

Mark Scheme (Results)

January 2012

GCE Further Pure FP1 (6667) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
 - ft – follow through
 - the symbol \checkmark will be used for correct ft
 - cao – correct answer only
 - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - \square The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

General Principals for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to } x = \dots$$

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to } x = \dots$$

2. Formula

Attempt to use correct formula (with values for a , b and c), leading to $x = \dots$

3. Completing the square

$$\text{Solving } x^2 + bx + c = 0 : \quad \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0, \quad \text{leading to } x = \dots$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

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6667 Further Pure Mathematics FP1
Mark Scheme

Question Number	Scheme	Notes	Marks	
1(a)	$\arg z_1 = -\arctan(1)$	$-\arctan(1)$ or $\arctan(1)$ or $\arctan(-1)$	M1	
	$= -\frac{\pi}{4}$	or -45 or awrt -0.785 (oe e.g $\frac{7\pi}{4}$)	A1	
	Correct answer only 2/2		(2)	
(b)	$z_1 z_2 = (1-i)(3+4i) = 3-3i+4i-4i^2$	At least 3 correct terms (Unsimplified)	M1	
	$= 7+i$	cao	A1	
			(2)	
(c)	$\frac{z_2}{z_1} = \frac{(3+4i)}{(1-i)} = \frac{(3+4i) \cdot (1+i)}{(1-i) \cdot (1+i)}$	Multiply top and bottom by $(1+i)$	M1	
	$= \frac{(3+4i) \cdot (1+i)}{2}$	$(1+i)(1-i) = 2$	A1	
	$= -\frac{1}{2} + \frac{7}{2}i$	or $\frac{-1+7i}{2}$	A1	
	Special case $\frac{z_1}{z_2} = \frac{(1-i)}{(3+4i)} = \frac{(1-i) \cdot (3-4i)}{(3+4i) \cdot (3-4i)}$ Allow M1A0A0			
			(3)	
Correct answers only in (b) and (c) scores no marks			Total 7	

Question Number	Scheme	Notes	Marks
2	$f(x) = x^4 + x - 1$		
(a)	$f(0.5) = -0.4375 \quad (-\frac{7}{16})$ $f(1) = 1$	Either any one of $f(0.5) = \text{awrt } -0.4$ or $f(1) = 1$	M1
	Sign change (positive, negative) (and $f(x)$ is continuous) therefore (a root) α is between $x = 0.5$ and $x = 1.0$	$f(0.5) = \text{awrt } -0.4$ and $f(1) = 1$, sign change and conclusion	A1
			(2)
(b)	$f(0.75) = 0.06640625(\frac{17}{256})$	Attempt $f(0.75)$	M1
	$f(0.625) = -0.222412109375(-\frac{911}{4096})$	$f(0.75) = \text{awrt } 0.07$ and $f(0.625) = \text{awrt } -0.2$	A1
	$0.625, \alpha, 0.75$	$0.625, \alpha, 0.75$ or $0.625 < \alpha < 0.75$ or $[0.625, 0.75]$ or $(0.625, 0.75)$. or equivalent in words.	A1
	In (b) there is no credit for linear interpolation and a correct answer with no working scores no marks.		(3)
(c)	$f'(x) = 4x^3 + 1$	Correct derivative (May be implied later by e.g. $4(0.75)^3 + 1$)	B1
	$x_1 = 0.75$		
	$x_2 = 0.75 - \frac{f(0.75)}{f'(0.75)} = 0.75 - \frac{0.06640625}{2.6875(43/16)}$	Attempt Newton-Raphson	M1
	$x_2 = 0.72529(06976...) = \frac{499}{688}$	Correct first application – a correct numerical expression e.g. $0.75 - \frac{17/256}{43/16}$	A1
	$x_3 = 0.724493\left(\frac{499}{688} - \frac{0.002015718978}{2.562146811}\right)$	Awrt 0.724	A1
	$(\alpha) = 0.724$	cao	A1
		A final answer of 0.724 with evidence of NR applied twice with no incorrect work should score 5/5	
			Total 10

Question Number	Scheme	Notes	Marks
3(a)	Focus (4,0)		B1
	Directrix $x+4=0$	$x+4=0$ or $x=-4$	M1
		$x+4=0$ or $x=-4$	A1
			(3)
(b)	$y = 4x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = 2x^{-\frac{1}{2}}$ $y^2 = 16x \Rightarrow 2y \frac{dy}{dx} = 16$ or $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = 8 \cdot \frac{1}{8t}$	$\frac{dy}{dx} = k x^{-\frac{1}{2}}$ $ky \frac{dy}{dx} = c$ their $\frac{dy}{dt} \times \left(\frac{1}{\text{their } \frac{dx}{dt}} \right)$	M1
	$\frac{dy}{dx} = 2x^{-\frac{1}{2}}$ or $2y \frac{dy}{dx} = 16$ or $\frac{dy}{dx} = 8 \cdot \frac{1}{8t}$	Correct differentiation	A1
	At P, gradient of normal = -t	Correct normal gradient with no errors seen.	A1
	$y - 8t = -t(x - 4t^2)$	Applies $y - 8t = \text{their } m_N (x - 4t^2)$ or $y = (\text{their } m_N)x + c$ using $x = 4t^2$ and $y = 8t$ in an attempt to find c. Their m_N must be different from their m_T and must be a function of t.	M1
	$y + tx = 8t + 4t^3$ *	cso **given answer**	A1
	Special case – if the correct gradient is <u>quoted</u> could score M0A0A0M1A1		
			Total 8

Question Number	Scheme	Notes	Marks
4(a)	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 & 2 \\ 1 & 1 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \end{pmatrix}$	Attempt to multiply the right way round with at least 4 correct elements	M1
	T' has coordinates (1,1), (1,2) and (4,2) or $\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ NOT just $\begin{pmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \end{pmatrix}$	Correct coordinates or vectors	A1
			(2)
(b)	Reflection in the line $y = x$	Reflection	B1
		$y = x$	B1
	Allow 'in the axis' 'about the line' $y = x$ etc. Provided both features are mentioned ignore any reference to the origin unless there is a clear contradiction.		
			(2)
(c)	$\mathbf{QR} = \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}$	2 correct elements	M1
		Correct matrix	A1
	Note that $\mathbf{RQ} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 10 & -4 \\ 24 & -10 \end{pmatrix}$ scores M0A0 in (c) but allow all the marks in (d) and (e)		
			(2)
(d)	$\det(\mathbf{QR}) = -2 \times 2 - 0 = -4$	"-2"x"2" - "0"x"0"	M1
		-4	A1
	Answer only scores 2/2 $\frac{1}{\det(\mathbf{QR})}$ scores M0		(2)
(e)	Area of $T = \frac{1}{2} \times 1 \times 3 = \frac{3}{2}$	Correct area for T	B1
	Area of $T'' = \frac{3}{2} \times 4 = 6$	Attempt at " $\frac{3}{2}$ "x"4"	M1
		6 or follow through their $\det(\mathbf{QR})$ x Their triangle area provided area > 0	A1ft
			(3)
			Total 11

Question Number	Scheme	Notes	Marks
6(a)	$n = 1, \text{LHS} = 1^3 = 1, \text{RHS} = \frac{1}{4} \times 1^2 \times 2^2 = 1$	Shows both LHS = 1 and RHS = 1	B1
	Assume true for $n = k$		
	When $n = k + 1$ $\sum_{r=1}^{k+1} r^3 = \frac{1}{4}k^2(k+1)^2 + (k+1)^3$	Adds $(k+1)^3$ to the given result	M1
	$= \frac{1}{4}(k+1)^2[k^2 + 4(k+1)]$	Attempt to factorise out $\frac{1}{4}(k+1)^2$	dM1
		Correct expression with $\frac{1}{4}(k+1)^2$ factorised out.	A1
	$= \frac{1}{4}(k+1)^2(k+2)^2$ Must see 4 things: <u>true for $n = 1$</u> , <u>assumption true for $n = k$</u> , <u>said true for $n = k + 1$</u> and therefore <u>true for all n</u>	Fully complete proof with no errors and comment. All the previous marks must have been scored.	A1cso
See extra notes for alternative approaches			(5)
(b)	$\sum (r^3 - 2) = \sum r^3 - \sum 2$	Attempt two sums	M1
	$\sum r^3 - \sum 2n$ is M0		
	$= \frac{1}{4}n^2(n+1)^2 - 2n$	Correct expression	A1
	$= \frac{n}{4}(n^3 + 2n^2 + n - 8) *$	Completion to printed answer with no errors seen.	A1
			(3)
(c)	$\sum_{r=20}^{50} (r^3 - 2) = \frac{50}{4} \times 130042 - \frac{19}{4} \times 7592$ $(= 1625525 - 36062)$	Attempt $S_{50} - S_{20}$ or $S_{50} - S_{19}$ and substitutes into a correct expression at least once.	M1
		Correct numerical expression (unsimplified)	A1
	$= 1\ 589\ 463$	cao	A1
			(3)
(c) Way 2	$\sum_{r=20}^{50} (r^3 - 2) = \sum_{r=20}^{50} r^3 - \sum_{r=20}^{50} (2) = \frac{50^2}{4} \times 51^2 - \frac{19^2}{4} \times 20^2 - 2 \times 31$	M1 for $(S_{50} - S_{20}$ or $S_{50} - S_{19}$ for cubes) - $(2 \times 30$ or $2 \times 31)$	Total 11
		A1 correct numerical expression	
	$= 1\ 589\ 463$	A1	

Question Number	Scheme	Notes	Marks
7(a)	$u_2 = 3, u_3 = 7$		B1, B1
			(2)
(b)	At $n = 1, u_1 = 2^1 - 1 = 1$ and so result true for $n = 1$		B1
	Assume true for $n = k; u_k = 2^k - 1$		
	and so $u_{k+1} (= 2u_k + 1) = 2(2^k - 1) + 1$	Substitutes u_k into u_{k+1} (must see this line)	M1
		Correct expression	A1
	$u_{k+1} (= 2^{k+1} - 2 + 1) = 2^{k+1} - 1$	Correct completion to $u_{k+1} = 2^{k+1} - 1$	A1
	Must see 4 things: <u>true for $n = 1$</u> , <u>assumption true for $n = k$</u> , <u>said true for $n = k + 1$</u> and therefore <u>true for all n</u>	Fully complete proof with no errors and comment. All the previous marks in (b) must have been scored.	A1cso
Ignore any subsequent attempts e.g. $u_{k+2} = 2u_{k+1} + 1 = 2(2^{k+1} - 1) + 1$ etc.		(5)	
			Total 7

Question Number	Scheme	Notes	Marks
8(a)	$\det(\mathbf{A}) = 3 \times 0 - 2 \times 1 (= -2)$	Correct attempt at the determinant	M1
	$\det(\mathbf{A}) \neq 0$ (so \mathbf{A} is non singular)	$\det(\mathbf{A}) = -2$ and some reference to zero	A1
	$\frac{1}{\det(\mathbf{A})}$ scores M0		
(b)	$\mathbf{BA}^2 = \mathbf{A} \Rightarrow \mathbf{BA} = \mathbf{I} \Rightarrow \mathbf{B} = \mathbf{A}^{-1}$	Recognising that \mathbf{A}^{-1} is required	M1
	$\mathbf{B} = -\frac{1}{2} \begin{pmatrix} 3 & -1 \\ -2 & 0 \end{pmatrix}$	At least 3 correct terms in $\begin{pmatrix} 3 & -1 \\ -2 & 0 \end{pmatrix}$	M1
		$\frac{1}{\text{their } \det(\mathbf{A})} \begin{pmatrix} * & * \\ * & * \end{pmatrix}$	B1ft
		Fully correct answer	A1
Correct answer only score 4/4			(4)
Ignore poor matrix algebra notation if the intention is clear			Total 6
(b) Way 2	$\mathbf{A}^2 = \begin{pmatrix} 2 & 3 \\ 6 & 11 \end{pmatrix}$	Correct matrix	B1
	$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 6 & 11 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} \Rightarrow \begin{matrix} 2a+6b=0 & 2c+6d=2 \\ 3a+11b=1 & 3c+11d=3 \end{matrix}$ <i>or</i>	2 equations in a and b or 2 equations in c and d	M1
	$a = -\frac{3}{2}, b = \frac{1}{2}, c = 1, d = 0$	M1 Solves for a and b or c and d	M1A1
		A1 All 4 values correct	
(b) Way 3	$\mathbf{A}^2 = \begin{pmatrix} 2 & 3 \\ 6 & 11 \end{pmatrix}$	Correct matrix	B1
	$(\mathbf{A}^2)^{-1} = \frac{1}{"2" \times "11" - "3" \times "6"} \begin{pmatrix} "11" & "-3" \\ "-6" & "2" \end{pmatrix}$ see note	Attempt inverse of \mathbf{A}^2	M1
	$\mathbf{A}(\mathbf{A}^2)^{-1} = \frac{1}{4} \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 11 & -3 \\ -6 & 2 \end{pmatrix}$ <i>or</i> $\frac{1}{4} \begin{pmatrix} 11 & -3 \\ -6 & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$	Attempts $\mathbf{A}(\mathbf{A}^2)^{-1}$ <i>or</i> $(\mathbf{A}^2)^{-1} \mathbf{A}$	M1
	$\mathbf{B} = -\frac{1}{2} \begin{pmatrix} 3 & -1 \\ -2 & 0 \end{pmatrix}$	Fully correct answer	A1
(b) Way 4	$\mathbf{BA} = \mathbf{I}$	Recognising that $\mathbf{BA} = \mathbf{I}$	B1
	$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \begin{matrix} 2b=1 & 2d=0 \\ a+3b=0 & c+3d=1 \end{matrix}$ <i>or</i>	2 equations in a and b or 2 equations in c and d	M1
	$a = -\frac{3}{2}, b = \frac{1}{2}, c = 1, d = 0$	M1 Solves for a and b or c and d	M1A1
		A1 All 4 values correct	

Question Number	Scheme	Notes	Marks
9 (a)	$y = 9x^{-1} \Rightarrow \frac{dy}{dx} = -9x^{-2}$ $xy = 9 \Rightarrow x \frac{dy}{dx} + y = 0$ or $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{-3}{p^2} \cdot \frac{1}{3}$	$\frac{dy}{dx} = k x^{-2}$ Correct use of product rule. The sum of two terms, one of which is correct. their $\frac{dy}{dt} \times \left(\frac{1}{\text{their } \frac{dx}{dt}} \right)$	M1
	$\frac{dy}{dx} = -9x^{-2}$ or $x \frac{dy}{dx} + y = 0$ or $\frac{dy}{dx} = \frac{-3}{p^2} \cdot \frac{1}{3}$	Correct differentiation.	A1
	$y - \frac{3}{p} = -\frac{1}{p^2}(x - 3p)$	Applies $y - \frac{3}{p} = (\text{their } m)(x - 3p)$ or $y = (\text{their } m)x + c$ using $x = 3p$ and $y = \frac{3}{p}$ in an attempt to find c. Their m must be a function of p and come from their dy/dx.	M1
	$x + p^2 y = 6p$ *	Cso **given answer**	A1
Special case – if the correct gradient is <u>quoted</u> could score M0A0M1A1			(4)
(b)	$x + q^2 y = 6q$	Allow this to score here or in (c)	B1
			(1)
(c)	$6p - p^2 y = 6q - q^2 y$	Attempt to obtain an equation in one variable x or y	M1
	$y(q^2 - p^2) = 6(q - p) \Rightarrow y = \frac{6(q - p)}{q^2 - p^2}$ $x(q^2 - p^2) = 6pq(q - p) \Rightarrow x = \frac{6pq(q - p)}{q^2 - p^2}$	Attempt to isolate x or y – must reach x or y = f(p, q) or f(p) or f(q)	M1
	$y = \frac{6}{p + q}$	One correct simplified coordinate	A1
	$x = \frac{6pq}{p + q}$	Both coordinates correct and simplified	A1
			(4)
			Total 9

6(a) To show equivalence between $\frac{1}{4}k^2(k+1)^2 + (k+1)^3$ and $\frac{1}{4}(k+1)^2(k+2)^2$

$$\frac{1}{4}k^2(k+1)^2 + (k+1)^3 = \frac{1}{4}k^4 + \frac{3}{2}k^3 + \frac{13}{4}k^2 + 3k + 1$$

Attempt to expand one correct expression up to a quartic M1

$$\frac{1}{4}(k+1)^2(k+2)^2 = \frac{1}{4}k^4 + \frac{3}{2}k^3 + \frac{13}{4}k^2 + 3k + 1$$

Attempt to expand both correct expressions up to a quartic M1

One expansion completely correct (dependent on both M's) A1

Both expansions correct and conclusion A1

Or

To show $\frac{1}{4}(k+1)^2(k+2)^2 - \frac{1}{4}k^2(k+1)^2 = (k+1)^3$

$$\frac{1}{4}(k+1)^2(k+2)^2 - \frac{1}{4}k^2(k+1)^2 \quad \text{Attempt to subtract} \quad \text{M1}$$

$$\frac{1}{4}(k+1)^2(k+2)^2 - \frac{1}{4}k^2(k+1)^2 = k^3 + 3k^2 + 3k + 1 \quad \text{Obtains a cubic expression} \quad \text{M1}$$

Correct expression A1

$$\frac{1}{4}(k+1)^2(k+2)^2 - \frac{1}{4}k^2(k+1)^2 = (k+1)^3 \quad \text{Correct completion and comment} \quad \text{A1}$$

8(b) Way 3

Attempting inverse of \mathbf{A}^2 needs to be recognisable as an attempt at an inverse

E.g $(\mathbf{A}^2)^{-1} = \frac{1}{\text{Their Det}(\mathbf{A}^2)} (\text{A changed } \mathbf{A}^2)$

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