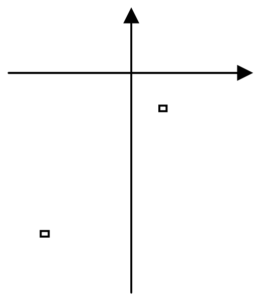
June 2010
Further Pure Mathematics FP1 6667
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $(2-3\mathrm{i})(2-3\mathrm{i}) = \ldots\ldots$ **Expand and use** $\mathrm{i}^2=-1$, getting completely correct expansion of 3 or 4 terms Reaches $-5-12\mathrm{i}$ after completely correct work (must see 4 – 9) (*)	M1 A1cso (2)
	(b) $\lvert z^2\rvert = \sqrt{(-5)^2+(-12)^2} = 13$ **or** $\lvert z^2\rvert = \sqrt{5^2+12^2} = 13$ **Alternative methods** for part (b) $\lvert z^2\rvert = \lvert z\rvert^2 = 2^2+(-3)^2 = 13$ Or: $\lvert z^2\rvert = zz^* = 13$	M1 A1 (2) M1 A1 (2)
	(c) $\tan\alpha = \frac{12}{5}$ (allow $-\frac{12}{5}$) or $\sin\alpha = \frac{12}{13}$ or $\cos\alpha = \frac{5}{13}$ $\arg(z^2) = -(\pi - 1.176\ldots) = -1.97$ (or 4.32) allow awrt **Alternative method** for part (c) $\alpha = 2\times\arctan\left(-\tfrac{3}{2}\right)$ (allow $\tfrac{3}{2}$) **or** use $\frac{\pi}{2}+\arctan\frac{5}{12}$ so $\arg(z^2) = -(\pi - 1.176\ldots) = -1.97$ (or 4.32) allow awrt	M1 A1 (2) M1 A1
	(d)  Both in correct quadrants. Approximate relative scale No labels needed Allow two diagrams if some indication of scale Allow points or arrows	B1 (1) **7 marks**
	Notes: (a) M1: for 4 – 9 – 12i or 4 – 9 – 6i – 6i or $4-3^2-12\mathrm{i}$ but must have correct statement seen and see i^2 replaced by -1 maybe later A1: Printed answer. Must see 4 – 9 in working. Jump from 4 – 6i – 6i + 9i^2 to -5-12i is M0A0 (b) Method may be implied by correct answer. NB $\lvert z^2\rvert = 169$ is M0 A0 (c) Allow $\arctan\frac{12}{5}$ for M1 or $\pm\frac{\pi}{2}\pm\arctan\frac{5}{12}$	

Question Number	Scheme	Marks
2.	(a) $\mathbf{M} = \begin{pmatrix} 4 & 3 \\ 6 & 2 \end{pmatrix}$ Determinant: $(8-18) = -10$	B1
	$\mathbf{M}^{-1} = \frac{1}{-10}\begin{pmatrix} 2 & -3 \\ -6 & 4 \end{pmatrix}$ $\left[= \begin{pmatrix} -0.2 & 0.3 \\ 0.6 & -0.4 \end{pmatrix}\right]$	M1 A1 (3)
	(b) Setting $\Delta = 0$ and using $2a^2 \pm 18 = 0$ to obtain $a = .$	M1
	$a = \pm 3$	A1 cao (2) **5 marks**
	Notes: (a) B1: must be -10 M1: for correct attempt at changing elements in major diagonal and changing signs in minor diagonal. Three or four of the numbers in the matrix should be correct – eg allow one slip A1: for any form of the correct answer, with correct determinant then isw. Special case: *a* not replaced is B0M1A0 (b) Two correct answers, $a = \pm 3$, with no working is M1A1 Just *a* = 3 is M1A0, and also one of these answers rejected is A0. Need 3 to be simplified (not $\sqrt{9}$).	

Question Number	Scheme	Marks
3.	(a) $f(1.4) = ...$ and $f(1.5) = ...$ Evaluate both	M1
	$f(1.4) = -0.256$ (or $-\frac{32}{125}$), $f(1.5) = 0.708...$ (or $\frac{17}{24}$) **Change of sign, $\therefore$ root**	A1 (2)
	Alternative method: **Graphical method** could earn M1 if 1.4 and 1.5 are both indicated. A1 then needs correct graph and conclusion, i.e. change of sign $\therefore$ root	
	(b) $f(1.45) = 0.221...$ or 0.2 [$\therefore$ root is in [1.4, 1.45]]	M1
	$f(1.425) = -0.018...$ or -0.019 or -0.02	M1
	$\therefore$ root is in [1.425, 1.45]	A1cso (3)
	(c) $f'(x) = 3x^2 + 7x^{-2}$	M1 A1
	$f'(1.45) = 9.636...$ (Special case: $f'(x) = 3x^2 + 7x^{-2} + 2$ then $f'(1.45) = 11.636...$)	A1ft
	$x_1 = 1.45 - \frac{f(1.45)}{f'(1.45)} = 1.45 - \frac{0.221...}{9.636...} = 1.427$	M1 A1cao (5) **10 marks**

Notes

(a) M1: Some attempt at two evaluations
A1: needs accuracy to 1 figure truncated or rounded and conclusion including **sign change** indicated (One figure accuracy sufficient)

(b) M1: See f(1.45) attempted and positive
M1: See f(1.425) attempted and negative
A1: is cso – any slips in numerical work are penalised here even if correct region found.
Answer may be written as $1.425 \le \alpha \le 1.45$ or $1.425 < \alpha < 1.45$ or (1.425, 1.45) must be correct way round. Between is sufficient.
There is no credit for linear interpolation. This is M0 M0 A0
Answer with no working is also M0M0A0

(c) M1: for attempt at differentiation (decrease in power) A1 is cao
Second A1may be implied by correct answer (do not need to see it)
ft is limited to special case given.
2nd M1: for attempt at Newton Raphson with their values for f(1.45) and $f'(1.45)$.
A1: is cao and needs to be correct to 3dp
Newton Raphson used more than once – isw.
Special case: $f'(x) = 3x^2 + 7x^{-2} + 2$ then $f'(1.45) = 11.636...$) is M1 A0 A1ft M1 A0 This mark can also be given by implication from final answer of 1.43

Question Number	Scheme	Marks
4.	(a) $a = -2, \quad b = 50$	B1, B1 (2)
	(b) -3 is a root	B1
	Solving 3-term quadratic $x = \dfrac{2 \pm \sqrt{4-200}}{2}$ or $(x-1)^2 - 1 + 50 = 0$	M1
	$= 1 + 7\text{i}, \quad$ 1-7i	A1, A1ft (4)
	(c) $(-3) + (1+7\text{i}) + (1-7\text{i}) = -1$	B1ft (1) **7 marks**
	Notes (a) Accept $x^2 - 2x + 50$ as evidence of values of *a* and *b*. (b) B1: –3 must be seen in part (b) M1: for solving quadratic following usual conventions A1: for a correct root (simplified as here) and A1ft: for conjugate of first answer. Accept correct answers with no working here. If answers are written down as factors then isw. Must see roots for marks. (c) ft requires the sum of two non-real conjugate roots and a real root resulting in a real number. Answers including *x* are B0	

Question Number	Scheme	Marks
5.	(a) $y^2 = (10t)^2 = 100t^2$ and $20x = 20 \times 5t^2 = 100t^2$	B1 (1)
	Alternative method: Compare with $y^2 = 4ax$ and identify $a = 5$ to give answer.	B1 (1)
	(b) Point A is (80, 40) (stated or seen on diagram). May be given in part (a) Focus is (5, 0) (stated or seen on diagram) or (a , 0) with a = 5 May be given in part (a). Gradient: $\frac{40-0}{80-5} = \frac{40}{75} \left(= \frac{8}{15} \right)$	B1 B1 M1 A1 (4) **5 marks**
	Notes: (a) Allow substitution of x to obtain $y = \pm 10t$ (or just $10t$) or of y to obtain x (b) M1: requires use of gradient formula correctly, for their values of x and y. This mark may be implied by correct answer. Differentiation is M0 A0 A1: Accept 0.533 or awrt	

Question Number	Scheme	Marks
6.	(a) $\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$	B1 (1)
	(b) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	B1 (1)
	(c) $\mathbf{T} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & -8 \end{pmatrix}$	M1 A1 (2)
	(d) $\mathbf{AB} = \begin{pmatrix} 6 & 1 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} k & 1 \\ c & -6 \end{pmatrix} = \begin{pmatrix} 6k+c & 0 \\ 4k+2c & -8 \end{pmatrix}$	M1 A1 A1 (3)
	(e) "$6k + c = 8$" and "$4k + 2c = 0$" Form equations and solve simultaneously $k = 2$ and $c = -4$	M1 A1 (2) **9 marks**
	Alternative method for (e) **M1: AB = T $\Rightarrow$ B = A^{-1}T =** and compare elements to find k and c. Then A1 as before.	
	<u>Notes</u> (c) M1: Accept multiplication of their matrices either way round (this can be implied by correct answer) A1: cao (d) M1: Correct matrix multiplication method implied by one or two correct terms in correct positions. A1: for three correct terms in correct positions 2nd A1: for all four terms correct and simplified (e) M1: follows their previous work but must give two equations from which k and c can be found and there must be attempt at solution getting to k = or c =. A1: is cao (but not cso - may follow error in position of $4k + 2c$ earlier).	

Question Number	Scheme		Marks
7.	(a) LHS = $\mathrm{f}(k+1)=2^{k+1}+6^{k+1}$	OR RHS =	M1
		$=6\mathrm{f}(k)-4(2^k)=6(2^k+6^k)-4(2^k)$	
	$=2(2^k)+6(6^k)$	$=2(2^k)+6(6^k)$	A1
	$=6(2^k+6^k)-4(2^k)=6\mathrm{f}(k)-4(2^k)$	$=2^{k+1}+6^{k+1}=\mathrm{f}(k+1)$ (*)	A1 (3)
	OR $\mathrm{f}(k+1)-6f(k)=2^{k+1}+6^{k+1}-6(2^k+6^k)$		M1
	$=(2-6)(2^k)=-4.2^k$, and so $\mathrm{f}(k+1)=6\mathrm{f}(k)-4(2^k)$		A1, A1 (3)
	(b) n = 1: $\mathrm{f}(1)=2^1+6^1=8$, **which is divisible** by 8		B1
	Either Assume f(k) divisible by 8 and try to use f(k + 1) = 6f(k) - $4(2^k)$	**Or** Assume f(k) divisible by 8 and try to use f (k + 1)– f (k) or f (k + 1) + f (k) including factorising $6^k=2^k3^k$	M1
	Show $4(2^k)=4\times2(2^{k-1})=8(2^{k-1})$ or $8(\frac{1}{2}2^k)$ Or valid statement	$=2^32^{k-3}(1+5.3^k)$ or $=2^32^{k-3}(3+7.3^k)$ o.e.	A1
	Deduction that result is implied for $n=k+1$ and so is true for positive integers by induction (may include n = 1 true here)	Deduction that result is implied for $n=k+1$ and so is true for positive integers by induction (must include explanation of why n = 2 is also true here)	A1cso (4) **7 marks**

Notes

(a) M1: for substitution into LHS (or RHS) or $\mathrm{f}(k+1)-6f(k)$

A1: for correct split of the two separate powers cao

A1: for completion of proof with no error or ambiguity (needs (for example) to start with one side of equation and reach the other or show that each side separately is $2(2^k)+6(6^k)$ and **conclude** LHS = RHS)

(b) B1: for substitution of n = 1 and **stating** "true for n = 1" or "divisible by 8" or tick. (This statement may appear in the concluding statement of the proof)

M1: Assume f(k) divisible by 8 and consider f(k + 1) = 6f(k) - $4(2^k)$ or equivalent expression that could lead to proof – not merely f (k+1) – f(k) unless deduce that 2 is a factor of 6 (see right hand scheme above).

A1: Indicates each term divisible by 8 **OR** takes out factor 8 or 2^3

A1: Induction statement . Statement n = 1 here could contribute to B1 mark earlier.

NB: $f(k+1)-f(k)=2^{k+1}-2^k+6^{k+1}-6^k=2^k+5.6^k$ **only** is M0 A0 A0

(b) "**Otherwise" methods**

Could use: $\mathrm{f}(k+1)=2\mathrm{f}(k)+4(6^k)$ or $\mathrm{f}(k+2)=36\mathrm{f}(k)-32(6^k)$ or $\mathrm{f}(k+2)=4\mathrm{f}(k)+32(2^k)$ in a similar way to given expression and Left hand mark scheme is applied.

Special Case: Otherwise Proof **not involving induction**: This can only be awarded the B1 for checking n = 1.

Question Number	Scheme	Marks
8.	(a) $\frac{c}{3}$	B1 (1)
	(b) $y=\frac{c^2}{x}\Rightarrow\frac{dy}{dx}=-c^2x^{-2}$,	B1
	or $y+x\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=-\frac{y}{x}$ **or** $\dot{x}=c,\ \dot{y}=-\frac{c}{t^2}$ so $\frac{dy}{dx}=-\frac{1}{t^2}$	
	and at A $\frac{dy}{dx}=-\frac{c^2}{(3c)^2}=-\frac{1}{9}$ so gradient of normal is 9	M1 A1
	Either $y-\frac{c}{3}=9(x-3c)$ **or** $y=9x+k$ and use $x=3c$, $y=\frac{c}{3}$	M1
	$\Rightarrow$ $3y=27x-80c$ (*)	A1 (5)
	(c) $\frac{c^2}{x}=\frac{27x-80c}{3}$ / $\frac{c^2}{y}=\frac{3y+80c}{27}$ / $3\frac{c}{t}=27ct-80c$	M1
	$3c^2=27x^2-80cx$ / $27c^2=3y^2+80cy$ / $3c=27ct^2-80ct$	A1
	$(x-3c)(27x+c)=0$ so $x=$ / $(y+27c)(3y-c)=0$ so $y=$ / $(t-3)(27t+1)=0$ so $t=$	M1
	$x=-\frac{c}{27}$, $y=-27c$ / $x=-\frac{c}{27}$, $y=-27c$ / ($t=-\frac{1}{27}$ and so) $x=-\frac{c}{27}$, $y=-27c$	A1 , A1 (5) **11 marks**

Notes

(b) B1: Any valid method of differentiation but must get to correct expression for $\frac{dy}{dx}$

M1 : Substitutes values and uses negative reciprocal (**needs to follow calculus**)
A1: 9 cao (needs to follow calculus)
M1: Finds equation of line through A with any gradient (other than 0 and ∞)
A1: Correct work throughout – **obtaining printed answer**.

(c) M1: Obtains equation in one variable (x, y or t)
A1: Writes as correct three term quadratic (any equivalent form)
M1: Attempts to solve three term quadratic to obtain $x=$ or $y=$ or $t=$
A1: x coordinate, A1: y coordinate. (cao but allow recovery following slips)

Question Number	Scheme	Marks
9.	(a) If $n=1$, $\sum_{r=1}^{n} r^2 = 1$ and $\frac{1}{6}n(n+1)(2n+1) = \frac{1}{6}\times 1\times 2\times 3 = 1$, so true for n = 1.	B1
	Assume result true for $n = k$	M1
	$\sum_{r=1}^{k+1} r^2 = \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$	M1
	$= \frac{1}{6}(k+1)(2k^2+7k+6)$ or $= \frac{1}{6}(k+2)(2k^2+5k+3)$ or $= \frac{1}{6}(2k+3)(k^2+3k+2)$	A1
	$= \frac{1}{6}(k+1)(k+2)(2k+3)$ $= \frac{1}{6}(k+1)(\{k+1\}+1)(2\{k+1\}+1)$ or equivalent	dM1
	True for $n = k + 1$ if true for $n = k$, (and true for n = 1) so true by induction for all n.	A1cso (6)
	Alternative for (a) After first three marks B M M1 as earlier :	B1M1M1
	May state RHS $= \frac{1}{6}(k+1)(\{k+1\}+1)(2\{k+1\}+1) = \frac{1}{6}(k+1)(k+2)(2k+3)$ for third M1	dM1
	Expands to $\frac{1}{6}(k+1)(2k^2+7k+6)$ and show equal to $\sum_{r=1}^{k+1} r^2 = \frac{1}{6}k(k+1)(2k+1)+(k+1)^2$ for A1	A1
	So true for $n = k + 1$ if true for $n = k$, and true for n = 1, so true by induction for all n.	A1cso (6)
	(b) $\sum_{r=1}^{n}(r^2+5r+6) = \sum_{r=1}^{n} r^2 + 5\sum_{r=1}^{n} r +$ ($\sum_{r=1}^{n} 6$)	M1
	$\frac{1}{6}n(n+1)(2n+1) + \frac{5}{2}n(n+1),\quad +6n$	A1, B1
	$= \frac{1}{6}n[(n+1)(2n+1)+15(n+1)+36]$	M1
	$= \frac{1}{6}n[2n^2+18n+52] = \frac{1}{3}n(n^2+9n+26)$ or $a = 9$, $b = 26$	A1 (5)
	(c) $\sum_{r=n+1}^{2n}(r+2)(r+3) = \frac{1}{3}2n(4n^2+18n+26) - \frac{1}{3}n(n^2+9n+26)$	M1 A1ft
	$\frac{1}{3}n(8n^2+36n+52-n^2-9n-26) = \frac{1}{3}n(7n^2+27n+26)$ (*)	A1cso (3) **14 marks**
	Notes: (a) B1: Checks n = 1 on both sides and states true for n = 1 here or in conclusion M1: **Assumes true** for $n = k$ (should use one of these **two** words) M1: Adds (k+1)th term to sum of k terms A1: Correct work to support proof M1: Deduces $\frac{1}{6}n(n+1)(2n+1)$ with $n = k + 1$ A1: Makes induction statement. Statement true for n = 1 here could contribute to B1 mark earlier	

	Question 9 Notes continued: (b) M1: Expands and splits (but allow 6 rather than sigma 6 for this mark) A1: first two terms correct B1: for 6*n* M1: Take out factor *n*/6 or *n*/3 correctly – no errors factorising A1: for correct factorised cubic or for identifying *a* and *b* (c) M1: Try to use $\sum_{1}^{2n}(r+2)(r+3)-\sum_{1}^{n}(r+2)(r+3)$ with previous result used **at least once** A1ft Two correct expressions for their a and b values A1: Completely correct work to printed answer