



Mark Scheme (Results)

June 2013

GCE Further Pure Mathematics FP1
(6667/01)
Original Paper

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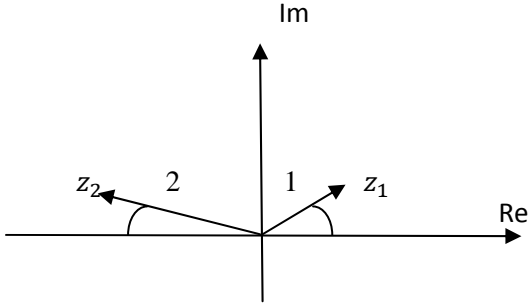
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Question Number	Scheme	Marks
1(a)	$\det \mathbf{M} = a(2 - a) - 1$	M1A1
1(b)	$\det \mathbf{M} = 0$ $a^2 - 2a + 1 = 0$ $(a - 1)^2 = 0$ $a = 1$	M1 M1 A1 (2) (3) [5]
(a)	Notes M for “ $ad - bc$ “	
(b)	First M for their $\det \mathbf{M} = 0$ Second M for attempt to solve their 3 term quadratic Method mark for solving 3 term quadratic: 1. <u>Factorisation</u> $(x^2 + bx + c) = (x + p)(x + q)$, where $ pq = c $, leading to $x =$ $=$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $ pq = c $ and $ mn = a $,leading to $x =$ 2. <u>Formula</u> Attempt to use <u>correct</u> formula (with values for a , b and c). 3. <u>Completing the square</u> Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$	

Question Number	Scheme	Marks
2	<p> $z = -2i - 1$ is also a root $(z - (2i - 1))(z - (-2i - 1)) = z^2 + 2z + 5$ $(z + 3)(z^2 + 2z + 5) = 0$ $z = -3$ </p> <p>Alternative</p> <p> $f(-3) = 0$ so $z = -3$ is also a root $(z + 3)(z^2 + 2z + 5) = 0$ $(z - (2i - 1))(z - (-2i - 1)) = 0$ $z = -2i - 1$ is also a root </p> <p>Notes</p> <p>First M for expanding their $(z - \alpha)(z - \beta)$</p> <p>Second M for inspection or long division.</p>	<p> B1 M1A1 M1 A1 (5) [5] </p> <p>M1A1</p> <p>M1A1</p> <p>B1</p>

Question Number	Scheme	Marks
3(a)	$z_1 = \frac{1}{2} + i\frac{\sqrt{3}}{2}$ $r = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1, \quad \tan \theta = \sqrt{3} \text{ so } \theta = \frac{\pi}{3}, \text{ both } r \text{ values}$ $z_2 = -\sqrt{3} + i$ $r = \sqrt{3+1} = 2, \quad \tan \theta = \frac{-1}{\sqrt{3}} \text{ so } \theta = \frac{5\pi}{6}$	M1A1 M1A1 (4)
3(b)	$ z_1 z_2 = z_1 z_2 = 2$	M1A1 (2)
3(c)		M1 A1ft (2) [8]
(a)	<p>Notes</p> <p>First M for use of Pythagoras, A1 for $r = 1$ and 2</p> <p>Second M for use of \tan or \tan^{-1}, A1 for $\theta = \frac{\pi}{3}$ and $\frac{5\pi}{6}$</p>	
(b)	<p>M for their $r_1 r_2$</p>	
(c)	<p>M for either of their numbers plotted correctly on a single diagram.</p> <p>A for both their numbers plotted correctly on a single diagram</p>	

Question Number	Scheme	Marks
4(a)	$xy = 3 \text{ or } y = \frac{3}{x}$ $x \frac{dy}{dx} + y = 0$ $\frac{dy}{dx} = -\frac{y}{x} \text{ or } \frac{dy}{dx} = -\frac{3}{x^2}$ <p>Gradient of normal is $\frac{x}{y}$ or $\frac{x^2}{3}$</p> $y - 3 = \frac{1}{3}(x - 1)$ $y = \frac{1}{3}x + \frac{8}{3}$	<p>M1A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;">(5)</p>
4(b)	<p>At R, $y = \frac{3}{x}$</p> $\frac{9}{x} - x = 8$ $x^2 + 8x - 9 = 0$ $(x + 9)(x - 1) = 0$ $x = -9, \quad y = -\frac{1}{3}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1,A1</p> <p style="text-align: right;">(5)</p>
(a)	<p>Notes</p> <p>First M: Use of the product rule: The sum of two terms including dy/dx, one of which is correct or</p> $\frac{dy}{dx} = kx^{-2}$ <p>First A for correct derivative</p> $-3x^{-2} \text{ or } -\frac{y}{x}$ <p>Second M for use of Perpendicular gradient rule $m_N m_T = -1$</p> <p>Third M for</p> <p>$y - 3 = \text{their } m_N(x - 1)$ or</p> <p>$y = mx + c$ with their m_N and (1,3) in an attempt to find 'c'.</p>	
(b)	<p>First M for substituting $y = \frac{3}{x}$ in their normal.</p> <p>First A for correct 3 term quadratic</p> <p>Second M for attempt to solve their 3 term quadratic</p>	<p style="text-align: right;">(5)</p> <p style="text-align: right;">[10]</p>

Question Number	Scheme	Marks
5	<p> $f(1)=3^2 + 7 = 16 = 8 \times 2$ True for $n = 1$ Assume true for $n = k$, $f(k) = 3^{2k} + 7 = 8p$ where p is a positive integer When $n = k + 1$ $f(k + 1) - f(k) = 3^{2(k+1)} + 7 - (3^{2k} + 7)$ $= 9 \times 3^{2k} + 7 - 3^{2k} - 7$ $= 8 \times 3^{2k}$ $f(k + 1) = 8(3^{2k} + p) = 8q$ where q is a positive integer <u>True for $n = k + 1$</u> <u>True for $n = 1$, if true for $n = k$ then true for $n = k + 1$</u> So <u>$3^{2n} + 7$ divisible by 8 for all n</u> by Induction. </p> <p>Notes B for $f(1)=3^2 + 7 = 16$ seen First M for substituting into $f(k + 1) - f(k)$ or showing $f(k + 1) = 9 \times 3^{2k} + 7$ Second M for using $f(k + 1) - f(k)$ or equivalent First A for $f(k + 1) = f(k) + 8 \times 3^{2k}$ or equivalent. Third M for showing divisible by 8. Accept ' $f(k)$ divisible by 8 and 8×3^{2k} divisible by 8'. Second A for conclusion with all 4 underlined elements that can be seen anywhere in the solution</p>	<p>B1</p> <p>M1</p> <p>dM1</p> <p>A1</p> <p>dM1</p> <p>A1cso</p> <p>(6)</p> <p>[6]</p>

Question Number	Scheme	Marks
6(a)	$y^2 = 4x$ $2y \frac{dy}{dx} = 4$ $\text{At } P, \frac{dy}{dx} = \frac{1}{p}$ $y - 2p = \frac{1}{p}(x - p^2)$	M1A1 A1 M1A1 (5)
6(b)(i)	$\text{At } (-1, 2)$ $2 - 2p = \frac{1}{p}(-1 - p^2)$ $p^2 - 2p - 1 = 0$ $p = 1 \pm \sqrt{2}$ $p = 1 + \sqrt{2}, \quad q = 1 - \sqrt{2}$	M1 A1 M1 A1 (4)
6(b)(ii)	$PR^2 = 32 + 16\sqrt{2}, \quad QR^2 = 32 - 16\sqrt{2}$ $\text{Area of } PQR = \frac{1}{2} PR \cdot QR = 8\sqrt{2}$	M1A1 M1A1 (4) [13]
(a)	<p>Notes</p> <p>First M for $x^{\frac{1}{2}} \rightarrow x^{-\frac{1}{2}}$ or $ky \frac{dy}{dx} = c$ or $\frac{dy}{dt} \times \frac{1}{\frac{dx}{dt}}$; can be a function of p or t.</p> <p>First A for accurate differentiation</p> <p>Second M applies $y - 2p = \text{their } m(x - p^2)$ or $y = (\text{their } m)x + c$ using $x = p^2$ and $y = 2p$ in an attempt to find c. Their m must be a function of p from calculus.</p>	
(b)i	<p>First M substitute coordinates of the point R into their tangent</p> <p>Second M for solving 3 term quadratic</p>	
(b)ii	<p>Second A for $1 \pm \sqrt{2}$ seen</p> <p>First M for attempt to find distance between their P and R or Q and R using formula or sketch and Pythagoras.</p> <p>Second M for using $\frac{1}{2}bh$ on their PQR</p> <p>Second A accept awrt 11.3</p>	

Question Number	Scheme	Marks
7(a)	$\sum_{r=1}^n r^2(r-1) = \sum_{r=1}^n r^3 - \sum_{r=1}^n r^2$ $= \frac{n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{6}$ $= \frac{n(n+1)}{12} (3n(n+1) - 2(2n+1))$ $= \frac{n(n+1)(3n^2 - n - 2)}{12}$ $= \frac{n(n+1)(3n+2)(n-1)}{12}$	M1 A1 M1 A1 A1cso (5)
7(b)	$\sum_{r=10}^{r=50} r^2(r-1) = \sum_{r=1}^{50} r^2(r-1) - \sum_{r=1}^{r=9} r^2(r-1)$ $= \frac{1}{12} (50 \times 51 \times 152 \times 49) - \frac{1}{12} (9 \times 10 \times 29 \times 8)$ $= 1582700 - 1740 = 1580960$	M1 A1 A1 (3) [8]
(a)	<p>Notes</p> <p>First M for expanding brackets</p> <p>First A for correct expressions for $\sum r^3$ and $\sum r^2$</p> <p>Second M for factorising by $n(n+1)$</p> <p>Second A for $(3n^2 - n - 2)$ or equivalent factor</p>	
(b)	<p>First M for f(49 or 50) – f(9 or 10) and attempt to use part (a).</p>	

Question Number	Scheme	Marks																		
8(a)	$(f(1) =) -4 (< 0)$ $(f(2) =) 1 (> 0)$ Changes sign so root (in $[1, 2]$)	B1 B1 B1 (3)																		
8(b)	<table border="1" data-bbox="240 439 1086 600"> <thead> <tr> <th>a</th> <th>$f(a)$</th> <th>b</th> <th>$f(b)$</th> <th>$\frac{a+b}{2}$</th> <th>$f\left(\frac{a+b}{2}\right)$</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>-4</td> <td>2</td> <td>1</td> <td>1.5</td> <td>-2.625</td> </tr> <tr> <td>1.5</td> <td>-2.625</td> <td>2</td> <td>1</td> <td>1.75</td> <td>-1.140625</td> </tr> </tbody> </table> <p>Interval is $[1.75, 2]$</p>	a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$	1	-4	2	1	1.5	-2.625	1.5	-2.625	2	1	1.75	-1.140625	B1M1 A1 (3)
a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$															
1	-4	2	1	1.5	-2.625															
1.5	-2.625	2	1	1.75	-1.140625															
8(c)	$f'(x) = 3x^2 - 2$ $x_1 = 1.8 - \frac{1.8^3 - 2 \times 1.8 - 3}{3 \times 1.8^2 - 2}$ $x_1 = 1.90$ to 3sf.	M1A1 M1A1 A1 (5) [11]																		
(b)	<p>Notes</p> <p>B for awrt -2.6 M for attempt to find $f(1.75)$ A for $f(1.75) =$ awrt -1.1 with $1.75 \leq \alpha \leq 2$ or $1.75 < \alpha < 2$ or $[1.75, 2]$ or $(1.75, 2)$.</p>																			
(c)	<p>First M for at least one of the two terms differentiated correctly. First A for correct derivative Second M for correct application of Newton-Raphson using their values. Second A for $f(1.8) = -0.768$ Third A for 1.90 cao</p>																			

Question Number	Scheme	Marks
9(a)	$\begin{pmatrix} 3 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 10 & 1 \\ 1 & 5 \end{pmatrix}$	M1A1 (2)
(b)	$\det \mathbf{A} = -7 \neq 0$ so \mathbf{A} is non-singular	M1A1 (2)
(c)	$\mathbf{A}^{-1} = -\frac{1}{7} \begin{pmatrix} -2 & -1 \\ -1 & 3 \end{pmatrix}$	M1A1 (2)
(d)	$-\frac{1}{7} \begin{pmatrix} -2 & -1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} k-1 \\ 2-k \end{pmatrix} = -\frac{1}{7} \begin{pmatrix} -2(k-1) - 1(2-k) \\ -1(k-1) + 3(2-k) \end{pmatrix}$ $= \begin{pmatrix} \frac{1}{7}k \\ \frac{4}{7}k - 1 \end{pmatrix}$ <p>(p lies on $y = 4x - 1$)</p>	M1 A1,A1 (3) [9]
(d)	<p>Notes</p> <p>Alt</p> $\begin{pmatrix} 3 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} k-1 \\ 2-k \end{pmatrix}$ <p>then multiply out and attempt to solve simultaneous equations for x or y in terms of k. M1</p> <p>$x = \frac{1}{7}k$ A1</p> <p>$y = \frac{4}{7}k - 1$ A1</p>	

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