January 2010
6667 Further Pure Mathematics FP1
Mark Scheme

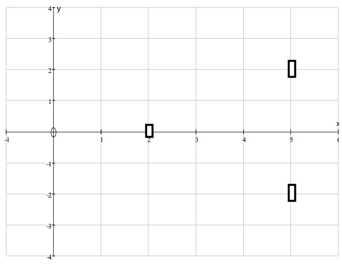
Question Number	Scheme	Marks
Q1	(a) $\frac{z_1}{z_2} = \frac{2+8i}{1-i} \times \frac{1+i}{1+i}$	M1
	$= \frac{2+2i+8i-8}{2} = -3+5i$	A1 A1 (3)
	(b) $\left\lvert \frac{z_1}{z_2} \right\rvert = \sqrt{(-3)^2+5^2} = \sqrt{34}$ (or awrt 5.83)	M1 A1ft (2)
	(c) $\tan\alpha = -\frac{5}{3}$ or $\frac{5}{3}$	M1
	$\arg\frac{z_1}{z_2} = \pi - 1.03... = 2.11$	A1 (2) **[7]**
	Notes (a) $\times\frac{1+i}{1+i}$ and attempt to multiply out for M1 -3 for first A1, +5i for second A1 (b) Square root required without i for M1 $\frac{\lvert z_1\rvert}{\lvert z_2\rvert}$ award M1 for attempt at Pythagoras for both numerator and denominator (c) tan or $\tan^{-1}$, $\pm\frac{5}{3}$ or $\pm\frac{3}{5}$ seen with their 3 and 5 award M1 2.11 correct answer only award A1	

Question Number	Scheme	Marks
Q2	(a) $\mathrm{f}(1.3) = -1.439$ and $\mathrm{f}(1.4) = 0.268$ (allow awrt)	B1 (1)
	(b) $\mathrm{f}(1.35) < 0$ $(-0.568...)$ $\Rightarrow$ $1.35 < \alpha < 1.4$ $\mathrm{f}(1.375) < 0$ $(-0.146...)$ $\Rightarrow$ $1.375 < \alpha < 1.4$	M1 A1 A1 (3)
	(c) $\mathrm{f}'(x) = 6x + 22x^{-3}$ $x_1 = x_0 - \dfrac{\mathrm{f}(x_0)}{\mathrm{f}'(x_0)} = 1.4 - \dfrac{0.268}{16.417}$, $= 1.384$	M1 A1 M1 A1, A1 (5) **[9]**
	Notes (a) Both answers required for B1. Accept anything that rounds to 3dp values above. (b) f(1.35) or awrt -0.6 M1 (f(1.35) and awrt -0.6) AND (f(1.375) and awrt -0.1) for first A1 $1.375 < \alpha < 1.4$ or expression using brackets or equivalent in words for second A1 (c) One term correct for M1, both correct for A1 Correct formula seen or implied and attempt to substitute for M1 awrt 16.4 for second A1 which can be implied by correct final answer awrt 1.384 correct answer only A1	

Question Number	Scheme	Marks
Q3	For $n = 1$: $u_1 = 2,\ u_1 = 5^0 + 1 = 2$	B1
	Assume true for $n = k$:	
	$u_{k+1} = 5u_k - 4 = 5(5^{k-1} + 1) - 4 = 5^k + 5 - 4 = 5^k + 1$	M1 A1
	$\therefore$ True for $n = k + 1$ if true for $n = k$.	
	True for $n = 1$,	
	$\therefore$ true for all n.	A1 cso
		[4]
	Notes Accept $u_1 = 1 + 1 = 2$ or above B1 $5(5^{k-1} + 1) - 4$ seen award M1 $5^k + 1$ or $5^{(k+1)-1} + 1$ award first A1 All three elements stated somewhere in the solution award final A1	

Question Number	Scheme	Marks
Q4	(a) (3, 0) cao	B1 (1)
	(b) P: $x = \frac{1}{3} \Rightarrow y = 2$ A and B lie on $x = -3$ $PB = PS$ or a correct method to find both PB and PS Perimeter = $6 + 2 + 3\frac{1}{3} + 3\frac{1}{3} = 14\frac{2}{3}$	B1 B1 M1 M1 A1 (5) **[6]**
	Notes (b) Both B marks can be implied by correct diagram with lengths labelled or coordinates of vertices stated. Second M1 for their four values added together. $14\frac{2}{3}$ or awrt 14.7 for final A1	

Question Number	Scheme	Marks
Q5	(a) det $\mathbf{A} = a(a+4)-(-5\times 2) = a^2+4a+10$	M1 A1 (2)
	(b) $a^2+4a+10=(a+2)^2+6$ Positive for all values of a, so $\mathbf{A}$ is non-singular	M1 A1ft A1cso (3)
	(c) $\mathbf{A}^{-1}=\frac{1}{10}\begin{pmatrix}4 & 5\\ -2 & 0\end{pmatrix}$ B1 for $\frac{1}{10}$	B1 M1 A1 (3) **[8]**
	Notes (a) Correct use of $ad-bc$ for M1 (b) Attempt to complete square for M1 Alt 1 Attempt to establish turning point (e.g. calculus, graph) M1 Minimum value 6 for A1ft Positive for all values of a, so **A** is non-singular for A1 cso Alt 2 Attempt at b^2-4ac for M1. Can be part of quadratic formula Their correct -24 for first A1 No real roots or equivalent, so **A** is non-singular for final A1cso (c) Swap leading diagonal, and change sign of other diagonal, with numbers or a for M1 Correct matrix independent of 'their $\frac{1}{10}$ award' final A1	

Question Number	Scheme	Marks
Q6	(a) $5-2\mathrm{i}$ is a root	B1 (1)
	(b) $(x-(5+2\mathrm{i}))(x-(5-2\mathrm{i})) = x^2 - 10x + 29$ $x^3 - 12x^2 + cx + d = (x^2 - 10x + 29)(x-2)$ $c = 49,\qquad d = -58$	M1 M1 M1 A1, A1 (5)
	(c)  Conjugate pair in 1st and 4th quadrants (symmetrical about real axis) Fully correct, labelled	B1 B1 (2) **[8]**
	(b) 1st M: Form brackets using $(x-\alpha)(x-\beta)$ and expand. 2nd M: Achieve a 3-term quadratic with no i's. (b) <u>Alternative</u>: Substitute a complex root (usually 5+2i) and expand brackets — M1 $(5+2\mathrm{i})^3 - 12(5+2\mathrm{i})^2 + c(5+2\mathrm{i}) + d = 0$ $(125+150\mathrm{i}-60-8\mathrm{i}) - 12(25+20\mathrm{i}-4) + (5c+2c\mathrm{i}) + d = 0$ — M1 (2nd M for achieving an expression with no powers of i) Equate real and imaginary parts — M1 $c = 49,\qquad d = -58$ — A1, A1	

Question Number	Scheme	Marks
Q7	(a) $y=\frac{c^2}{x}$ $\quad \frac{dy}{dx}=-c^2x^{-2}$	B1
	$\frac{dy}{dx}=-\frac{c^2}{(ct)^2}=-\frac{1}{t^2}$ without x or y	M1
	$y-\frac{c}{t}=-\frac{1}{t^2}(x-ct) \quad \Rightarrow \quad t^2y+x=2ct$ (*)	M1 A1cso (4)
	(b) Substitute $(15c,-c)$: $\quad -ct^2+15c=2ct$	M1
	$t^2+2t-15=0$	A1
	$(t+5)(t-3)=0 \quad \Rightarrow \quad t=-5 \quad t=3$	M1 A1
	Points are $\left(-5c,-\frac{c}{5}\right)$ and $\left(3c,\frac{c}{3}\right)$ both	A1 (5) **[9]**
	Notes (a) Use of $y-y_1=m(x-x_1)$ where m is their gradient expression in terms of c and / or t only for second M1. Accept $y=mx+k$ and attempt to find k for second M1. (b) Correct absolute factors for their constant for second M1. Accept correct use of quadratic formula for second M1. <u>Alternatives:</u> (a) $\frac{dx}{dt}=c$ and $\frac{dy}{dt}=-ct^{-2}$ B1 $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}=-\frac{1}{t^2}$ M1, then as in main scheme. (a) $y+x\frac{dy}{dx}=0$ B1 $\frac{dy}{dx}=-\frac{y}{x}=-\frac{1}{t^2}$ M1, then as in main scheme.	

Question Number	Scheme	Marks
Q8	(a) $\sum_{r=1}^{1} r^3 = 1^3 = 1$ and $\frac{1}{4}\times 1^2 \times 2^2 = 1$	B1
	Assume true for $n = k$:	
	$\sum_{r=1}^{k+1} r^3 = \frac{1}{4}k^2(k+1)^2 + (k+1)^3$	B1
	$\frac{1}{4}(k+1)^2\left[k^2 + 4(k+1)\right] = \frac{1}{4}(k+1)^2(k+2)^2$	M1 A1
	$\therefore$ True for $n = k + 1$ if true for $n = k$. True for $n = 1$, $\therefore$ true for all n.	A1cso (5)
	(b) $\sum r^3 + 3\sum r + \sum 2 = \frac{1}{4}n^2(n+1)^2 + 3\left(\frac{1}{2}n(n+1)\right), \; + 2n$	B1, B1
	$= \frac{1}{4}n\left[n(n+1)^2 + 6(n+1) + 8\right]$	M1
	$= \frac{1}{4}n\left[n^3 + 2n^2 + 7n + 14\right] = \frac{1}{4}n(n+2)(n^2+7)$ (*)	A1 A1cso (5)
	(c) $\sum_{15}^{25} = \sum_{1}^{25} - \sum_{1}^{14}$ with attempt to sub in answer to part (b)	M1
	$= \frac{1}{4}(25\times 27\times 632) - \frac{1}{4}(14\times 16\times 203) = 106650 - 11368 = 95282$	A1 (2)
		[12]
	Notes (a) Correct method to identify $(k+1)^2$ as a factor award M1 $\frac{1}{4}(k+1)^2(k+2)^2$ award first A1 All three elements stated somewhere in the solution award final A1 (b) Attempt to factorise by n for M1 $\frac{1}{4}$ and $n^3 + 2n^2 + 7n + 14$ for first A1 (c) no working 0/2	

Question Number	Scheme	Marks
Q9	(a) $45°$ or $\frac{\pi}{4}$ rotation (anticlockwise), about the origin	B1, B1 (2)
	(b) $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 3\sqrt{2} \\ 4\sqrt{2} \end{pmatrix}$ $p - q = 6$ and $p + q = 8$ or equivalent $p = 7$ and $q = 1$ both correct	M1 M1 A1 A1 (4)
	(c) Length of OA (= length of OB) = $\sqrt{7^2 + 1^2}$, $= \sqrt{50} = 5\sqrt{2}$	M1, A1 (2)
	(d) $\mathrm{M}^2 = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	M1 A1 (2)
	(e) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3\sqrt{2} \\ 4\sqrt{2} \end{pmatrix}$ so coordinates are $(-4\sqrt{2}, 3\sqrt{2})$	M1 A1 (2) **[12]**
	Notes Order of matrix multiplication needs to be correct to award Ms (a) More than one transformation 0/2 (b) Second M1 for correct matrix multiplication to give two equations <u>Alternative</u>: (b) $\mathbf{M}^{-1} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ First M1 A1 $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 3\sqrt{2} \\ 4\sqrt{2} \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$ Second M1 A1 (c) Correct use of their p and their q award M1 (e) Accept column vector for final A1.	