{2k+1}) - 7(2^{k+2})$	or $8(2^{k+2} + 3^{2k+1})$ or $8f(k); -7(2^{k+2})$	
	$= f(k) + 7(3^{2k+1})$ **or** $= 8f(k) - 7(2^{k+2})$		
	$\therefore f(k+1) = 2f(k) + 7(3^{2k+1})$ **or** $f(k+1) = 9f(k) - 7(2^{k+2})$	**Dependent on at least one of the previous accuracy marks being awarded.** Makes $f(k+1)$ the subject	dM1
	$\{\therefore f(k+1) = 2f(k) + 7(3^{2k+1})$ is divisible by 7 as both $2f(k)$ and $7(3^{2k+1})$ are both divisible by 7}		
	If the result is **true for** $n = k$, then it is now **true for** $n = k+1$. As the result has shown to be **true for** $n = 1$, then the result is true **for all *n*** $(\in \mathbb{Z}^+)$.	Correct conclusion seen at the end. Condone true for n = 1 stated earlier**.**	A1 cso
			[6]
ALT	$f(k+1) - \alpha f(k) = 2^{k+3} + 3^{2k+3} - \alpha(2^{k+2} + 3^{2k+1})$	Applies $f(k+1)$ with at least 1 power correct	M1
	$f(k+1) - \alpha f(k) = (2-\alpha)2^{k+2} + (9-\alpha)3^{2k+1}$		
	$f(k+1) - \alpha f(k) = (2-\alpha)(2^{k+2} + 3^{2k+1}) + 7.3^{2k+1}$ or	$(2-\alpha)(2^{k+2} + 3^{2k+1})$ or $(2-\alpha)f(k); 7.3^{2k+1}$ or	A1;A1
	$f(k+1) - \alpha f(k) = (9-\alpha)(2^{k+2} + 3^{2k+1}) - 7.2^{k+2}$	$(9-\alpha)(2^{k+2} + 3^{2k+1})$ or $(9-\alpha)f(k); -7.2^{k+2}$	
		NB: Choosing $\alpha = 0, \alpha = 2, \alpha = 9$ will make relevant terms disappear, but marks should be awarded accordingly.	
			Total 6

Question Number	Scheme		Marks
9.(i) (a)	$\frac{3w+7}{5} = \frac{(p-4\mathrm{i})}{(3-\mathrm{i})} \times \frac{(3+\mathrm{i})}{(3+\mathrm{i})}$	Multiplies by $\frac{(3+\mathrm{i})}{(3+\mathrm{i})}$ or divide by $(9-3\mathrm{i})$ then multiply by $\frac{(9+3\mathrm{i})}{(9+3\mathrm{i})}$	M1
	$= \left(\frac{3p+4}{10}\right) + \left(\frac{p-12}{10}\right)\mathrm{i}$	Evidence of $(3-\mathrm{i})(3+\mathrm{i}) = 10$ or 3^2+1^2 or 9^2+3^2	B1
	So, $w = \left(\frac{3p-10}{6}\right) + \left(\frac{p-12}{6}\right)\mathrm{i}$	Rearranges to $w = ...$	dM1
		At least one of either the real or imaginary part of *w* is correct in any equivalent form.	A1
		Correct *w* in the form $a+b\mathrm{i}$. Accept $a+\mathrm{i}b$.	A1
			[5]
ALT (i) (a)	$(3-\mathrm{i})(3w+7) = 5(p-4\mathrm{i})$		
	$9w + 21 - 3\mathrm{i}w - 7\mathrm{i} = 5p - 20\mathrm{i}$		
	$w(9-3\mathrm{i}) = 5p - 21 - 13\mathrm{i}$		
	Let $w = a+b\mathrm{i}$, so $(a+b\mathrm{i})(9-3\mathrm{i}) = 5p-21-13\mathrm{i}$		
	$9a+3b-3a\mathrm{i}+9b\mathrm{i} = 5p-21-13\mathrm{i}$		
	Real: $9a+3b = 5p-21$ Imaginary: $-3a+9b = -13$	Sets $w = a+b\mathrm{i}$ and equates at least either the real or imaginary part.	M1
		$9a+3b = 5p-21$	B1
	$b = \frac{p-12}{6}$, $a = \frac{3p-10}{6}$	Solves to finds $a = ...$ and $b = ...$	dM1
		At least one of *a* or *b* is correct in any equivalent form.	A1
	$w = \left(\frac{3p-10}{6}\right) + \left(\frac{p-12}{6}\right)\mathrm{i}$	Correct *w* in the form $a+b\mathrm{i}$. Accept $a+\mathrm{i}b$.	A1
			[5]
(b)	$\left\{\arg w = -\frac{\pi}{2} \Rightarrow \left(\frac{3p-10}{6}\right) = 0\right\} \Rightarrow p = \frac{10}{3}$	$p = \frac{10}{3}$ Follow through provided $p < 12$	B1ft
			[1]

(ii)	$(x+\mathrm{i}y+1-2\mathrm{i})^* = 4\mathrm{i}(x+\mathrm{i}y)$	Replaces z with $x+\mathrm{i}y$ on both sides of the equation	M1
	$x-\mathrm{i}y+1+2\mathrm{i} = 4\mathrm{i}(x+\mathrm{i}y)$ or $x+\mathrm{i}y+1-2\mathrm{i} = -4\mathrm{i}(x-\mathrm{i}y)$	Fully correct method for applying the conjugate	M1
	$x-\mathrm{i}y+1+2\mathrm{i} = 4\mathrm{i}x-4y$		
	Real: $x+1=-4y$ Imaginary: $-y+2=4x$	$x+1=-4y$ and $-y+2=4x$	A1
	$4x+16y=-4$ $4x+y=2$ $\Rightarrow 15y=-6 \Rightarrow y=\ldots$	Solves two equations in x and y to obtain at least one of x or y	ddM1
	So, $x=\frac{3}{5}, y=-\frac{2}{5}$ $\left\{z=\frac{3}{5}-\frac{2}{5}\mathrm{i}\right\}$	At least one of either x or y are correct	A1
		Both x and y are correct	A1
			[6]
			Total 12

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