Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Further Pure Mathematics FP1
(6667/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⎣ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q),\ \text{where} |pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q),\ \text{where} |pq| = |c| \text{ and } |mn| = |a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\ \ q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

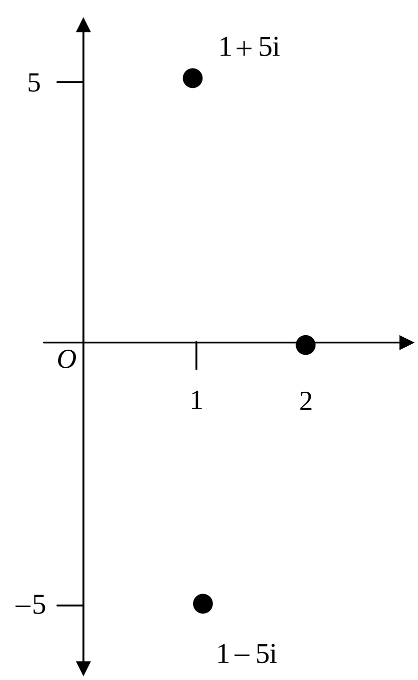
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Notes	Marks
1.(a)	$\frac{z_1}{z_2}=\frac{p+2i}{1-2i}.\frac{1+2i}{1+2i}$	Multiplying top and bottom by conjugate	M1
	$=\frac{p+2pi+2i-4}{5}$	At least 3 correct terms in the numerator, evidence that i^2 = -1 and denominator real.	M1
	$=\frac{p-4}{5}, \quad +\frac{2p+2}{5}\mathrm{i}$	Real + imaginary with i factored out. Accept single denominator with numerator in correct form. Accept 'a= ' and 'b='.	A1, A1
			(4)
(b)	$\left\|\frac{z_1}{z_2}\right\|^2=\left(\frac{p-4}{5}\right)^2+\left(\frac{2p+2}{5}\right)^2$	Accept their answers to part (a). Any erroneous i or i^2 award M0	M1
	$\left(\frac{p-4}{5}\right)^2+\left(\frac{2p+2}{5}\right)^2=13^2$ $or \ \sqrt{\left(\frac{p-4}{5}\right)^2+\left(\frac{2p+2}{5}\right)^2}=13$	$\left\|\frac{z_1}{z_2}\right\|^2=13^2$ or $\left\|\frac{z_1}{z_2}\right\|=13$	dM1
	$\frac{p^2-8p+16}{25}+\frac{4p^2+8p+4}{25}=169 \ or \ 13^2$		
	$5p^2+20=4225$		
	$p^2=841\Rightarrow p=\pm 29$	dM1:Attempt to solve their quadratic in *p*, dependent on both previous Ms. A1:**both** 29 **and** -29	dM1A1
	OR		
	$\frac{\|z_1\|}{\|z_2\|}=\frac{\sqrt{p^2+4}}{\sqrt{5}}$	Finding moduli Any erroneous i or i^2 award M0	M1
	$\frac{\sqrt{p^2+4}}{\sqrt{5}}=13$ oe	Equating to 13	dM1
	$\frac{p^2+4}{5}=169 \ or \ 13^2$ oe		
	$p^2=841\Rightarrow p=\pm 29$	dM1:Attempt to solve their quadratic in *p*, dependent on both previous Ms	dM1A1
		A1:**both** 29 **and** -29	
			(4)
			Total 8

Question Number	Scheme	Notes	Marks
2.	$\mathrm{f}(x)=x^3-\dfrac{5}{2x^{\frac{3}{2}}}+2x-3$		
(a)	$\mathrm{f}(1.1)=-1.6359604$, $\mathrm{f}(1.5)=2.0141723$	Attempts to evaluate both $\mathrm{f}(1.1)$ and $\mathrm{f}(1.5)$ and evaluates at least one of them correctly to awrt (or trunc.) 2 sf.	M1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore **a root / α** is between $x=1.1$ and $x=1.5$	Both values correct to awrt (or trunc.) 2 sf, sign change (or a statement which implies this e.g. -1.63.. < 0 < 2.014..) and conclusion.	A1
			(2)
(b)	$\mathrm{f}(x)=x^3-\frac{5}{2}x^{-\frac{3}{2}}+2x-3$ $\Rightarrow \mathrm{f}'(x)=3x^2+\frac{15}{4}x^{-\frac{5}{2}}+2$	M1: $x^n\to x^{n-1}$ for at least one term A1:Correct derivative oe	M1A1
			(2)
(c)	$\mathrm{f}'(1.1)=3(1.1)^2+\frac{15}{4}(1.1)^{-\frac{5}{2}}+2\,(=8.585)$	Attempt to find $\mathrm{f}'(1.1)$. Accept $\mathrm{f}'(1.1)$ seen and their value.	M1
	$\alpha_2=1.1-\left(\dfrac{"-1.6359604"}{"8.585"}\right)$	Correct application of N-R	M1
	$\alpha_2=1.291$	cao	A1
			(3)
			Total 7

Question Number	Scheme	Notes	Marks
3.	$x^3 + px^2 + 30x + q = 0$		
(a)	$1+5i$		B1
			(1)
(b)	$((x-(1+5i))(x-(1-5i))) = x^2 - 2x + 26$ $((x-2)(x-(1\pm 5i))) = x^2 - (3\pm 5i)x + 2(1\pm 5i)$	M1: 1. Attempt to expand or 2. Use sum and product of the complex roots. A1: Correct expression	M1A1
	$(x^2 - 2x + 26)(x-2) = x^3 + px^2 + 30x + q$	Uses their third factor with their quadratic with at least 4 terms in the expansion	M1
	$p = -4, \quad q = -52$	May be seen in cubic	A1, A1
OR	f(1+5i)=0 or f(1-5i)=0	Substitute one complex root to achieve 2 equations in *p* and / or q	M1
	$q - 24p - 44 = 0$ and $10p + 40 = 0$	Both equations correct oe	A1
		Solving for *p* and *q*	M1
	$p = -4, \quad q = -52$	May be seen in cubic	A1, A1
			(5)
(c)		B1: Conjugate pair correctly positioned and labelled with 1+5i, 1-5i or (1,5),(1,-5) or axes labelled 1 and 5. B1: The 2 correctly positioned relative to conjugate pair and labelled.	B1 B1
			(2) **Total 8**

Question Number	Scheme	Notes	Marks
4.	$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix}$		
(i)(a)	$\begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 5 & 6 \\ 5 & -6 & 11 \\ 13 & 11 & 21 \end{pmatrix}$	M1: 3x3 matrix with a number or numerical expression for each element A2:cao (**-**1 each error) Only 1 error award A1A0	M1A2
(b)	$\mathbf{BA} = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 15 & 25 \\ 14 & 4 \end{pmatrix}$	Allow any convincing argument. E.g.s BA is a 2x2 matrix (so AB ≠ BA) or dimensionally different. Attempt to evaluate product not required. NB 'Not commutative' only is B0	B1
			(4)
(ii)	$(\det \mathbf{C} =) 2k \times k - 3 \times (-2)$	Correct attempt at determinant	M1
	$\mathbf{C}^{-1} = \dfrac{1}{2k^2 + 6}\begin{pmatrix} k & 2 \\ -3 & 2k \end{pmatrix}$	M1: $\dfrac{\mathbf{1}}{\text{their} \det \mathbf{C}}\begin{pmatrix} k & 2 \\ -3 & 2k \end{pmatrix}$ A1:cao oe	M1A1
			(3)
			Total 7

Question Number	Scheme	Notes	Marks
5.(a)	$\left(\left(2r-1\right)^2=\right)4r^2-4r+1$		B1
	Proof by induction will usually score no more marks without use of standard results		
	$\sum_{r=1}^{n}\left(2r-1\right)^2=\sum_{r=1}^{n}\left(4r^2-4r+1\right)$		
	$=4\sum r^2-4\sum r+\sum 1$		
	$=\underline{4.\frac{1}{6}n\left(n+1\right)\left(2n+1\right)-4.\frac{1}{2}n\left(n+1\right)},+n$	M1: An attempt to use at least one of the standard results correctly in summing at least 2 terms of their expansion of $\left(2r-1\right)^2$ A1: Correct underlined expression oe B1: $\sum 1=n$	M1A1B1
	$=\frac{1}{3}n\left[4n^2+6n+2-6n-6+3\right]$	Attempt to factor out $\frac{1}{3}n$ before given answer	M1
	$=\frac{1}{3}n\left[4n^2-1\right]$	cso	A1
			(6)
(b)	$\sum_{r=2n+1}^{4n}\left(2r-1\right)^2=\mathrm{f}(4n)-\mathrm{f}(2n)$ or $\mathrm{f}(2n+1)$	Require some use of the result in part (a) for method.	M1
	$=\frac{1}{3}4n\left(4.\left(4n\right)^2-1\right)-\frac{1}{3}.2n\left(4.\left(2n\right)^2-1\right)$	Correct expression	A1
	$=\frac{2}{3}n\left[128n^2-2-16n^2+1\right]$		
	$=\frac{2}{3}n\left[112n^2-1\right]$	Accept $a=\frac{2}{3},b=112$	A1
			(3)
			Total 9

Question Number	Scheme	Notes	Marks
6.	$xy = c^2$ at $\left(ct, \frac{c}{t}\right)$.		
(a)	$y = \dfrac{c^2}{x} = c^2x^{-1} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = -c^2x^{-2} = -\dfrac{c^2}{x^2}$	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = kx^{-2}$	
	$xy = c^2 \Rightarrow x\dfrac{\mathrm{d}y}{\mathrm{d}x} + y = 0$	Correct use of product rule. The sum of two terms, one of which is correct **and** rhs = 0	M1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{dy}{dt}.\dfrac{dt}{dx} = -\dfrac{c}{t^2}.\dfrac{1}{c}$	their $\dfrac{dy}{dt} \times \left(\dfrac{1}{\text{their } \frac{dx}{dt}}\right)$	
	$\dfrac{dy}{dx} = -c^2x^{-2}$ or $x\dfrac{dy}{dx} + y = 0$ or $\dfrac{dy}{dx} = \dfrac{-c}{t^2}.\dfrac{1}{c}$ or equivalent expressions	Correct differentiation	A1
	$y - \dfrac{c}{t} = -\dfrac{1}{t^2}(x - ct)$ $\qquad (\times t^2)$	$y - \dfrac{c}{t} =$ their $m_T(x - ct)$ or $y = mx + c$ with their m_T and $(ct, \frac{c}{t})$ in an attempt to find 'c'. **Their m_T must have come from calculus and should be a function of t or c or both c and t.**	dM1
	$t^2y + x = 2ct$ (Allow $x + t^2y = 2ct$)	Correct solution only.	A1*
			(4)
	(a) Candidates who derive $x + t^2y = 2ct$, by stating that $m_T = -\dfrac{1}{t^2}$, with no justification score <u>no</u> marks in (a).		
(b)	$y = 0 \Rightarrow x = \dfrac{ct^4 - c}{t^3} \Rightarrow A\left(\dfrac{ct^4 - c}{t^3}, 0\right)$	$\dfrac{ct^4 - c}{t^3}$ or equivalent form	B1
	$y = 0 \Rightarrow x = 2ct \Rightarrow B(2ct, 0)$.	$2ct$	B1
			(2)
(c)	AB = $\text{"}2ct\text{"} - \text{"}\dfrac{ct^4 - c}{t^3}\text{"}$ **or** PA= $ct^{-3}\sqrt{t^4 + 1}$ and PB= $ct^{-1}\sqrt{t^4 + 1}$	Attempt to subtract their x-coordinates either way around.	M1
	Area APB = $\dfrac{1}{2} \times \textit{their AB} \times \dfrac{c}{t}$	Valid complete method for the area of the triangle in terms of t or c and t.	M1
	$= \dfrac{1}{2}\left(2ct - \dfrac{ct^4 - c}{t^3}\right)\dfrac{c}{t} = \dfrac{c^2(t^4 + 1)}{2t^4}$		
	$= 8\left(1 + \dfrac{1}{t^4}\right)$ or $\dfrac{8(t^4 + 1)}{t^4}$ or $\dfrac{8t^4 + 8}{t^4}$ or $8 + \dfrac{8}{t^4}$	Use of $c = 4$ and completes to one of the given forms oe simplest form. Final answer should be positive for A mark.	A1
			(3)
			Total 9

Question Number	Scheme	Notes	Marks
7.(i)(a)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$		B1
(b)	$\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$		B1
(c)	$\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$	M1: Multiplies their (b) x their (a) in the correct order A1: Correct matrix Correct matrix seen M1A1	M1A1
			(4)
(ii)	Area triangle T = $\frac{1}{2}\times(11-3)\times k = 4k$	M1: Correct method for area for *T* A1: 4k	M1A1
	$\det\begin{pmatrix} 6 & -2 \\ 1 & 2 \end{pmatrix} = 6\times 2 - 1\times(-2)(=14)$	M1: Correct method for determinant A1: 14	M1A1
	Area triangle $T = \frac{364}{"14"}(=26) \Rightarrow 4k = 26$	Uses 364 and their determinant correctly to form an equation in *k*.	M1
	$k = \frac{26}{4}\left(=\frac{13}{2}\right)$	Accept $k = \pm\frac{13}{2}$ or $k = -\frac{13}{2}$	A1
			(6)
			Total 10

Question Number	Scheme	Notes	Marks
8.(a)	$m=\dfrac{4k-8k}{k^2-4k^2}\left(=\dfrac{4}{3k}\right)$	Valid attempt to find gradient in terms of *k*	M1
	$y-8k=\dfrac{4}{3k}\left(x-4k^2\right)$ or $y-4k=\dfrac{4}{3k}\left(x-k^2\right)$ or $y=\dfrac{4}{3k}x+\dfrac{8k}{3}$	M1: Correct straight line method with their gradient in terms of *k*. If using $y=mx+c$ then award M provided they attempt to find *c* A1: Correct equation. If using $y=mx+c$, awardwhen they obtain $c=\dfrac{8k}{3}$ oe	M1A1
	$3ky-24k^2=4x-16k^2\Rightarrow 3ky-4x=8k^2$ * or $3ky-12k^2=4x-4k^2\Rightarrow 3ky-4x=8k^2$ *	Correct completion to printed answer with at least one intermediate step.	A1*
			(4)
(b)	(Focus) (4, 0)	Seen or implied as a number	B1
	(Directrix) x = -4	Seen or implied as a number	B1
	Gradient of l_2 is $-\dfrac{3k}{4}$	Attempt negative reciprocal of grad l_1 as a function of *k*	M1
	$y-0=\dfrac{-3k}{4}(x-4)$	Use of their changed gradient and numerical Focus in either formula, as printed oe	M1, A1
	$x=-4\Rightarrow y=\dfrac{-3k}{4}(-4-4)$	Substitute numerical directrix into their line	M1
	$(y=)6k$	oe	A1
			(7)
			Total 11

Question Number	Scheme	Notes	Marks
9.	$f(n) = 8^n - 2^n$ is divisible by 6.		
	$f(1) = 8^1 - 2^1 = 6$,	Shows that $f(1) = 6$	B1
	Assume that for $n = k$, $f(k) = 8^k - 2^k$ is divisible by 6.		
	$f(k+1) - f(k) = 8^{k+1} - 2^{k+1} - \left(8^k - 2^k\right)$	Attempt $f(k+1) - f(k)$	M1
	$= 8^k(8-1) + 2^k(1-2) = 7\times 8^k - 2^k$		
	$= 6\times 8^k + 8^k - 2^k \left(= 6\times 8^k + f(k)\right)$	M1: Attempt $f(k+1) - f(k)$ as a multiple of 6 A1: rhs a correct multiple of 6	M1A1
	$f(k+1) = 6\times 8^k + 2f(k)$	Completes to $f(k+1)$ = a multiple of 6	A1
	If the result is **true for** $n = k$, then it is now **true for** $n = k+1$. As the result has been shown to be **true for** $n = 1$, then the result is **true for all** n $(\in \mathbb{Z}^+.)$		A1cso
		Do not award final A if n defined incorrctly e.g. 'n is an integer' award A0	
			(6)
			Total 6
Way 2	$f(1) = 8^1 - 2^1 = 6$,	Shows that $f(1) = 6$	B1
	Assume that for $n = k$, $f(k) = 8^k - 2^k$ is divisible by 6.		
	$f(k+1) = 8^{k+1} - 2^{k+1} = 8\left(8^k - 2^k + 2^k\right) - 2.2^k$	Attempts $f(k+1)$ in terms of 2^k and 8^k	M1
	$f(k+1) = 8^{k+1} - 2^{k+1} = 8\left(f(k) + 2^k\right) - 2.2^k$	M1:Attempts $f(k+1)$ in terms of $f(k)$ A1: rhs correct and a multiple of 6	M1A1
	$f(k+1) = 8f(k) + 6.2^k$	Completes to $f(k+1)$ = a multiple of 6	A1
	If the result is **true for** $n = k$, then it is now **true for** $n = k+1$. As the result has been shown to b**e true for** $n = 1$, then the result is **true for all** n $(\in \mathbb{Z}^+.)$		A1cso
Way 3	$f(1) = 8^1 - 2^1 = 6$,	Shows that $f(1) = 6$	B1
	Assume that for $n = k$, $f(k) = 8^k - 2^k$ is divisible by 6.		
	$f(k+1) - 8f(k) = 8^{k+1} - 2^{k+1} - 8.8^k + 8.2^k$	Attempt $f(k+1) - 8f(k)$	M1
		Any multiple m replacing 8 award M1	
	$f(k+1) - 8f(k) = 8^{k+1} - 8^{k+1} + 8.2^k - 2.2^k = 6.2^k$	M1: Attempt $f(k+1) - f(k)$ as a multiple of 6 A1: rhs a correct multiple of 6	M1A1
	$f(k+1) = 8f(k) + 6.2^k$	Completes to $f(k+1)$ = a multiple of 6	A1
		General Form for multiple m $f(k+1) = 6.8^k + (2-m)(8^k - 2^k)$	
	If the result is **true for** $n = k$, then it is now **true for** $n = k+1$. As the result has been shown to be **true for** $n = 1$, then the result is **true for all** n $(\in \mathbb{Z}^+.)$		A1cso

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