Mark Scheme (Results)

Summer 2017

Pearson Edexcel GCE
In Further Pure Mathematics FP1 (6667/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper or ag- answer given
- $\sqsubset$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to x = ...

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2 \pm q \pm c = 0,\ \ q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

<table>
<tr><th>Question Number</th><th>Scheme</th><th></th><th>Marks</th></tr>
<tr><td>1.</td><td>$\mathrm{f}(x) = \frac{1}{3}x^2 + \frac{4}{x^2} - 2x - 1,\ x > 0$</td><td></td><td></td></tr>
<tr><td>(a)</td><td>$\mathrm{f}(6) = -0.88888888...$
$\mathrm{f}(7) = 1.414965986...$</td><td>Either any one of $\mathrm{f}(6) = \text{awrt } -0.9$ or $\mathrm{f}(7) = \text{awrt } 1.4$</td><td>M1</td></tr>
<tr><td></td><td>Sign change or $\mathrm{f}(6) = -\text{ve}$ and $\mathrm{f}(7) = +\text{ve}$ or $\mathrm{f}(6)\times\mathrm{f}(7) = -\text{ve}$ o.e. (and $\mathrm{f}(x)$ is continuous) therefore a root / α (exists between $x = 6$ and $x = 7$) o.e.</td><td>Both $\mathrm{f}(6) = \text{awrt } -0.9$ and $\mathrm{f}(7) = \text{awrt } 1.4$, sign change and conclusion.
Allow $\mathrm{f}(6) = -\frac{8}{9}$ and $\mathrm{f}(7) = \frac{208}{147}$.</td><td>A1</td></tr>
<tr><td></td><td></td><td></td><td>[2]</td></tr>
<tr><td>(b)</td><td>$\mathrm{f}'(x) = \frac{2}{3}x - \frac{8}{x^3} - 2$</td><td>$\frac{1}{3}x^2 \to \pm Ax$ or $\frac{4}{x^2} \to \pm Bx^{-3}$ or $-2x - 1 \to -2$</td><td>M1</td></tr>
<tr><td></td><td></td><td>At least two of these terms differentiated correctly.</td><td>A1</td></tr>
<tr><td></td><td></td><td>Correct derivative.</td><td>A1</td></tr>
<tr><td></td><td>$\{\mathrm{f}'(6) = 1.962962963...\}$</td><td>$\mathrm{f}'(6) = \frac{53}{27}$</td><td></td></tr>
<tr><td></td><td>$\alpha \simeq 6 - \left(\frac{\text{"}-0.88888888...\text{"}}{\text{"}1.962962963...\text{"}}\right)$</td><td>Correct application of Newton-Raphson using their values.</td><td>M1</td></tr>
<tr><td></td><td>$= 6.452830189...$</td><td>Exact form of α is $\frac{342}{53}$</td><td></td></tr>
<tr><td></td><td>$= 6.45\ (2\,\text{dp})$</td><td>6.45</td><td>A1 cso</td></tr>
<tr><td></td><td></td><td></td><td>[5]
7</td></tr>
<tr><td></td><td colspan="3">Question 1 Notes</td></tr>
<tr><td>1. (a)</td><td colspan="3">Note Accept at least ‘sign change therefore root’ o.e. for A1.
Any incorrect statements made in the conclusion award A0.</td></tr>
<tr><td>(b)</td><td colspan="3">Note Denominator in NR calculation may contain evidence for first 3 marks.
Correct answer of 6.45 with minimal working will imply earlier marks for elements not explicitly stated. However, incorrect values leading to a correct final answer should be marked accordingly.</td></tr>
</table>

Question Number	Scheme		Marks
2.	$\mathbf{A}=\begin{pmatrix}2 & -1\\ 4 & 3\end{pmatrix}$, $\mathbf{P}=\begin{pmatrix}3 & 6\\ 11 & -8\end{pmatrix}$		
(a)	$\mathbf{A}^{-1}=\frac{1}{10}\begin{pmatrix}3 & 1\\ -4 & 2\end{pmatrix}$	Either $\frac{1}{10}$ or $\begin{pmatrix}3 & 1\\ -4 & 2\end{pmatrix}$	M1
		Correct matrix seen.	A1
			[2]
(b) **Way 1**	$\mathbf{P}=\mathbf{AB}$ $\Rightarrow\ \mathbf{A}^{-1}\mathbf{P}=\mathbf{A}^{-1}\mathbf{AB}\ \Rightarrow\ \mathbf{B}=\mathbf{A}^{-1}\mathbf{P}$		
	$\mathbf{B}\quad=\frac{1}{10}\begin{pmatrix}3 & 1\\ -4 & 2\end{pmatrix}\begin{pmatrix}3 & 6\\ 11 & -8\end{pmatrix}$	Multiplies **their** $\mathbf{A}^{-1}$ by **P** in correct order. This substituted statement is sufficient.	M1
	$=\begin{pmatrix}2 & 1\\ 1 & -4\end{pmatrix}$	At least 2 elements correct or $k\begin{pmatrix}20 & 10\\ 10 & -40\end{pmatrix}$ oe. May be unsimplified	A1
		Correct simplified matrix.	A1
			[3]
(b) **Way 2**	$\{\mathbf{P}=\mathbf{AB}\Rightarrow\}$ $\begin{pmatrix}3 & 6\\ 11 & -8\end{pmatrix}=\begin{pmatrix}2 & -1\\ 4 & 3\end{pmatrix}\begin{pmatrix}a & b\\ c & d\end{pmatrix}$		
	$\begin{pmatrix}3 & 6\\ 11 & -8\end{pmatrix}=\begin{pmatrix}2a-c & 2b-d\\ 4a+3c & 4b+3d\end{pmatrix}$	Attempt to multiply **A** by **B** in the correct order and puts equal to **P**	M1
	$\Rightarrow a=2,\ c=1,\ b=1,\ d=-4$		
	So, $\mathbf{B}=\begin{pmatrix}2 & 1\\ 1 & -4\end{pmatrix}$	At least 2 elements are correct.	A1
		Correct matrix.	A1
			[3] **5**

Question Number	Scheme		Marks
3. (a)	$x = 4t,\ y = \dfrac{4}{t},\ t \neq 0$		
	$t = \dfrac{1}{4} \Rightarrow P(1, 16),\quad t = 2 \Rightarrow Q(8, 2)$	Coordinates for either P or Q are correctly stated. (Can be implied).	B1
	$m(PQ) = \dfrac{2-16}{8-1}\ \{= -2\}$	Finds the gradient of the chord PQ with $\dfrac{y_2 - y_1}{x_2 - x_1}$ then uses in $y = -\dfrac{1}{m}x$. Condone incorrect sign of gradient.	M1
	$m(l) = \dfrac{1}{2}$		
	So, $l: y = \dfrac{1}{2}x$ or $2y = x$	$y = \dfrac{1}{2}x$ or $2y = x$	A1 oe
			[3]
(b)	$xy = 16$ or $y = \dfrac{16}{x}$ or $x = \dfrac{16}{y}$	Correct Cartesian equation. Accept $\dfrac{4}{y} = \dfrac{x}{4}$ or $xy = 4^2$	B1 oe
			[1]
(c)	**Way 1**: $\dfrac{1}{2}x = \dfrac{16}{x}$ $\{x^2 = 32\}$ **Way 2**: $\dfrac{4}{t} = \dfrac{1}{2}(4t)$ $\{t^2 = 2\}$ **Way 3**: $2y = \dfrac{16}{y}$ $\{y^2 = 8\}$	Attempts to substitute their l into either their Cartesian equation or parametric equations of H	M1
	$(4\sqrt{2},\ 2\sqrt{2}),\ (-4\sqrt{2},\ -2\sqrt{2})$	At least one set of coordinates (simplified or un-simplified) or $x = \pm 4\sqrt{2},\ y = \pm 2\sqrt{2}$	A1
		Both sets of simplified coordinates. Accept written in pairs as $x = 4\sqrt{2},\ y = 2\sqrt{2}$ $x = -4\sqrt{2},\ y = -2\sqrt{2}$	A1
			[3] **7**

Question Number	Scheme		Marks
4. (i)		Mark (i)(a) and (i)(b) together.	
	$w = \dfrac{p-4\text{i}}{2-3\text{i}} \qquad \arg w = \dfrac{\pi}{4}$		
(a) **Way 1**	$w = \dfrac{(p-4\text{i})}{(2-3\text{i})} \times \dfrac{(2+3\text{i})}{(2+3\text{i})}$	Multiplies by $\dfrac{(2+3\text{i})}{(2+3\text{i})}$	M1
.	$= \left(\dfrac{2p+12}{13}\right) + \left(\dfrac{3p-8}{13}\right)\text{i}$	At least one of either the real or imaginary part of *w* is correct. Must be expanded but could be unsimplified e.g. expressed as single fraction. Condone $a+\text{i}b$.	A1
		Correct *w* in its simplest form.	A1
			[3]
(a) **Way 2**	$(a+\text{i}b)(2-3\text{i}) = (p-4\text{i})$		
	$2a+3b=p$ $3a-2b=4$	Multiplies out to obtain 2 equations in two unknowns.	M1
	$= \left(\dfrac{2p+12}{13}\right) + \left(\dfrac{3p-8}{13}\right)\text{i}$	At least one of either the real or imaginary part of *w* is correct. Must be expanded but could be unsimplified e.g. expressed as single fraction. Condone $a+\text{i}b$.	A1
		Correct *w* in its simplest form.	A1
			[3]
(b)	$\left\{\arg w = \dfrac{\pi}{4} \Rightarrow\right\} \quad 2p+12 = 3p-8$ o.e. seen anywhere.	Sets the numerators of the real part of their *w* equal to the imaginary part of their *w* or if arctan used, require evidence of $\tan\dfrac{\pi}{4} = 1$	M1
	$\Rightarrow p = 20$	$p = 20$	A1
			[2]
(ii) **Way 1**	$z = (1-\lambda\text{i})(4+3\text{i})$ and $\lvert z\rvert = 45$		
	$\sqrt{1+\lambda^2}\,\sqrt{4^2+3^2}$	Attempts to apply $\lvert(1-\lambda\text{i})(4+3\text{i})\rvert = \sqrt{1+\lambda^2}\,\sqrt{4^2+3^2}$	M1
	$\sqrt{1+\lambda^2}\,\sqrt{4^2+3^2} = 45$	Correct equation.	A1
	$\{\lambda^2 = 9^2 - 1 \Rightarrow\}\ \lambda = \pm 4\sqrt{5}$	$\lambda = \pm 4\sqrt{5}$	A1
			[3]
Way 2	$z = (4+3\lambda) + (3-4\lambda)\text{i}$ $\sqrt{(4+3\lambda)^2 + (3-4\lambda)^2}$	Attempt to multiply out, group real and imaginary parts and apply the modulus.	M1
	$(4+3\lambda)^2 + (3-4\lambda)^2 = 45^2$ or $\sqrt{(4+3\lambda)^2 + (3-4\lambda)^2} = 45$	Correct equation.	A1
	$\{16+24\lambda+9\lambda^2+9-24\lambda+16\lambda^2 = 2025\}$	Condone if middle terms in expansions not explicitly stated.	
	$\{25\lambda^2 = 2000 \Rightarrow\}\ \lambda = \pm 4\sqrt{5}$	$\lambda = \pm 4\sqrt{5}$	A1
			[3] **8**

	Question 4 Notes
(ii)	**M1** Also allow $(1+\lambda^2)(4^2+3^2)$ for M1. **M1** Also allow $(4+3\lambda)^2 + (3-4\lambda)^2$ for M1.

Question Number	Scheme		Marks
5. (i)	$\mathbf{A}=\begin{pmatrix} p & 2 \\ 3 & p \end{pmatrix}$, $\mathbf{B}=\begin{pmatrix} -5 & 4 \\ 6 & -5 \end{pmatrix}$, $\mathbf{M}=\begin{pmatrix} a & -9 \\ 1 & 2 \end{pmatrix}$	p, a are constants.	
(a)	$\{\mathbf{AB}\}=\begin{pmatrix} -5p+12 & 4p-10 \\ -15+6p & 12-5p \end{pmatrix}$	At least 2 elements are correct.	M1
		Correct matrix.	A1
			[2]
(b)	$\{\mathbf{AB}+2\mathbf{A}=k\mathbf{I}\}$		
	$\begin{pmatrix} -5p+12 & 4p-10 \\ -15+6p & 12-5p \end{pmatrix}+2\begin{pmatrix} p & 2 \\ 3 & p \end{pmatrix}=k\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	If ‘simultaneous equations’ used, allocate marks as below.	
	$\begin{pmatrix} -3p+12 & 4p-6 \\ -9+6p & 12-3p \end{pmatrix}=\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$		
	$"4p-10"+4=0$ or $"-15+6p"+6=0$ or $"-9+6p"="4p-6"$	Forms an equation in p	M1
	$\Rightarrow p=\frac{3}{2}$	$p=\frac{3}{2}$ o.e.	A1
	$k=-5\left(\frac{3}{2}\right)+12+2\left(\frac{3}{2}\right)\Rightarrow k=...$	Substitutes their $p=\frac{3}{2}$ into "their $(-5p+12)$"$+2p$ to find a value for k or eliminates p to find k.	M1
	$k=\frac{15}{2}$	$k=\frac{15}{2}$ oe	A1
			[4]
(ii) **Way 1**	$\pm\frac{270}{15}$ $\{=\pm 18\}$	Can be implied from calculations.	B1
	$\det\mathbf{M}=(a)(2)-(-9)(1)$	Applies $ad-bc$ to $\mathbf{M}$. Require clear evidence of correct formula being used for M1 if errors seen.	M1
	$\Rightarrow 2a+9=18$ or $2a+9=-18$	Equates their $\det\mathbf{A}$ to either 18 or -18	M1
	$\Rightarrow a=4.5$ or $a=-13.5$	At least one of either $a=4.5$ or $a=-13.5$	A1
		Both $a=4.5$ and $a=-13.5$	A1
			[5]
(ii) **Way 2**	Consider vertices of triangle with area 15 units e.g. (0,0),(15,0) and (0,2) and attempting 2 values of a.		B1
	e.g. $\begin{pmatrix} a & -9 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 0 & 15 & 0 \\ 0 & 0 & 2 \end{pmatrix}=\begin{pmatrix} 0 & 15a & -18 \\ 0 & 15 & 4 \end{pmatrix}$	Pre-multiplies their matrix by $\mathbf{M}$ and obtains single matrix	M1
	e.g. $\frac{1}{2}\begin{vmatrix} 0 & 15a & -18 & 0 \\ 0 & 15 & 4 & 0 \end{vmatrix}=270$	Equates their determinant to 270 and attempts to solve.	M1
	$\Rightarrow a=4.5$ or $a=-13.5$	At least one of either $a=4.5$ or $a=-13.5$	A1
		Both $a=4.5$ and $a=-13.5$	A1
			[5]
			11

Question Number	Scheme		Marks
6.	$x^3+ax^2+bx-52=0$, $a, b\in\mathbb{R}$, 4 and $2\text{i}-3$ are roots		
(a)	$-2\text{i}-3$	$-2\text{i}-3$ seen anywhere in solution for Q6.	B1
			[1]
(b) **Way 1**	$(x-(2\text{i}-3))(x-"(-2\text{i}-3)")$; $=x^2+6x+13$ or $x=-3\pm2\text{i}\Rightarrow(x+3)^2=-4$; $=x^2+6x+13(=0)$ $(x-4)(x-(2\text{i}-3))$; $=x^2-(1+2\text{i})x+4(2\text{i}-3)$ $(x-4)(x-"(-2\text{i}-3)")$; $=x^2-(1-2\text{i})x+4(-2\text{i}-3)$	Must follow from their part (a). Any incorrect signs for their part (a) in initial statement award M0; accept any equivalent expanded expression for A1.	M1; A1
	$(x-4)(x^2+6x+13)$ $\{=x^3+ax^2+bx-52\}$	$(x-3^{\text{rd}}\text{ root})(\text{their quadratic})$. Could be found by comparing coefficients from long division.	M1
	$a=2$, $b=-11$ or $x^3+2x^2-11x-52$	At least one of $a=2$ or $b=-11$	A1
		Both $a=2$ and $b=-11$	A1
			[5]
(b) **Way 2**	Sum $=(2\text{i}-3)+"(-2\text{i}-3)"=-6$ Product $=(2\text{i}-3)\times"(-2\text{i}-3)"=13$ So quadratic is $x^2+6x+13$	Attempts to apply either $x^2-(\text{sum roots})x+(\text{product roots})=0$ or $x^2-2\text{Re}(\alpha)x+\lvert\alpha^2\rvert=0$	M1
		$x^2+6x+13$	A1
	$(x-4)(x^2+6x+13)$ $\{=x^3+ax^2+bx-52\}$	$(x-3^{\text{rd}}\text{ root})(\text{their quadratic})$	M1
	$a=2$, $b=-11$ or $x^3+2x^2-11x-52$	At least one of $a=2$ or $b=-11$	A1
		Both $a=2$ and $b=-11$	A1
			[5]
(b) **Way 3**	$(2\text{i}-3)^3+a(2\text{i}-3)^2+b(2\text{i}-3)-52=0$ $5a-3b=43$ (real parts) and $6a-b=23$ (imaginary parts) or uses f(4) = 0 and f(a complex root) = 0 to form equations in *a* and *b*.	Substitutes $2\text{i}-3$ into the displayed equation and equates both real and imaginary parts.	M1
		$5a-3b=43$ and $6a-b=23$ **or** $16a+4b=-12$ and $(2i-3)^3+a(2i-3)^2+b(2i-3)-52=0$ / $(-2i-3)^3+a(-2i-3)^2+b(-2i-3)-52=0$	A1
	So $a=2, b=-11$ or $x^3+2x^2-11x-52$	Solves these equations simultaneously to find at least one of either $a=\ldots$ or $b=\ldots$	M1
		At least one of $a=2$ or $b=-11$	A1
		Both $a=2$ and $b=-11$	A1
			[5]
(b) **Way 4**	$b=$ sum of product pairs	Attempts sum of product pairs.	M1
	$=4(2\text{i}-3)+4"(-2\text{i}-3)"+(2\text{i}-3)"(-2\text{i}-3)"$	All pairs correct o.e.	A1
	$a=-(\text{sum of 3 roots})=-(4+2\text{i}\ -3"-2\text{i}-3")$	Adds up all 3 roots	M1
	$a=2$, $b=-11$ or $x^3+2x^2-11x-52$	At least one of $a=2$ or $b=-11$	A1
		Both $a=2$ and $b=-11$	A1
			[5]

(b) **Way 5**	Uses f(4) = 0	M1
	$16a+4b=-12$	A1
	$a=-(\text{sum of 3 roots})=-(4+2\text{i}\ -3"-2\text{i}-3")$ Adds up all 3 roots	M1
	$a=2,\ b=-11$ or $x^3+2x^2-11x-52$ At least one of $a=2$ or $b=-11$	A1
	Both $a=2$ and $b=-11$	A1
		[5] **6**

Questin Number	Scheme		Marks
7.	$y^2 = 4ax$, at $Q(aq^2, 2aq)$		
(a)	$y = 2\sqrt{a}\,x^{\frac{1}{2}} \Rightarrow \dfrac{dy}{dx} = \sqrt{a}\,x^{-\frac{1}{2}}$ or $2y\dfrac{dy}{dx} = 4a$ or $\dfrac{dy}{dx} = 2a \times \dfrac{1}{2aq}$	$\dfrac{dy}{dx} = \pm k x^{-\frac{1}{2}}$ or $ky\dfrac{dy}{dx} = c$ or $\dfrac{\text{their } \frac{dy}{dq}}{\text{their } \frac{dx}{dq}}$	M1
	When $x = aq^2$, $m_T = \dfrac{dy}{dx} = \dfrac{\sqrt{a}}{\sqrt{aq^2}} = \dfrac{\sqrt{a}}{\sqrt{a}\,q} = \dfrac{1}{q}$ or when $y = 2aq$, $m_T = \dfrac{dy}{dx} = \dfrac{4a}{2(2aq)} = \dfrac{1}{q}$	$\dfrac{dy}{dx} = \dfrac{1}{q}$	A1
	T: $y - 2aq = \dfrac{1}{q}\left(x - aq^2\right)$	Applies $y - 2aq = (\text{their } m_T)\left(x - aq^2\right)$ or $y = (\text{their } m_T)x + c$ and an attempt to find c with gradient from calculus.	dM1
	T: $qy - 2aq^2 = x - aq^2$ **T**: $qy = x + aq^2$ *	**cso**	A1 *
			[4]
(b)	$X\left(-\frac{1}{4}a, 0\right) \Rightarrow 0 = -\frac{1}{4}a + aq^2$	Substitutes $x = -\frac{1}{4}a$ and $y = 0$ into **T**	M1
	$\Rightarrow \left\{q^2 = \dfrac{1}{4} \Rightarrow q = -\dfrac{1}{2}\text{ (reject)}\right\}\ q = \dfrac{1}{2}$	$q = \dfrac{1}{2}$ oe	A1
	So, $\dfrac{1}{2}y = -a + a\left(\dfrac{1}{2}\right)^2$	Substitutes their "$q = \frac{1}{2}$" and $x = -a$ in **T** or finds $y_D = \dfrac{1}{q}\left(-a + aq^2\right)$	M1
	giving, $y = -\dfrac{3a}{2}$. So $D\left(-a, -\frac{3}{2}a\right)$ o.e.	$D\left(-a, -\frac{3}{2}a\right)$ o.e.	A1
			[4]
(c)	$\{\text{focus } F(a, 0)\}$		
Way 1	$\text{Area}(FXD) = \dfrac{1}{2}\left(\dfrac{5a}{4}\right)\left(\dfrac{3a}{2}\right) = \dfrac{15a^2}{16}$	Applies $\dfrac{1}{2}(\text{their } \lvert FX\rvert)(\text{their } \lvert y_D\rvert)$. If their $\left\lvert y_D = \dfrac{1}{q}\left(-a + aq^2\right)\right\rvert$ then require an attempt to sub for q to award M.	M1
		$\dfrac{15a^2}{16}$ or $0.9375a^2$	A1 cso
			[2]

(c) **Way 2**	$\text{Area}(FXD) = \frac{1}{2}\begin{vmatrix} a & -\frac{1}{4}a & -a & a \\ 0 & 0 & -\frac{3}{2}a & 0 \end{vmatrix}$		
	$= \frac{1}{2}\left\|\left(0+\frac{3}{8}a^2+0\right)-\left(0+0-\frac{3}{2}a^2\right)\right\| = \frac{15}{16}a^2$	A correct attempt to apply the shoelace method.	M1
		$\frac{15a^2}{16}$ or $0.9375a^2$	A1cao
			[2]
(c) **Way 3**	Rectangle – triangle 1 – triangle 2		
	$= 2a.\frac{3a}{2} - \frac{1}{2}.\frac{3a}{4}.\frac{3a}{2} - \frac{1}{2}.2a.\frac{3a}{2} = 3a^2 - \frac{9a^2}{16} - \frac{3a^2}{2}$		M1
		$\frac{15a^2}{16}$ or $0.9375a^2$	A1cao
(c) **Way 4**	Attempts sine rule using appropriate choice from	Uses Area $= \frac{1}{2}ab\sin C$	M1
	$FX = \frac{5a}{4}$, $FD = \frac{5a}{2}$, $DX = \frac{3\sqrt{5}a}{4}$, $\sin F = \frac{3}{5}$, $\sin X = \frac{2}{\sqrt{5}}$		
		$\frac{15a^2}{16}$ or $0.9375a^2$	A1cao
			10

	Question 7 Notes
(c) **Way 1**	Do not award M1 if area of wrong triangle found e.g. $\frac{1}{2}.2a.\frac{3a}{2} = \frac{3a^2}{2}$

Question Number	Scheme		Marks
8. (a)	$\sum_{r=1}^{n}\left(3r^2+8r+3\right)$		
	$= \frac{3}{6}n(n+1)(2n+1)+\frac{8}{2}n(n+1)+3n$	An attempt to use at least one of the correct standard formulae for first two terms.	M1
		Correct first two terms.	A1
		$3 \to 3n$	B1
	$= \frac{1}{2}n(n+1)(2n+1)+4n(n+1)+3n$		
	$= \frac{1}{2}n\left((2n+1)(n+1)+8(n+1)+6\right)$	Factorise out at least n from all terms at any point. There must be a factor of n in every term.	M1
	$= \frac{1}{2}n\left(2n^2+3n+1+8n+8+6\right)$		
	$= \frac{1}{2}n\left(2n^2+11n+15\right)$		
	$= \frac{1}{2}n(2n+5)(n+3)$ (*)	Achieves the correct answer, no errors seen.	A1*cso
			[5]
	$\sum_{r=1}^{12}\left(3r^2+8r+3+k(2^{r-1})\right) = 3520$		
(b)	$\sum_{r=1}^{12}\left(3r^2+8r+3\right) = \frac{1}{2}(12)(29)(15)$ $\{= 2610\}$	Attempt to evaluate $\sum_{r=1}^{12}\left(3r^2+8r+3\right)$	M1
	$\sum_{r=1}^{12}(2^{r-1}) = \frac{1(1-2^{12})}{1-2}$ $\{= 4095\}$	Attempt to apply the sum to 12 terms of a GP or adds up all 12 terms.	M1
		$\frac{1(1-2^{12})}{1-2}$ o.e. or 4095.	A1
	So, $2610+4095k=3520 \Rightarrow 4095k=910$		
	giving, $k=\frac{2}{9}$	$k=\frac{2}{9}$ or $0.\dot{2}$	A1
			[4] **9**

	Question 8 Notes
8. (b)	**Note** 2nd M1 1st A1: These two marks can be implied by seeing 4095 or 4095k

Question Number	Scheme	Marks
9.	$u_{n+2} = 6u_{n+1} - 9u_n$, $n \geqslant 1$, $u_1 = 6$, $u_2 = 27$; $u_n = 3^n(n+1)$	
(i)	$n=1$; $u_1 = 3(2) = 6$ — Check that $u_1 = 6$ and $u_2 = 27$	B1
	$n=2$; $u_2 = 3^2(2+1) = 27$	
	So u_n is true when $n=1$ and $n=2$.	
	Assume that $u_k = 3^k(k+1)$ and $u_{k+1} = 3^{k+1}(k+2)$ are true. — Could assume for $n=k, n=k-1$ and show for $n=k+1$	
	Then $u_{k+2} = 6u_{k+1} - 9u_k$	
	$= 6(3^{k+1})(k+2) - 9(3^k)(k+1)$ — Substituting u_k and u_{k+1} into $u_{k+2} = 6u_{k+1} - 9u_k$	M1
	Correct expression	A1
	$= 2(3^{k+2})(k+2) - (3^{k+2})(k+1)$ — Achieves an expression in 3^{k+2}	M1
	$= (3^{k+2})(2k+4-k-1)$	
	$= (3^{k+2})(k+3)$	
	$= (3^{k+2})(k+2+1)$ — $(3^{k+2})(k+2+1)$ or $(3^{k+2})(k+3)$	A1
	If the result is **true for *n* = *k* and *n* = *k*+1** then it is now **true for *n* = *k*+2**. As it is **true for *n* = 1 and *n* = 2** then it is **true for all *n*** $(\in \mathbb{Z}^+)$. — Correct conclusion seen at the end. Condone true for *n* = 1 and *n* = 2 seen anywhere**.** This should be compatible with assumptions.	A1 cso
		[6]
(ii)	$\mathrm{f}(n) = 3^{3n-2} + 2^{3n+1}$ is divisible by 19	
	In all ways, first M is for applying $\mathrm{f}(k+1)$ **with at least 1 power correct. The second M is dependent on at least one accuracy being awarded and making** $\mathrm{f}(k+1)$ **the subject and the final A is correct solution only.**	
(ii) **Way 1**	$\mathrm{f}(1) = 3^1 + 2^4 = 19$ {which is divisible by 19}. — Shows $\mathrm{f}(1) = 19$	B1
	$\{\therefore \mathrm{f}(n)$ is divisible by 19 when $n=1$ }	
	{Assume that for $n=k$,	
	$\mathrm{f}(k) = 3^{3k-2} + 2^{3k+1}$ is divisible by 19 for $k \in \mathbb{Z}^+$.}	
	$\mathrm{f}(k+1) - \mathrm{f}(k) = 3^{3(k+1)-2} + 2^{3(k+1)+1} - (3^{3k-2} + 2^{3k+1})$ — Applies $\mathrm{f}(k+1)$ with at least 1 power correct	M1
	$\mathrm{f}(k+1) - \mathrm{f}(k) = 27(3^{3k-2}) + 8(2^{3k+1}) - (3^{3k-2} + 2^{3k+1})$	
	$\mathrm{f}(k+1) - \mathrm{f}(k) = 26(3^{3k-2}) + 7(2^{3k+1})$	
	$= 7(3^{3k-2} + 2^{3k+1}) + 19(3^{3k-2})$ — Either $7(3^{3k-2} + 2^{3k+1})$ or $7\mathrm{f}(k)$; $19(3^{3k-2})$	A1;
	or $= 26(3^{3k-2} + 2^{3k+1}) - 19(2^{3k+1})$ — or $26(3^{3k-2} + 2^{3k+1})$ or $26\mathrm{f}(k)$; $-19(2^{3k+1})$	A1
	$= 7\mathrm{f}(k) + 19(3^{3k-2})$	
	or $= 26\mathrm{f}(k) - 19(2^{3k+1})$	
	$\therefore \mathrm{f}(k+1) = 8\mathrm{f}(k) + 19(3^{3k-2})$ **or** $\mathrm{f}(k+1) = 27\mathrm{f}(k) - 19(2^{3k+1})$ — Dependent on at least one of the previous accuracy marks being awarded. Makes Applies $\mathrm{f}(k+1)$ with at least 1 power correct the subject	dM1
	$\{\therefore \mathrm{f}(k+1) = 8\mathrm{f}(k) + 19(3^{3k-2})$ is divisible by 19 as both $8\mathrm{f}(k)$ and $19(3^{3k-2})$ are both divisible by 19}	

	If the result is **true for $n = k$**, then it is now **true for $n = k + 1$**. As the result has shown to be **true for $n = 1$**, then the result is **true for all n** $(\in \mathbb{Z}^+)$.	Correct conclusion seen at the end. Condone true for $n = 1$ stated earlier.	A1 cso
			[6]
(ii) **Way 2**	$f(1) = 3^1 + 2^4 = 19$ {which is divisible by 19}.	Shows $f(1) = 19$	B1
	$\{\therefore f(n)$ is divisible by 19 when $n = 1$ }		
	Assume that for $n = k$,		
	$f(k) = 3^{3k-2} + 2^{3k+1}$ is divisible by 19 for $k \in \mathbb{Z}^+$.		
	$f(k+1) = 3^{3(k+1)-2} + 2^{3(k+1)+1}$	Applies $f(k+1)$ with at least 1 power correct	M1
	$f(k+1) = 27(3^{3k-2}) + 8(2^{3k+1})$		
	$= 8(3^{3k-2} + 2^{3k+1}) + 19(3^{3k-2})$	Either $8(3^{3k-2} + 2^{3k+1})$ or $8f(k)$; $19(3^{3k-2})$	A1;
	or $= 27(3^{3k-2} + 2^{3k+1}) - 19(2^{3k+1})$	or $27(3^{3k-2} + 2^{3k+1})$ or $27f(k)$; $-19(2^{3k+1})$	A1
	$\therefore f(k+1) = 8f(k) + 19(3^{3k-2})$	**Dependent on at least one of the previous accuracy marks being awarded.**	dM1
	or $f(k+1) = 27f(k) - 19(2^{3k+1})$		
	$\{\therefore f(k+1) = 8f(k) + 19(3^{3k-2})$ is divisible by 19 as both $8f(k)$ and $19(3^{3k-2})$ are both divisible by 19}		
	If the result is **true for $n = k$**, then it is now **true for $n = k + 1$**. As the result has shown to be **true for $n = 1$**, then the result is **true for all n** $(\in \mathbb{Z}^+)$.	Correct conclusion seen at the end. Condone true for $n = 1$ stated earlier.	A1 cso
			[6]
	$f(n) = 3^{3n-2} + 2^{3n+1}$ is divisible by 19		
(ii) **Way 3**	$f(1) = 3^1 + 2^4 = 19$ {which is divisible by 19}.	Shows $f(1) = 19$	B1
	$\{\therefore f(n)$ is divisible by 19 when $n = 1$ }		
	Assume that for $n = k$,		
	$f(k) = 3^{3k-2} + 2^{3k+1}$ is divisible by 19 for $k \in \mathbb{Z}^+$.		
	$f(k+1) - \alpha f(k) = 3^{3(k+1)-2} + 2^{3(k+1)+1} - \alpha(3^{3k-2} + 2^{3k+1})$	Applies $f(k+1)$ with at least 1 power correct	M1
	$f(k+1) - \alpha f(k) = (27-\alpha)(3^{3k-2}) + (8-\alpha)2^{3k+1}$		
	$= (8-\alpha)(3^{3k-2} + 2^{3k+1}) + 19(3^{3k-2})$	$(8-\alpha)(3^{3k-2} + 2^{3k+1})$ or $(8-\alpha)f(k)$; $19(3^{3k-2})$ NB choosing $\alpha = 8$ makes first term disappear.	A1;
	or $= (27-\alpha)(3^{3k-2} + 2^{3k+1}) - 19(2^{3k+1})$	$(27-\alpha)(3^{3k-2} + 2^{3k+1})$ or $(27-\alpha)f(k)$; $-19(2^{3k+1})$ NB choosing $\alpha = 27$ makes first term disappear.	A1
	$\therefore f(k+1) = 8f(k) + 19(3^{3k-2})$	**Dependent on at least one of the previous accuracy marks being awarded.**	dM1
	or $f(k+1) = 27f(k) - 19(2^{3k+1})$	Makes $f(k+1)$ the subject.	
	$\{\therefore f(k+1) = 27f(k) - 19(2^{3k+1})$ is divisible by 19 as both $27f(k)$ and $19(2^{3k+1})$ are both divisible by 19}		
	If the result is **true for $n = k$**, then it is now **true for $n = k + 1$**. As the result has shown to be **true for $n = 1$**, then the result is true **for all n** $(\in \mathbb{Z}^+)$.	Correct conclusion seen at the end. Condone true for $n = 1$ stated earlier.	A1 cso
			[6]
			12

	Question 9 Notes
(ii)	Accept use of $f(k) = 3^{3k-2} + 2^{3k+1} = 19m$ o.e. and award method and accuracy as above.

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