PMT

Edexcel Maths FP1

Mark Scheme Pack

2009-2014

January 2009
6667 Further Pure Mathematics FP1 (new)
Mark Scheme

Question Number	Scheme	Marks
1	$x - 3$ is a factor	B1
	$\mathrm{f}(x) = (x-3)(2x^2 - 2x + 1)$	M1 A1
	Attempt to solve quadratic i.e. $x = \dfrac{2 \pm \sqrt{4-8}}{4}$	M1
	$x = \dfrac{1 \pm \mathrm{i}}{2}$	A1 [5]

Notes:

First and last terms in second bracket required for first M1
Use of correct quadratic formula for their equation for second M1

Question Number	Scheme	Marks
2 (a)	$6\sum r^2 + 4\sum r - \sum 1 = 6\frac{n}{6}(n+1)(2n+1) + 4\frac{n}{2}(n+1), -n$	M1 A1, B1
	$= \frac{n}{6}(12n^2 + 18n + 6 + 12n + 12 - 6)$ or $n(n+1)(2n+1) + (2n+1)n$	M1
	$= \frac{n}{6}(12n^2 + 30n + 12) = n(2n^2 + 5n + 2) = n(n+2)(2n+1)$ *	A1 (5)
(b)	$\sum_{r=1}^{20}(6r^2 + 4r - 1) - \sum_{r=1}^{10}(6r^2 + 4r - 1) = 20 \times 22 \times 41 - 10 \times 12 \times 21$	M1
	$= 15520$	A1 (2) [7]

Notes:

(a) First M1 for first 2 terms, B1 for $-n$
Second M1 for attempt to expand and gather terms.
Final A1 for correct solution only

(b) Require (r from 1 to 20) subtract (r from 1 to 10) and attempt to substitute for M1

Question Number	Scheme	Marks
3 (a)	$xy = 25 = 5^2$ or $c = \pm 5$	B1 (1)
(b)	*A* has co-ords (5, 5) and *B* has co-ords (25, 1)	B1
	Mid point is at (15, 3)	M1A1 (3) [4]

Notes:

(a) $xy = 25$ only B1, $c^2 = 25$ only B1, $c = 5$ only B1

(b) Both coordinates required for B1
Add theirs and divide by 2 on both for M1

Question Number	Scheme	Marks
4	When n = 1, LHS = $\frac{1}{1\times 2}=\frac{1}{2}$, RHS = $\frac{1}{1+1}=\frac{1}{2}$. So LHS = RHS and result true for n = 1	B1
	Assume true for n = k; $\sum_{r=1}^{k}\frac{1}{r(r+1)}=\frac{k}{k+1}$ and so $\sum_{r=1}^{k+1}\frac{1}{r(r+1)}=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}$	M1
	$\sum_{r=1}^{k+1}\frac{1}{r(r+1)}=\frac{k(k+2)+1}{(k+1)(k+2)}=\frac{k^2+2k+1}{(k+1)(k+2)}=\frac{(k+1)^2}{(k+1)(k+2)}=\frac{k+1}{k+2}$	M1 A1
	and so result is true for n = k + 1 (and by induction true for $n\in\mathbf{Z}^+$)	B1 [5]

Notes:

Evaluate both sides for first B1
Final two terms on second line for first M1
Attempt to find common denominator for second M1.
Second M1 dependent upon first.
$\frac{k+1}{k+2}$ for A1
'Assume true for $n=k$ 'and 'so result true for $n=k+1$' and correct solution for final B1

Question Number	Scheme	Marks
5 (a)	attempt evaluation of f(1.1) and f(1.2) (– looking for sign change)	M1
	f(1.1) = 0.30875 , f(1.2) = -0.28199 Change of sign in f(x) $\Rightarrow$ root in the interval	A1 (2)
(b)	$\text{f}'(x) = \frac{3}{2}x^{-\frac{1}{2}} - 9x^{-1\frac{1}{2}}$	M1 A1 A1 (3)
(c)	f (1.1) = 0.30875.. f ′(1.1) = -6.37086…	B1 B1
	$x_1 = 1.1 - \frac{0.30875...}{-6.37086..}$	M1
	= 1.15(to 3 sig.figs.)	A1 (4) [9]

Notes:

(a) awrt 0.3 and -0.3 and indication of sign change for first A1
(b) Multiply by power and subtract 1 from power for evidence of differentiation and award of first M1
(c) awrt 0.309 B1and awrt -6.37 B1 if answer incorrect
Evidence of Newton-Raphson for M1
Evidence of Newton-Raphson and awrt 1.15 award 4/4

Question Number	Scheme	Marks
6	At n =1, $u_n = 5\times 6^0 + 1 = 6$ and so result true for n = 1	B1
	Assume true for n = k; $u_k = 5\times 6^{k-1} + 1$,and so $u_{k+1} = 6(5\times 6^{k-1} + 1) - 5$	M1, A1
	$\therefore u_{k+1} = 5\times 6^k + 6 - 5 \quad \therefore u_{k+1} = 5\times 6^k + 1$	A1
	and so result is true for n = k + 1 and by induction true for $n \geq 1$	B1
		[5]

Notes:
6 and so result true for n = 1 award B1
Sub u_k into u_{k+1} or M1 and A1 for correct expression on right hand of line 2
Second A1 for $\therefore u_{k+1} = 5\times 6^k + 1$
'Assume true for n = k' and 'so result is true for n = k + 1' and correct solution for final B1

Question Number	Scheme	Marks
7 (a)	The determinant is a - 2	M1
	$\mathbf{X}^{-1} = \dfrac{1}{a-2}\begin{pmatrix} -1 & -a \\ 1 & 2 \end{pmatrix}$	M1 A1 (3)
(b)	$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	B1
	Attempt to solve $2 - \dfrac{1}{a-2} = 1$, or $a - \dfrac{a}{a-2} = 0$, or $-1 + \dfrac{1}{a-2} = 0$, or $-1 + \dfrac{2}{a-2} = 1$	M1
	To obtain a =3 only	A1 cso (3) [6]
	Alternatives for (b) If they use $\mathbf{X}^2 + \mathbf{I} = \mathbf{X}$ they need to identify **I** for B1, then attempt to solve suitable equation for M1 and obtain a = 3 for A1 If they use $\mathbf{X}^2 + \mathbf{X}^{-1} = \mathbf{O}$, they can score the B1then marks for solving If they use $\mathbf{X}^3 + \mathbf{I} = \mathbf{O}$ they need to identify **I** for B1, then attempt to solve suitable equation for M1 and obtain a = 3 for A1	

Notes:

(a) Attempt *ad-bc* for first M1

$\dfrac{1}{\det}\begin{pmatrix} -1 & -a \\ 1 & 2 \end{pmatrix}$ for second M1

(b) Final A1 for correct solution only

Question Number	Scheme	Marks
8 (a)	$\frac{dy}{dx} = a^{\frac{1}{2}} x^{-\frac{1}{2}}$ or $2y\frac{\mathrm{d}y}{\mathrm{d}x} = 4a$	M1
	The gradient of the tangent is $\frac{1}{q}$	A1
	The equation of the tangent is $y - 2aq = \frac{1}{q}(x - aq^2)$	M1
	So $yq = x + aq^2$ *	A1 (4)
(b)	R has coordinates (0, aq)	B1
	The line l has equation $y - aq = -qx$	M1A1 (3)
(c)	When $y = 0$ $x = a$ (so line l passes through (a, 0) the focus of the parabola.)	B1 (1)
(d)	Line l meets the directrix when $x = -a$: Then $y = 2aq$. So coordinates are ($-a$, $2aq$)	M1:A1 (2) [10]

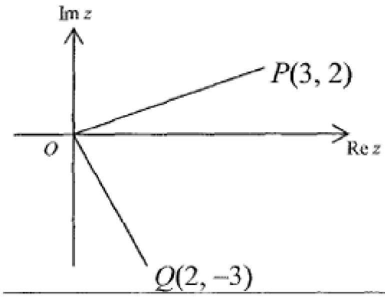
Notes:

(a) $\frac{dy}{dx} = \frac{2a}{2aq}$ OK for M1
Use of $y = mx + c$ to find c OK for second M1
Correct solution only for final A1

(b) -1/(their gradient in part a) in equation OK for M1

(c) They must attempt $y = 0$ or $x = a$ to show correct coordinates of R for B1

(d) Substitute $x = -a$ for M1.
Both coordinates correct for A1.

Question Number	Scheme	Marks
9 (a)	$z_2 = \frac{12-5i}{3+2i} \times \frac{3-2i}{3-2i} = \frac{36-24i-15i-10}{13}$	M1
	$= 2-3i$	A1 (2)
(b)		
	P: B1, Q: B1ft	B1, B1ft (2)
(c)	grad. OP × grad. $OQ = \frac{2}{3} \times -\frac{3}{2}$	
	$= -1 \quad \Rightarrow \angle POQ = \frac{\pi}{2} \quad (*)$	
OR	$\angle POX = \tan^{-1}\frac{2}{3}$, $\angle QOX = \tan^{-1}\frac{3}{2}$	
	$\mathrm{Tan}(\angle POQ) = \frac{\frac{2}{3}+\frac{3}{2}}{1-\frac{2}{3}\times\frac{3}{2}}$ M1	M1
	$\Rightarrow \angle POQ = \frac{\pi}{2} \quad (*)$ A1	A1 (2)
(d)	$z = \frac{3+2}{2} + \frac{2+(-3)}{2}i$	M1
	$= \frac{5}{2} - \frac{1}{2}i$	A1 (2)
(e)	$r = \sqrt{\left(\frac{5}{2}\right)^2 + \left(-\frac{1}{2}\right)^2}$	M1
	$= \frac{\sqrt{26}}{2}$ or exact equivalent	A1 (2) [10]

Notes:

(a) $\times\frac{3-2i}{3-2i}$ for M1

(b) Position of points not clear award B1B0

(c) Use of calculator / decimals award M1A0

(d) Final answer must be in complex form for A1

(e) Radius or diameter for M1

Question Number	Scheme	Marks
10 (a)	**A** represents an enlargement scale factor $3\sqrt{2}$ (centre O)	M1 A1
	B represents reflection in the line $y = x$	B1
	C represents a rotation of $\frac{\pi}{4}$, i.e.45° (anticlockwise) (about O)	B1 (4)
(b)	$\begin{pmatrix} 3 & -3 \\ 3 & 3 \end{pmatrix}$	M1 A1 (2)
(c)	$\begin{pmatrix} 3 & -3 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}$	B1 (1)
(d)	$\begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} 0 & -15 & 4 \\ 0 & 15 & 21 \end{pmatrix} = \begin{pmatrix} 0 & 90 & 51 \\ 0 & 0 & 75 \end{pmatrix}$ so (0, 0), (90 , 0) and (51 , 75)	M1A1A1A1 (4)
(e)	Area of $\Delta\, OR'S'$ is $\frac{1}{2}\times 90\times 75 = 3375$	B1
	Determinant of **E** is –18 or use area scale factor of enlargement So area of $\Delta\, ORS$ is $3375 \div 18 = 187.5$	M1A1 (3) [14]

Notes:

(a) Enlargement for M1
$3\sqrt{2}$ for A1

(b) Answer incorrect, require **CD** for M1

(c) Answer given so require **DB** as shown for B1

(d) Coordinates as shown or written as $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 90 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 51 \\ 75 \end{pmatrix}$ for each A1

(e) 3375 B1
Divide by theirs for M1

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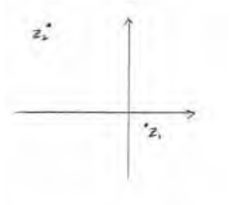
Question Number	Scheme	Marks
Q1 (a)		B1 (1)
(b)	$\lvert z_1 \rvert = \sqrt{2^2 + (-1)^2} = \sqrt{5}$ (or awrt 2.24)	M1 A1 (2)
(c)	$\alpha = \arctan\left(\frac{1}{2}\right)$ or $\arctan\left(-\frac{1}{2}\right)$	M1
	$\arg z_1 = -0.46$ or 5.82 (awrt) (answer in degrees is A0 unless followed by correct conversion)	A1 (2)
(d)	$\frac{-8+9\text{i}}{2-\text{i}} \times \frac{2+\text{i}}{2+\text{i}}$	M1
	$= \frac{-16-8\text{i}+18\text{i}-9}{5} = -5+2\text{i}$ i.e. a = -5 and b =2 or $-\frac{2}{5}a$	A1 A1ft (3) [8]
Notes	Alternative method to part (d)	
	$-8+9\text{i} = (2-i)(a+b\text{i})$, and so $2a+b=-8$ and $2b-a=9$ and attempt to solve as far as equation in one variable	M1
	So a = -5 and $b = 2$	A1 A1cao
	(a) B1 needs both complex numbers as either points or vectors, in correct quadrants and with 'reasonably correct' relative scale	
	(b) M1 Attempt at Pythagoras to find modulus of either complex number	
	A1 condone correct answer even if negative sign not seen in (-1) term	
	A0 for $\pm\sqrt{5}$	
	(c) arctan 2 is M0 unless followed by $\boxed{\frac{3\pi}{2} + \arctan 2}$ or $\boxed{\frac{\pi}{2} - \arctan 2}$ Need to be clear that argz = - 0.46 or 5.82 for A1	
	(d) M1 Multiply numerator and denominator by conjugate of their denominator	
	A1 for –5 and A1 for 2i (should be simplified)	
	Alternative scheme for (d) Allow slips in working for first M1	

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Question Number	Scheme	Marks
Q2 (a)	$r(r+1)(r+3) = r^3 + 4r^2 + 3r$, so **use** $\sum r^3 + 4\sum r^2 + 3\sum r$	M1
	$= \frac{1}{4}n^2(n+1)^2 + 4\left(\frac{1}{6}n(n+1)(2n+1)\right) + 3\left(\frac{1}{2}n(n+1)\right)$	A1 A1
	$= \frac{1}{12}n(n+1)\{3n(n+1) + 8(2n+1) + 18\}$ or $= \frac{1}{12}n\{3n^3 + 22n^2 + 45n + 26\}$	
	or $= = \frac{1}{12}(n+1)\{3n^3 + 19n^2 + 26n\}$	M1 A1
	$= \frac{1}{12}n(n+1)\{3n^2 + 19n + 26\} = \frac{1}{12}n(n+1)(n+2)(3n+13)$ $(k = 13)$	M1 A1cao (7)
(b)	$\sum_{21}^{40} = \sum_{1}^{40} - \sum_{1}^{20}$	M1
	$= \frac{1}{12}(40 \times 41 \times 42 \times 133) - \frac{1}{12}(20 \times 21 \times 22 \times 73) = 763420 - 56210 = 707210$	A1 cao (2) [9]
Notes	(a) M1 expand and must start to use at least one standard formula First 2 A marks: One wrong term A1 A0, two wrong terms A0 A0. M1: Take out factor $kn(n+1)$ or kn or $k(n+1)$ directly or from quartic A1: See scheme (cubics must be simplified) M1: Complete method including a quadratic factor and attempt to factorise it A1 Completely correct work. Just gives $k = 13$, no working is **0** marks for the question. **Alternative method** Expands $(n+1)(n+2)(3n+k)$ and confirms that it equals $\{3n^3 + 22n^2 + 45n + 26\}$ together with statement $k = 13$ can earn last **M1A1** The previous **M1A1** can be implied if they are using a quartic**.** **(b)** M 1 is for substituting 40 and 20 into their **answer** to (a) and subtracting. (NB not 40 and 21) Adding terms is M0A0 as the question said "Hence"	

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Question Number	Scheme	Marks
Q3 (a)	$x^2 + 4 = 0 \quad \Rightarrow \quad x = k\text{i}, \quad x = \pm 2\text{i}$	M1, A1
	Solving 3-term quadratic	M1
	$x = \dfrac{-8 \pm \sqrt{64-100}}{2} \quad = -4+3\text{i}$ and $-4-3\text{i}$	A1 A1ft
		(5)
(b)	$2\text{i} + (-2\text{i}) + (-4+3\text{i}) + (-4-3\text{i}) = -8$	M1 A1cso
		(2)
		[7]
	Alternative method : Expands f(x) as quartic and chooses $\pm$ coefficient of x^3	M1
	-8	A1 cso
Notes	(a) Just $x = 2\text{i}$ is M1 A0 $x = \pm 2$ is M0A0 M1 for solving quadratic follows usual conventions, then A1 for a correct root (simplified as here) and A1ft for conjugate of first answer. Accept correct answers with no working here. Do not give accuracy marks for factors unless followed by roots. (b) M1 for adding four roots of which at least two are complex conjugates and getting a real answer. A1 for –8 following **correct** roots or the alternative method. If any incorrect working in part (a) this A mark will be A0	

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Question Number	Scheme	Marks
Q4 (a)	$f(2.2) = 2.2^3 - 2.2^2 - 6 \quad (= -0.192)$ $f(2.3) = 2.3^3 - 2.3^2 - 6 \quad (= 0.877)$ Change of sign $\Rightarrow$ Root need numerical values correct (to 1 s.f.).	M1 A1 (2)
(b)	$f'(x) = 3x^2 - 2x$ $f'(2.2) = 10.12$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.2 - \frac{-0.192}{10.12}$ $= 2.219$	B1 B1 M1 A1ft A1cao (5)
(c)	$\frac{\alpha - 2.2}{\pm'0.192'} = \frac{2.3 - \alpha}{\pm'0.877'}$ (or equivalent such as $\frac{k}{\pm'0.192'} = \frac{0.1 - k}{\pm'0.877'}$.) $\alpha(0.877 + 0.192) = 2.3 \times 0.192 + 2.2 \times 0.877$ or $k(0.877 + 0.192) = 0.1 \times 0.192$, where $\alpha = 2.2 + k$ so $\alpha \approx 2.218$ (2.21796…) (Allow awrt)	M1 A1 A1 (3) [10]
Alternative	Uses equation of line joining (2.2, -0.192) to (2.3, 0.877) and substitutes $y = 0$ $y + 0.192 = \frac{0.192 + 0.877}{0.1}(x - 2.2)$ and $y = 0$, so $\alpha \approx 2.218$ or awrt as before (NB Gradient = 10.69)	M1 A1, A1
Notes	(a) M1 for attempt at f(2.2) and f(2.3) A1 need indication that there is a change of sign – (could be –0.19<0, 0.88>0) and need conclusion. (These marks may be awarded in other parts of the question if not done in part (a)) (b) B1 for seeing correct derivative (but may be implied by later correct work) B1 for seeing 10.12 or this may be implied by later work M1 Attempt Newton-Raphson with their values A1ft may be implied by the following answer (but does not require an evaluation) Final A1 must 2.219 exactly as shown. So answer of 2.21897 would get 4/5 If done twice ignore second attempt (c) M1 Attempt at ratio with their values of ± f(2.2) and ± f(2.3). N.B. If you see $0.192 - \alpha$ or $0.877 - \alpha$ in the fraction then this is M0 A1 correct linear expression and definition of variable if not α (may be implied by final correct answer- does not need 3 dp accuracy) A1 for awrt 2.218 If done twice ignore second attempt	

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Question Number	Scheme	Marks
Q5 (a)	$\mathbf{R}^2 = \begin{pmatrix} a^2+2a & 2a+2b \\ a^2+ab & 2a+b^2 \end{pmatrix}$	M1 A1 A1 (3)
(b)	Puts their $a^2+2a = 15$ or their $2a+b^2 = 15$ or their $(a^2+2a)(2a+b^2)-(a^2+ab)(2a+2b) = 225$ (or to 15) ,	M1,
	Puts their $a^2+ab=0$ or their $2a+2b=0$	M1
	Solve to find either a or b	M1
	$a=3, \quad b=-3$	A1, A1 (5) [8]
Alternative for (b)	Uses $\mathbf{R}^2\times$ column vector = $15\times$ column vector, and equates rows to give two equations in a and b only Solves to find either a or b as above method	M1, M1 M1 A1 A1
Notes	(a) 1 term correct: M1 A0 A0 2 or 3 terms correct: M1 A1 A0 (b) M1 M1 as described in scheme (In the alternative scheme column vector can be general or specific for first M1 but must be specific for 2nd M1) M1 requires solving equations to find a and/or b (though checking that correct answer satisfies the equations will earn this mark) This mark can be given independently of the first two method marks. So solving $\mathbf{M}^2 = 15\mathbf{M}$ for example gives M0M0M1A0A0 in part (b) Also putting leading diagonal = 0 and other diagonal = 15 is M0M0M1A0A0 (No possible solutions as $a > 0$) A1 A1 for correct answers only **Any Extra answers given**, e.g. a = -5 and b = 5 or wrong answers – **deduct last A1 awarded** So the two sets of answers would be A1 A0 Just the answer . a = -5 and b = 5 is A0 A0 Stopping at two values for a or for b – no attempt at other is A0A0 Answer with no working at all is 0 marks	

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Question Number	Scheme	Marks
Q6 (a)	$y^2 = (8t)^2 = 64t^2$ and $16x = 16 \times 4t^2 = 64t^2$ Or identifies that $a = 4$ and uses general coordinates $(at^2, 2at)$	B1 (1)
(b)	$(4, 0)$	B1 (1)
(c)	$y = 4x^{\frac{1}{2}}$ $\quad \frac{\mathrm{d}y}{\mathrm{d}x} = 2x^{-\frac{1}{2}}$	B1
	Replaces x by $4t^2$ to give **gradient** $\quad [2(4t^2)^{-\frac{1}{2}} = \frac{2}{2t} = \frac{1}{t}]$	M1,
	Uses Gradient of normal is $-\frac{1}{\text{gradient of curve}}$ $\quad [-t]$	M1
	$y - 8t = -t(x - 4t^2) \quad \Rightarrow \quad y + tx = 8t + 4t^3$ $\quad$ (*)	M1 A1cso (5)
(d)	At N, $y = 0$, so $x = 8 + 4t^2$ or $\frac{8t + 4t^3}{t}$	B1
	Base $SN = (8 + 4t^2) - 4$ $(= 4 + 4t^2)$	B1ft
	Area of Δ $PSN = \frac{1}{2}(4 + 4t^2)(8t) = 16t(1 + t^2)$ or $16t + 16t^3$ for $t > 0$	M1 A1 (4) [11]
	{Also Area of Δ $PSN = \frac{1}{2}(4 + 4t^2)(-8t) = -16t(1 + t^2)$ for $t < 0$ } *this is not required*	
	<u>Alternatives:</u> (c) $\frac{\mathrm{d}x}{\mathrm{d}t} = 8t$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 8$ $\quad$ B1 $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{t}$ $\quad$ M1 , then as in main scheme. (c) $2y\frac{\mathrm{d}y}{\mathrm{d}x} = 16$ $\quad$ B1 (or uses $x = \frac{y^2}{8}$ to give $\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{2y}{8}$) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{8}{y} = \frac{8}{8t} = \frac{1}{t}$ $\quad$ M1 , then as in main scheme.	
Notes	(c) Second M1 – need not be function of t Third M1 requires linear equation (not fraction) and should include the parameter t but could be given for equation of tangent (So tangent equation loses 2 marks only and could gain B1M1M0M1A0) (d) Second B1 does not require simplification and may be a constant rather than an expression in t. M1 needs correct area of triangle formula using ½ 'their SN' $\times 8t$ Or may use two triangles in which case need $(4t^2 - 4)$ and $(4t^2 + 8 - 4t^2)$ for B1ft Then Area of Δ $PSN = \frac{1}{2}(4t^2 - 4)(8t) + \frac{1}{2}(4t^2 + 8 - 4t^2)(8t) = 16t(1 + t^2)$ or $16t + 16t^3$	

Question Number	Scheme	Marks
Q7 (a)	Use $4a-(-2\times-1)=0 \quad \Rightarrow \quad a,=\frac{1}{2}$	M1, A1 (2)
(b)	Determinant: $(3\times 4)-(-2\times -1)=10 \quad (\Delta)$ $\mathbf{B}^{-1}=\frac{1}{10}\begin{pmatrix}4 & 2\\ 1 & 3\end{pmatrix}$	M1 M1 A1cso (3)
(c)	$\frac{1}{10}\begin{pmatrix}4 & 2\\ 1 & 3\end{pmatrix}\begin{pmatrix}k-6\\ 3k+12\end{pmatrix}, \quad =\frac{1}{10}\begin{pmatrix}4(k-6)+2(3k+12)\\ (k-6)+3(3k+12)\end{pmatrix}$ $\begin{pmatrix}k\\ k+3\end{pmatrix}$ Lies on $y=x+3$	M1, A1ft A1 (3) [8]
Notes	Alternatives: (c) $\begin{pmatrix}3 & -2\\ -1 & 4\end{pmatrix}\begin{pmatrix}x\\ x+3\end{pmatrix}, \quad =\begin{pmatrix}3x-2(x+3)\\ -x+4(x+3)\end{pmatrix}$, $=\begin{pmatrix}x-6\\ 3x+12\end{pmatrix}$**,** which was of the form $(k - 6, 3k +12)$ Or $\begin{pmatrix}3 & -2\\ -1 & 4\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}, \quad =\begin{pmatrix}3x-2y\\ -x+4y\end{pmatrix}=\begin{pmatrix}k-6\\ 3k+12\end{pmatrix},$ and solves simultaneous equations Both equations correct and eliminate one letter to get $x = k$ or $y = k + 3$ or $10x-10y=-30$ or equivalent. Completely correct work (to $x = k$ and $y = k + 3$), and conclusion lies on $y=x+3$ (a) Allow sign slips for first M1 (b) Allow sign slip for determinant for first M1 (This mark may be awarded for 1/10 appearing in inverse matrix.) Second M1 is for correctly treating the 2 by 2 matrix, ie for $\begin{pmatrix}4 & 2\\ 1 & 3\end{pmatrix}$ Watch out for determinant (3 + 4) – (-1 + -2) = 10 – M0 then final answer is A0 (c) M1 for multiplying matrix by appropriate column vector A1 correct work (ft wrong determinant) A1 for conclusion	 M1, A1, A1 M1 A1 A1

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Question Number	Scheme	Marks
Q8 (a)	$f(1) = 5 + 8 + 3 = 16$, (which is divisible by 4). ($\therefore$ True for $n = 1$).	B1
	Using the formula to write down f(k + 1), $f(k+1) = 5^{k+1} + 8(k+1) + 3$	M1 A1
	$f(k+1) - f(k) = 5^{k+1} + 8(k+1) + 3 - 5^k - 8k - 3$ $= 5(5^k) + 8k + 8 + 3 - 5^k - 8k - 3 = 4(5^k) + 8$	M1 A1
	$f(k+1) = 4(5^k + 2) + f(k)$, **which is divisible by** 4	A1ft
	$\therefore$ True for $n = k + 1$ **if** true for $n = k$. True for $n = 1$, $\therefore$ true for all n.	A1cso (7)
(b)	For $n = 1$, $\begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^1$ ($\therefore$ True for $n = 1$.)	B1
	$\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k+1 & -2k \\ 2k & 1-2k \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 2k+3 & -2k-2 \\ 2k+2 & -2k-1 \end{pmatrix}$	M1 A1 A1
	$= \begin{pmatrix} 2(k+1)+1 & -2(k+1) \\ 2(k+1) & 1-2(k+1) \end{pmatrix}$	M1 A1
	$\therefore$ True for $n = k + 1$ **if** true for $n = k$. **True for n** = 1, $\therefore$ **true for all n**	A1 cso (7) [14]
(a) Alternative for 2nd M:	$f(k+1) = 5(5^k) + 8k + 8 + 3$ M1 $= 4(5^k) + 8 + (5^k + 8k + 3)$ A1 or $= 5(5^k + 8k + 3) - 32k - 4$ $= 4(5^k + 2) + f(k)$, or $= 5f(k) - 4(8k+1)$ which is divisible by 4 A1 (or similar methods)	

Notes	(a) B1 Correct values of 16 or 4 for n = 1 or for n = 0 (Accept "is a multiple of") M1 Using the formula to write down f(k + 1) A1 Correct expression of f(k+1) (or for f(n +1) M1 Start method to connect f(k+1) with f(k) as shown A1 correct working toward multiples of 4, A1 ft result including f(k +1) as subject, A1cso conclusion (b) B1 correct statement for n = 1 or n = 0 First M1: Set up product of two appropriate matrices – product can be either way round A1 A0 for one or two slips in simplified result A1 A1 all correct simplified A0 A0 more than two slips M1: States in terms of (k + 1) A1 Correct statement A1 for induction conclusion
Part (b) Alternative	May write $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k+3 & -2k-2 \\ 2k+2 & -2k-1 \end{pmatrix}$. Then may or may not complete the proof. This can be awarded the second M (substituting k + 1)and following A (simplification) in part (b). The first three marks are awarded as before . Concluding that they have reached the same matrix and therefore a result will then be part of final A1 cso but also need other statements as in the first method.

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January 2010
6667 Further Pure Mathematics FP1
Mark Scheme

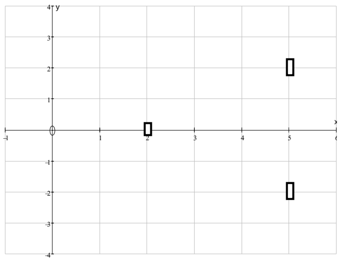
Question Number	Scheme	Marks
Q1	(a) $\frac{z_1}{z_2} = \frac{2+8\text{i}}{1-\text{i}} \times \frac{1+\text{i}}{1+\text{i}}$	M1
	$= \frac{2+2\text{i}+8\text{i}-8}{2} = -3+5\text{i}$	A1 A1 (3)
	(b) $\left\lvert\frac{z_1}{z_2}\right\rvert = \sqrt{(-3)^2+5^2} = \sqrt{34}$ (or awrt 5.83)	M1 A1ft (2)
	(c) $\tan\alpha = -\frac{5}{3}$ or $\frac{5}{3}$	M1
	$\arg\frac{z_1}{z_2} = \pi - 1.03... = 2.11$	A1 (2) [7]
	Notes (a) $\times\frac{1+\text{i}}{1+\text{i}}$ and attempt to multiply out for M1 -3 for first A1, +5i for second A1 (b) Square root required without i for M1 $\frac{\lvert z_1\rvert}{\lvert z_2\rvert}$ award M1 for attempt at Pythagoras for both numerator and denominator (c) tan or $\tan^{-1}$, $\pm\frac{5}{3}$ or $\pm\frac{3}{5}$ seen with their 3 and 5 award M1 2.11 correct answer only award A1	

Question Number	Scheme	Marks
Q2	(a) $f(1.3) = -1.439$ and $f(1.4) = 0.268$ (allow awrt)	B1 (1)
	(b) $f(1.35) < 0$ $(-0.568...)$ $\Rightarrow$ $1.35 < \alpha < 1.4$ $f(1.375) < 0$ $(-0.146...)$ $\Rightarrow$ $1.375 < \alpha < 1.4$	M1 A1 A1 (3)
	(c) $f'(x) = 6x + 22x^{-3}$ $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 1.4 - \dfrac{0.268}{16.417}$, $= 1.384$	M1 A1 M1 A1, A1 (5) [9]
	Notes (a) Both answers required for B1. Accept anything that rounds to 3dp values above. (b) f(1.35) or awrt -0.6 M1 (f(1.35) and awrt -0.6) AND (f(1.375) and awrt -0.1) for first A1 $1.375 < \alpha < 1.4$ or expression using brackets or equivalent in words for second A1 (c) One term correct for M1, both correct for A1 Correct formula seen or implied and attempt to substitute for M1 awrt 16.4 for second A1 which can be implied by correct final answer awrt 1.384 correct answer only A1	

Question Number	Scheme	Marks
Q3	For n = 1: $u_1 = 2,\ u_1 = 5^0 + 1 = 2$	B1
	Assume true for $n = k$:	
	$u_{k+1} = 5u_k - 4 = 5(5^{k-1} + 1) - 4 = 5^k + 5 - 4 = 5^k + 1$	M1 A1
	$\therefore$ True for $n = k + 1$ if true for $n = k$.	
	True for $n = 1$,	
	$\therefore$ true for all n.	A1 cso
		[4]
	Notes Accept $u_1 = 1 + 1 = 2$ or above B1 $5(5^{k-1} + 1) - 4$ seen award M1 $5^k + 1$ or $5^{(k+1)-1} + 1$ award first A1 All three elements stated somewhere in the solution award final A1	

Question Number	Scheme	Marks
Q4	(a) (3, 0) cao	B1 (1)
	(b) P: $x = \frac{1}{3} \Rightarrow y = 2$ A and B lie on $x = -3$ $PB = PS$ or a correct method to find both PB and PS Perimeter = $6 + 2 + 3\frac{1}{3} + 3\frac{1}{3} = 14\frac{2}{3}$	B1 B1 M1 M1 A1 (5) [6]
	Notes (b) Both B marks can be implied by correct diagram with lengths labelled or coordinates of vertices stated. Second M1 for their four values added together. $14\frac{2}{3}$ or awrt 14.7 for final A1	

Question Number	Scheme	Marks
Q5	(a) det $\mathbf{A} = a(a+4)-(-5\times 2) = a^2+4a+10$	M1 A1 (2)
	(b) $a^2+4a+10 = (a+2)^2+6$ Positive for all values of a, so $\mathbf{A}$ is non-singular	M1 A1ft A1cso (3)
	(c) $\mathbf{A}^{-1} = \frac{1}{10}\begin{pmatrix} 4 & 5 \\ -2 & 0 \end{pmatrix}$ B1 for $\frac{1}{10}$	B1 M1 A1 (3) [8]
	Notes (a) Correct use of $ad-bc$ for M1 (b) Attempt to complete square for M1 Alt 1 Attempt to establish turning point (e.g. calculus, graph) M1 Minimum value 6 for A1ft Positive for all values of a, so $\mathbf{A}$ is non-singular for A1 cso Alt 2 Attempt at b^2-4ac for M1. Can be part of quadratic formula Their correct -24 for first A1 No real roots or equivalent, so $\mathbf{A}$ is non-singular for final A1cso (c) Swap leading diagonal, and change sign of other diagonal, with numbers or a for M1 Correct matrix independent of 'their $\frac{1}{10}$ award' final A1	

Question Number	Scheme	Marks
Q6	(a) $5-2\text{i}$ is a root	B1 (1)
	(b) $(x-(5+2\text{i}))(x-(5-2\text{i}))=x^2-10x+29$	M1 M1
	$x^3-12x^2+cx+d=(x^2-10x+29)(x-2)$	M1
	$c=49,\qquad d=-58$	A1, A1 (5)
	(c)  Conjugate pair in 1st and 4th quadrants (symmetrical about real axis)	B1
	Fully correct, labelled	B1 (2)
		[8]
	(b) 1st M: Form brackets using $(x-\alpha)(x-\beta)$ and expand. 2nd M: Achieve a 3-term quadratic with no i's. (b) <u>Alternative</u>: Substitute a complex root (usually 5+2i) and expand brackets M1 $(5+2\text{i})^3-12(5+2\text{i})^2+c(5+2\text{i})+d=0$ $(125+150\text{i}-60-8\text{i})-12(25+20\text{i}-4)+(5c+2c\text{i})+d=0$ M1 (2nd M for achieving an expression with no powers of i) Equate real and imaginary parts M1 $c=49,\qquad d=-58$ A1, A1	

Question Number	Scheme		Marks
Q7	(a) $y=\dfrac{c^2}{x}$ $\quad\dfrac{dy}{dx}=-c^2x^{-2}$		B1
	$\dfrac{dy}{dx}=-\dfrac{c^2}{(ct)^2}=-\dfrac{1}{t^2}$	without x or y	M1
	$y-\dfrac{c}{t}=-\dfrac{1}{t^2}(x-ct) \quad\Rightarrow\quad t^2y+x=2ct$	(*)	M1 A1cso (4)
	(b) Substitute $(15c,-c)$: $\quad -ct^2+15c=2ct$		M1
	$t^2+2t-15=0$		A1
	$(t+5)(t-3)=0 \quad\Rightarrow\quad t=-5 \quad t=3$		M1 A1
	Points are $\left(-5c,-\dfrac{c}{5}\right)$ and $\left(3c,\dfrac{c}{3}\right)$	both	A1 (5) [9]

Notes

(a) Use of $y-y_1=m(x-x_1)$ where m is their gradient expression in terms of c and / or t only for second M1. Accept $y=mx+k$ and attempt to find k for second M1.

(b) Correct absolute factors for their constant for second M1.
Accept correct use of quadratic formula for second M1.

Alternatives:

(a) $\dfrac{dx}{dt}=c$ and $\dfrac{dy}{dt}=-ct^{-2}$ B1

$\dfrac{dy}{dx}=\dfrac{dy}{dt}\div\dfrac{dx}{dt}=-\dfrac{1}{t^2}$ M1, then as in main scheme.

(a) $y+x\dfrac{dy}{dx}=0$ B1

$\dfrac{dy}{dx}=-\dfrac{y}{x}=-\dfrac{1}{t^2}$ M1, then as in main scheme.

Question Number	Scheme	Marks
Q8	(a) $\sum_{r=1}^{1} r^3 = 1^3 = 1$ and $\frac{1}{4} \times 1^2 \times 2^2 = 1$	B1
	Assume true for $n = k$:	
	$\sum_{r=1}^{k+1} r^3 = \frac{1}{4}k^2(k+1)^2 + (k+1)^3$	B1
	$\frac{1}{4}(k+1)^2\left[k^2 + 4(k+1)\right] = \frac{1}{4}(k+1)^2(k+2)^2$	M1 A1
	$\therefore$ True for $n = k + 1$ if true for $n = k$. True for $n = 1$, $\therefore$ true for all n.	A1cso (5)
	(b) $\sum r^3 + 3\sum r + \sum 2 = \frac{1}{4}n^2(n+1)^2 + 3\left(\frac{1}{2}n(n+1)\right), + 2n$	B1, B1
	$= \frac{1}{4}n\left[n(n+1)^2 + 6(n+1) + 8\right]$	M1
	$= \frac{1}{4}n\left[n^3 + 2n^2 + 7n + 14\right] = \frac{1}{4}n(n+2)(n^2+7)$ (*)	A1 A1cso (5)
	(c) $\sum_{15}^{25} = \sum_{1}^{25} - \sum_{1}^{14}$ with attempt to sub in answer to part (b)	M1
	$= \frac{1}{4}(25 \times 27 \times 632) - \frac{1}{4}(14 \times 16 \times 203) = 106650 - 11368 = 95282$	A1 (2) [12]
	Notes (a) Correct method to identify $(k+1)^2$ as a factor award M1 $\frac{1}{4}(k+1)^2(k+2)^2$ award first A1 All three elements stated somewhere in the solution award final A1 (b) Attempt to factorise by n for M1 $\frac{1}{4}$ and $n^3 + 2n^2 + 7n + 14$ for first A1 (c) no working 0/2	

Question Number	Scheme	Marks
Q9	(a) 45° or $\frac{\pi}{4}$ rotation (anticlockwise), about the origin	B1, B1 (2)
	(b) $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 3\sqrt{2} \\ 4\sqrt{2} \end{pmatrix}$	M1
	$p - q = 6$ and $p + q = 8$ or equivalent	M1 A1
	$p = 7$ and $q = 1$ both correct	A1 (4)
	(c) Length of OA (= length of OB) = $\sqrt{7^2 + 1^2}$, $= \sqrt{50} = 5\sqrt{2}$	M1, A1 (2)
	(d) $\mathrm{M}^2 = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	M1 A1 (2)
	(e) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3\sqrt{2} \\ 4\sqrt{2} \end{pmatrix}$ so coordinates are $(-4\sqrt{2}, 3\sqrt{2})$	M1 A1 (2) [12]
	Notes Order of matrix multiplication needs to be correct to award Ms (a) More than one transformation 0/2 (b) Second M1 for correct matrix multiplication to give two equations <u>Alternative</u>: (b) $\mathbf{M}^{-1} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ First M1 A1 $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 3\sqrt{2} \\ 4\sqrt{2} \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$ Second M1 A1 (c) Correct use of their p and their q award M1 (e) Accept column vector for final A1.	

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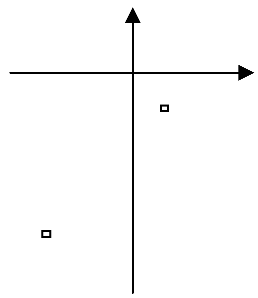
June 2010
Further Pure Mathematics FP1 6667
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $(2-3\text{i})(2-3\text{i}) = ...$ **Expand and use** $\text{i}^2 = -1$, getting completely correct expansion of 3 or 4 terms Reaches $-5-12\text{i}$ after completely correct work (must see 4 – 9) (*)	M1 A1cso (2)
	(b) $\|z^2\| = \sqrt{(-5)^2 + (-12)^2} = 13$ **or** $\|z^2\| = \sqrt{5^2 + 12^2} = 13$ **Alternative methods** for part (b) $\|z^2\| = \|z\|^2 = 2^2 + (-3)^2 = 13$ Or: $\|z^2\| = zz^* = 13$	M1 A1 (2) M1 A1 (2)
	(c) $\tan\alpha = \frac{12}{5}$ (allow $-\frac{12}{5}$) or $\sin\alpha = \frac{12}{13}$ or $\cos\alpha = \frac{5}{13}$ $\arg(z^2) = -(\pi - 1.176...) = -1.97$ (or 4.32) allow awrt **Alternative method** for part (c) $\alpha = 2\times\arctan\left(-\tfrac{3}{2}\right)$ (allow $\tfrac{3}{2}$) **or** use $\frac{\pi}{2} + \arctan\frac{5}{12}$ so $\arg(z^2) = -(\pi - 1.176...) = -1.97$ (or 4.32) allow awrt	M1 A1 (2) M1 A1
	(d)  Both in correct quadrants. Approximate relative scale No labels needed Allow two diagrams if some indication of scale Allow points or arrows	B1 (1) **7 marks**
	Notes: (a) M1: for 4 – 9 – 12i or 4 – 9 – 6i – 6i or $4-3^2-12\text{i}$ but must have correct statement seen and see i^2 replaced by -1 maybe later A1: Printed answer. Must see 4 – 9 in working. Jump from 4 – 6i – 6i + 9i^2 to -5-12i is M0A0 (b) Method may be implied by correct answer. NB $\|z^2\| = 169$ is M0 A0 (c) Allow $\arctan\frac{12}{5}$ for M1 or $\pm\frac{\pi}{2} \pm \arctan\frac{5}{12}$	

Question Number	Scheme	Marks
2.	(a) $\mathbf{M} = \begin{pmatrix} 4 & 3 \\ 6 & 2 \end{pmatrix}$ Determinant: $(8-18) = -10$	B1
	$\mathbf{M}^{-1} = \frac{1}{-10}\begin{pmatrix} 2 & -3 \\ -6 & 4 \end{pmatrix}$ $\left[= \begin{pmatrix} -0.2 & 0.3 \\ 0.6 & -0.4 \end{pmatrix} \right]$	M1 A1 (3)
	(b) Setting $\Delta = 0$ and using $2a^2 \pm 18 = 0$ to obtain $a = .$	M1
	$a = \pm 3$	A1 cao (2) **5 marks**
	Notes: (a) B1: must be -10 M1: for correct attempt at changing elements in major diagonal and changing signs in minor diagonal. Three or four of the numbers in the matrix should be correct – eg allow one slip A1: for any form of the correct answer, with correct determinant then isw. Special case: *a* not replaced is B0M1A0 (b) Two correct answers, $a = \pm 3$, with no working is M1A1 Just *a* = 3 is M1A0, and also one of these answers rejected is A0. Need 3 to be simplified (not $\sqrt{9}$).	

Question Number	Scheme	Marks
3.	(a) $\mathrm{f}(1.4) = \ldots$ and $\mathrm{f}(1.5) = \ldots$ Evaluate both	M1
	$\mathrm{f}(1.4) = -0.256$ (or $-\frac{32}{125}$), $\mathrm{f}(1.5) = 0.708\ldots$ (or $\frac{17}{24}$) **Change of sign, $\therefore$ root**	A1 (2)
	Alternative method: **Graphical method** could earn M1 if 1.4 and 1.5 are both indicated A1 then needs correct graph and conclusion, i.e. change of sign $\therefore$ root	
	(b) $\mathrm{f}(1.45) = 0.221\ldots$ or 0.2 [$\therefore$ root is in [1.4, 1.45]]	M1
	$\mathrm{f}(1.425) = -0.018\ldots$ or -0.019 or -0.02	M1
	$\therefore$ root is in [1.425, 1.45]	A1cso (3)
	(c) $\mathrm{f}'(x) = 3x^2 + 7x^{-2}$	M1 A1
	$\mathrm{f}'(1.45) = 9.636\ldots$ (Special case: $\mathrm{f}'(x) = 3x^2 + 7x^{-2}$ +2 then $\mathrm{f}'(1.45) = 11.636\ldots$)	A1ft
	$x_1 = 1.45 - \frac{\mathrm{f}(1.45)}{\mathrm{f}'(1.45)} = 1.45 - \frac{0.221\ldots}{9.636\ldots} = 1.427$	M1 A1cao (5) **10 marks**

Notes

(a) M1: Some attempt at two evaluations
A1: needs accuracy to 1 figure truncated or rounded and conclusion including **sign change** indicated (One figure accuracy sufficient)

(b) M1: See f(1.45) attempted and positive
M1: See f(1.425) attempted and negative
A1: is cso – any slips in numerical work are penalised here even if correct region found.
Answer may be written as $1.425 \le \alpha \le 1.45$ or $1.425 < \alpha < 1.45$ or (1.425, 1.45) must be correct way round. Between is sufficient.
There is no credit for linear interpolation. This is M0 M0 A0
Answer with no working is also M0M0A0

(c) M1: for attempt at differentiation (decrease in power) A1 is cao
Second A1may be implied by correct answer (do not need to see it)
ft is limited to special case given.
2nd M1: for attempt at Newton Raphson with their values for f(1.45) and $\mathrm{f}'(1.45)$.
A1: is cao and needs to be correct to 3dp
Newton Raphson used more than once – isw.
Special case: $\mathrm{f}'(x) = 3x^2 + 7x^{-2} + 2$ then $\mathrm{f}'(1.45) = 11.636\ldots$) is M1 A0 A1ft M1 A0 This mark can also be given by implication from final answer of 1.43

Question Number	Scheme	Marks
4.	(a) $a=-2, \quad b=50$	B1, B1 (2)
	(b) -3 is a root	B1
	Solving 3-term quadratic $x=\dfrac{2\pm\sqrt{4-200}}{2}$ or $(x-1)^2-1+50=0$	M1
	$=1+7\text{i}$, 1-7i	A1, A1ft (4)
	(c) $(-3)+(1+7\text{i})+(1-7\text{i})=-1$	B1ft (1) **7 marks**
	Notes (a) Accept $x^2-2x+50$ as evidence of values of a and b. (b) B1: –3 must be seen in part (b) M1: for solving quadratic following usual conventions A1: for a correct root (simplified as here) and A1ft: for conjugate of first answer. Accept correct answers with no working here. If answers are written down as factors then isw. Must see roots for marks. (c) ft requires the sum of two non-real conjugate roots and a real root resulting in a real number. Answers including x are B0	

Question Number	Scheme	Marks
5.	(a) $y^2 = (10t)^2 = 100t^2$ and $20x = 20 \times 5t^2 = 100t^2$	B1 (1)
	Alternative method: Compare with $y^2 = 4ax$ and identify $a = 5$ to give answer.	B1 (1)
	(b) Point A is (80, 40) (stated or seen on diagram). May be given in part (a) Focus is (5, 0) (stated or seen on diagram) or (a , 0) with a = 5 May be given in part (a). Gradient: $\frac{40-0}{80-5} = \frac{40}{75} \left(= \frac{8}{15}\right)$	B1 B1 M1 A1 (4) **5 marks**
	Notes: (a) Allow substitution of x to obtain $y = \pm 10t$ (or just $10t$) or of y to obtain x (b) M1: requires use of gradient formula correctly, for their values of x and y. This mark may be implied by correct answer. Differentiation is M0 A0 A1: Accept 0.533 or awrt	

Question Number	Scheme	Marks
6.	(a) $\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$	B1 (1)
	(b) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	B1 (1)
	(c) $\mathbf{T} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & -8 \end{pmatrix}$	M1 A1 (2)
	(d) $\mathbf{AB} = \begin{pmatrix} 6 & 1 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} k & 1 \\ c & -6 \end{pmatrix} = \begin{pmatrix} 6k+c & 0 \\ 4k+2c & -8 \end{pmatrix}$	M1 A1 A1 (3)
	(e) "$6k+c=8$" and "$4k+2c=0$" Form equations and solve simultaneously $k=2$ and $c=-4$	M1 A1 (2) **9 marks**
	Alternative method for (e) **M1: AB = T $\Rightarrow$ B = A^{-1}T =** and compare elements to find k and c. Then A1 as before.	
	<u>Notes</u> (c) M1: Accept multiplication of their matrices either way round (this can be implied by correct answer) A1: cao (d) M1: Correct matrix multiplication method implied by one or two correct terms in correct positions. A1: for three correct terms in correct positions 2nd A1: for all four terms correct and simplified (e) M1: follows their previous work but must give two equations from which k and c can be found and there must be attempt at solution getting to k = or c =. A1: is cao (but not cso - may follow error in position of $4k$ + $2c$ earlier).	

Question Number	Scheme		Marks
7.	(a) LHS = $\text{f}(k+1) = 2^{k+1} + 6^{k+1}$ $= 2(2^k) + 6(6^k)$ $= 6(2^k + 6^k) - 4(2^k) = 6\text{f}(k) - 4(2^k)$	OR RHS = $= 6\text{f}(k) - 4(2^k) = 6(2^k + 6^k) - 4(2^k)$ $= 2(2^k) + 6(6^k)$ $= 2^{k+1} + 6^{k+1} = \text{f}(k+1)$ (*)	M1 A1 A1 (3)
	OR $\text{f}(k+1) - 6f(k) = 2^{k+1} + 6^{k+1} - 6(2^k + 6^k)$		M1
	$= (2-6)(2^k) = -4.2^k$, and so $\text{f}(k+1) = 6\text{f}(k) - 4(2^k)$		A1, A1 (3)
	(b) n = 1: $\text{f}(1) = 2^1 + 6^1 = 8$, **which is divisible** by 8		B1
	Either Assume f(k) divisible by 8 and try to use f(k + 1) = 6f(k) - $4(2^k)$ Show $4(2^k) = 4 \times 2(2^{k-1}) = 8(2^{k-1})$ or $8(\frac{1}{2}2^k)$ Or valid statement Deduction that result is implied for $n = k + 1$ and so is true for positive integers by induction (may include n = 1 true here)	**Or** Assume f(k) divisible by 8 and try to use f (k + 1)– f (k) or f (k + 1) + f (k) including factorising $6^k = 2^k 3^k$ $= 2^3 2^{k-3}(1 + 5.3^k)$ or $= 2^3 2^{k-3}(3 + 7.3^k)$ o.e. Deduction that result is implied for $n = k + 1$ and so is true for positive integers by induction (must include explanation of why n = 2 is also true here)	M1 A1 A1cso (4) **7 marks**
	Notes (a) M1: for substitution into LHS (or RHS) or $\text{f}(k+1) - 6f(k)$ A1: for correct split of the two separate powers cao A1: for completion of proof with no error or ambiguity (needs (for example) to start with one side of equation and reach the other or show that each side separately is $2(2^k) + 6(6^k)$ and **conclude** LHS = RHS) (b) B1: for substitution of n = 1 and **stating** "true for n = 1" or "divisible by 8" or tick. (This statement may appear in the concluding statement of the proof) M1: Assume f(k) divisible by 8 and consider f(k + 1) = 6f(k) - $4(2^k)$ or equivalent expression that could lead to proof – not merely f (k+1) – f(k) unless deduce that 2 is a factor of 6 (see right hand scheme above). A1: Indicates each term divisible by 8 **OR** takes out factor 8 or 2^3 A1: Induction statement . Statement n = 1 here could contribute to B1 mark earlier. NB: $f(k+1) - f(k) = 2^{k+1} - 2^k + 6^{k+1} - 6^k = 2^k + 5.6^k$ **only** is M0 A0 A0 (b) "**Otherwise" methods** Could use: $\text{f}(k+1) = 2\text{f}(k) + 4(6^k)$ or $\text{f}(k+2) = 36\text{f}(k) - 32(6^k)$ or $\text{f}(k+2) = 4\text{f}(k) + 32(2^k)$ in a similar way to given expression and Left hand mark scheme is applied. Special Case: Otherwise Proof **not involving induction**: This can only be awarded the B1 for checking n = 1.		

Question Number	Scheme			Marks
8.	(a) $\frac{c}{3}$			B1 (1)
	(b) $y=\frac{c^2}{x} \Rightarrow \frac{dy}{dx}=-c^2x^{-2}$, **or** $y+x\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=-\frac{y}{x}$ **or** $\dot{x}=c,\ \dot{y}=-\frac{c}{t^2}$ so $\frac{dy}{dx}=-\frac{1}{t^2}$			B1
	and at A $\frac{dy}{dx}=-\frac{c^2}{(3c)^2}=-\frac{1}{9}$ so gradient of normal is 9			M1 A1
	Either $y-\frac{c}{3}=9(x-3c)$ **or** y=9x + k and use x=3c, y= $\frac{c}{3}$			M1
	$\Rightarrow$ $3y=27x-80c$ (*)			A1 (5)
	(c) $\frac{c^2}{x}=\frac{27x-80c}{3}$	$\frac{c^2}{y}=\frac{3y+80c}{27}$	$3\frac{c}{t}=27ct-80c$	M1
	$3c^2=27x^2-80cx$	$27c^2=3y^2+80cy$	$3c=27ct^2-80ct$	A1
	$(x-3c)(27x+c)=0$ so $x=$	$(y+27c)(3y-c)=0$ so $y=$	$(t-3)(27t+1)=0$ so $t=$	M1
	$x=-\frac{c}{27}$, $y=-27c$	$x=-\frac{c}{27}$, $y=-27c$	($t=-\frac{1}{27}$ and so) $x=-\frac{c}{27}$, $y=-27c$	A1 , A1 (5) **11 marks**
	Notes (b) B1: Any valid method of differentiation but must get to correct expression for $\frac{dy}{dx}$ M1 : Substitutes values and uses negative reciprocal (**needs to follow calculus**) A1: 9 cao (needs to follow calculus) M1: Finds equation of line through A with any gradient (other than 0 and ∞) A1: Correct work throughout – **obtaining printed answer**. (c) M1: Obtains equation in one variable (x, y or t) A1: Writes as correct three term quadratic (any equivalent form) M1: Attempts to solve three term quadratic to obtain x = or y = or t = A1: x coordinate, A1: y coordinate. (cao but allow recovery following slips)			

Question Number	Scheme	Marks
9.	(a) If $n=1$, $\sum_{r=1}^{n} r^2 = 1$ and $\frac{1}{6}n(n+1)(2n+1) = \frac{1}{6}\times 1\times 2\times 3 = 1$, so true for n = 1.	B1
	Assume result true for $n = k$	M1
	$\sum_{r=1}^{k+1} r^2 = \frac{1}{6}k(k+1)(2k+1)+(k+1)^2$	M1
	$=\frac{1}{6}(k+1)(2k^2+7k+6)$ or $=\frac{1}{6}(k+2)(2k^2+5k+3)$ or $=\frac{1}{6}(2k+3)(k^2+3k+2)$	A1
	$=\frac{1}{6}(k+1)(k+2)(2k+3)$ $=\frac{1}{6}(k+1)\left(\{k+1\}+1\right)\left(2\{k+1\}+1\right)$ or equivalent	dM1
	True for $n = k + 1$ if true for $n = k$, (and true for n = 1) so true by induction for all n.	A1cso (6)
	Alternative for (a) After first three marks B M M1 as earlier :	B1M1M1
	May state RHS $=\frac{1}{6}(k+1)\left(\{k+1\}+1\right)\left(2\{k+1\}+1\right)=\frac{1}{6}(k+1)(k+2)(2k+3)$ for third M1	dM1
	Expands to $\frac{1}{6}(k+1)(2k^2+7k+6)$ and show equal to $\sum_{r=1}^{k+1} r^2 = \frac{1}{6}k(k+1)(2k+1)+(k+1)^2$ for A1	A1
	So true for $n = k + 1$ if true for $n = k$, and true for n = 1, so true by induction for all n.	A1cso (6)
	(b) $\sum_{r=1}^{n}(r^2+5r+6)=\sum_{r=1}^{n}r^2+5\sum_{r=1}^{n}r+\ \left(\sum_{r=1}^{n}6\right)$	M1
	$\frac{1}{6}n(n+1)(2n+1)+\frac{5}{2}n(n+1),\quad +6n$	A1, B1
	$=\frac{1}{6}n\left[(n+1)(2n+1)+15(n+1)+36\right]$	M1
	$=\frac{1}{6}n\left[2n^2+18n+52\right]=\frac{1}{3}n(n^2+9n+26)$ or a = 9, b = 26	A1 (5)
	(c) $\sum_{r=n+1}^{2n}(r+2)(r+3)=\frac{1}{3}2n(4n^2+18n+26)-\frac{1}{3}n(n^2+9n+26)$	M1 A1ft
	$\frac{1}{3}n(8n^2+36n+52-n^2-9n-26)=\frac{1}{3}n(7n^2+27n+26)$ (*)	A1cso (3) **14 marks**
	Notes: (a) B1: Checks n = 1 on both sides and states true for n = 1 here or in conclusion M1: **Assumes true** for $n = k$ (should use one of these **two** words) M1: Adds (k+1)th term to sum of k terms A1: Correct work to support proof M1: Deduces $\frac{1}{6}n(n+1)(2n+1)$ with $n = k + 1$ A1: Makes induction statement. Statement true for n = 1 here could contribute to B1 mark earlier	

	Question 9 Notes continued: (b) M1: Expands and splits (but allow 6 rather than sigma 6 for this mark) A1: first two terms correct B1: for 6*n* M1: Take out factor *n*/6 or *n*/3 correctly – no errors factorising A1: for correct factorised cubic or for identifying *a* and *b* (c) M1: Try to use $\sum_{1}^{2n}(r+2)(r+3)-\sum_{1}^{n}(r+2)(r+3)$ with previous result used **at least once** A1ft Two correct expressions for their a and b values A1: Completely correct work to printed answer

Mark Scheme (Results)

January 2011

GCE

GCE Further Pure Mathematics FP1 (6667) Paper 1

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January 2011
Further Pure Mathematics FP1 6667
Mark Scheme

Question Number	Scheme		Marks
1.	$z = 5 - 3\mathrm{i},\ \ w = 2 + 2\mathrm{i}$		
(a)	$z^2 = (5 - 3\mathrm{i})(5 - 3\mathrm{i})$		
	$= 25 - 15\mathrm{i} - 15\mathrm{i} + 9\mathrm{i}^2$ $= 25 - 15\mathrm{i} - 15\mathrm{i} - 9$	An attempt to multiply out the brackets to give four terms (or four terms implied). zw is M0	M1
	$= 16 - 30\mathrm{i}$	$16 - 30\mathrm{i}$ Answer only 2/2	A1 (2)
(b)	$\frac{z}{w} = \frac{(5 - 3\mathrm{i})}{(2 + 2\mathrm{i})}$		
	$= \frac{(5 - 3\mathrm{i})}{(2 + 2\mathrm{i})} \times \frac{(2 - 2\mathrm{i})}{(2 - 2\mathrm{i})}$	Multiplies $\frac{z}{w}$ by $\frac{(2 - 2\mathrm{i})}{(2 - 2\mathrm{i})}$	M1
	$= \frac{10 - 10\mathrm{i} - 6\mathrm{i} - 6}{4 + 4}$	Simplifies realising that a real number is needed on the denominator and applies $\mathrm{i}^2 = -1$ on their numerator expression and denominator expression.	M1
	$= \frac{4 - 16\mathrm{i}}{8}$		
	$= \frac{1}{2} - 2\mathrm{i}$	$\frac{1}{2} - 2\mathrm{i}$ or $a = \frac{1}{2}$ and $b = -2$ or equivalent Answer as a single fraction A0	A1 (3) [5]

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Question Number	Scheme		Marks
2.			
(a)	$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$		
	$\mathbf{AB} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}\begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$		
	$= \begin{pmatrix} 2(-3)+0(5) & 2(-1)+0(2) \\ 5(-3)+3(5) & 5(-1)+3(2) \end{pmatrix}$	A correct method to multiply out two matrices. Can be implied by two out of four correct elements.	M1
	$= \begin{pmatrix} -6 & -2 \\ 0 & 1 \end{pmatrix}$	Any three elements correct	A1
		Correct answer	A1
		Correct answer only 3/3	(3)
(b)	Reflection; about the y-axis.	<u>Reflection</u>	M1
		<u>y-axis</u> (or $x = 0$.)	A1
			(2)
(c)	$\mathbf{C}^{100} = \mathbf{I} = \underline{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$	$\underline{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$ or $\mathbf{I}$	B1
			(1)
			[6]

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Question Number	Scheme		Marks
3.	$f(x) = 5x^2 - 4x^{\frac{3}{2}} - 6, \quad x \geqslant 0$		
(a)	$f(1.6) = -1.29543081...$	awrt -1.30	B1
	$f(1.8) = 0.5401863372...$	awrt 0.54	B1
	$\dfrac{\alpha - 1.6}{"1.29543081..."} = \dfrac{1.8 - \alpha}{"0.5401863372..."}$	Correct linear interpolation method with signs correct. Can be implied by working below.	M1
	$\alpha = 1.6 + \left(\dfrac{"1.29543081..."}{"0.5401863372..." + "1.29543081..."}\right) 0.2$		
	$= 1.741143899...$	awrt 1.741	A1
		Correct answer seen 4/4	(4)
(b)	$f'(x) = 10x - 6x^{\frac{1}{2}}$	At least one of $\pm ax$ or $\pm bx^{\frac{1}{2}}$ correct.	M1
		Correct differentiation.	A1
			(2)
(c)	$f(1.7) = -0.4161152711...$	$f(1.7) = \text{awrt } -0.42$	B1
	$f'(1.7) = 9.176957114...$	$f'(1.7) = \text{awrt } 9.18$	B1
	$\alpha_2 = 1.7 - \left(\dfrac{"-0.4161152711..."}{"9.176957114..."}\right)$	Correct application of Newton-Raphson formula using their values.	M1
	$= 1.745343491...$		
	$= 1.745 \text{ (3dp)}$	1.745	A1 cao
		Correct answer seen 4/4	(4)
			[10]

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Question Number	Scheme		Marks
4.	$z^2 + pz + q = 0, \quad z_1 = 2 - 4\text{i}$		
(a)	$z_2 = 2 + 4\text{i}$	$2 + 4\text{i}$	B1 (1)
(b)	$(z - 2 + 4\text{i})(z - 2 - 4\text{i}) = 0$		
	$\Rightarrow z^2 - 2z - 4\text{i}z - 2z + 4 - 8\text{i} + 4\text{i}z - 8\text{i} + 16 = 0$	An attempt to multiply out brackets of two complex factors and no i^2.	M1
	$\Rightarrow z^2 - 4z + 20 = 0$	Any one of $p = -4$, $q = 20$.	A1
		Both $p = -4$, $q = 20$.	A1
		$\Rightarrow z^2 - 4z + 20 = 0$ only 3/3	(3) [4]

Question Number	Scheme		Marks
5	$\sum_{r=1}^{n} r(r+1)(r+5)$		
(a)	$= \sum_{r=1}^{n} r^3 + 6r^2 + 5r$	Multiplying out brackets and an attempt to use at least one of the standard formulae correctly.	M1
	$= \underline{\frac{1}{4}n^2(n+1)^2 + 6.\frac{1}{6}n(n+1)(2n+1) + 5.\frac{1}{2}n(n+1)}$	Correct expression.	A1
	$= \frac{1}{4}n^2(n+1)^2 + n(n+1)(2n+1) + \frac{5}{2}n(n+1)$		
	$= \frac{1}{4}n(n+1)\left(n(n+1) + 4(2n+1) + 10\right)$	Factorising out at least $n(n+1)$	dM1
	$= \frac{1}{4}n(n+1)\left(n^2 + n + 8n + 4 + 10\right)$		
	$= \frac{1}{4}n(n+1)\left(n^2 + 9n + 14\right)$	Correct 3 term quadratic factor	A1
	$= \frac{1}{4}n(n+1)(n+2)(n+7)$ *	Correct proof. No errors seen.	A1 (5)
(b)	$S_n = \sum_{r=20}^{50} r(r+1)(r+5)$		
	$= S_{50} - S_{19}$		
	$= \frac{1}{4}(50)(51)(52)(57) - \frac{1}{4}(19)(20)(21)(26)$	Use of $S_{50} - S_{19}$	M1
	$= 1889550 - 51870$		
	$= 1837680$	1837680 Correct answer only 2/2	A1 (2) [7]

Question Number	Scheme		Marks
6.	$C: y^2 = 36x \Rightarrow a = \frac{36}{4} = 9$		
(a)	$S(9, 0)$	$(9, 0)$	B1 (1)
(b)	$x + 9 = 0$ or $x = -9$	$x + 9 = 0$ or $x = -9$ or ft using their a from part (a).	B1$\sqrt{}$ (1)
(c)	$PS = 25 \Rightarrow \underline{QP = 25}$	Either 25 by itself or $PQ = 25$. Do not award if just $PS = 25$ is seen.	B1 (1)
(d)	x-coordinate of $P \Rightarrow x = 25 - 9 = 16$	$x = 16$	B1$\sqrt{}$
	$y^2 = 36(16)$	Substitutes their x-coordinate into equation of C.	M1
	$\underline{y} = \sqrt{576} = \underline{24}$	$\underline{y = 24}$	A1 (3)
	Therefore $P(16, 24)$		
(e)	Area $OSPQ = \frac{1}{2}(9 + 25)24$	$\frac{1}{2}(\text{their } a + 25)(\text{their } y)$ or rectangle and 2 distinct triangles, correct for their values.	M1
	$= \underline{408}$ (units)^2	408	A1 (2) [8]

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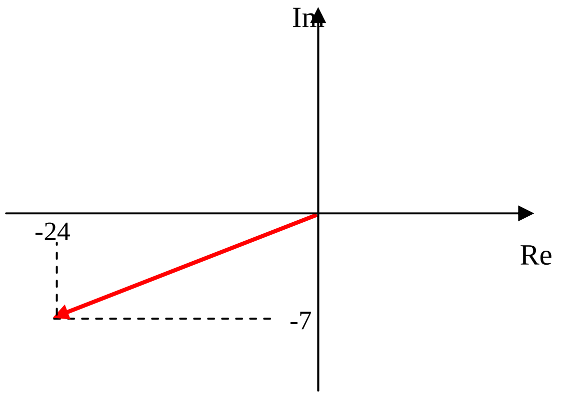
Question Number	Scheme		Marks
7. (a)		Correct quadrant with $(-24, -7)$ indicated.	B1 (1)
(b)	$\arg z = -\pi + \tan^{-1}\left(\frac{7}{24}\right)$	$\tan^{-1}\left(\frac{7}{24}\right)$ or $\tan^{-1}\left(\frac{24}{7}\right)$	M1
	$= -2.857798544... = -2.86$ (2 dp)	awrt -2.86 or awrt 3.43	A1 (2)
(c)	$\lvert w\rvert = 4$, $\arg w = \frac{5\pi}{6} \Rightarrow r = 4$, $\theta = \frac{5\pi}{6}$		
	$w = r\cos\theta + \mathrm{i}\, r\sin\theta$		
	$w = 4\cos\left(\frac{5\pi}{6}\right) + 4\mathrm{i}\sin\left(\frac{5\pi}{6}\right)$	Attempt to apply $r\cos\theta + \mathrm{i}\, r\sin\theta$. Correct expression for w.	M1 A1
	$= 4\left(\frac{-\sqrt{3}}{2}\right) + 4\mathrm{i}\left(\frac{1}{2}\right)$		
	$= -2\sqrt{3} + 2\mathrm{i}$	either $-2\sqrt{3} + 2\mathrm{i}$ or awrt $-3.5 + 2\mathrm{i}$	A1 (3)
	$a = -2\sqrt{3},\ b = 2$		
(d)	$\lvert z\rvert = \sqrt{(-24)^2 + (-7)^2} = \underline{25}$	$\underline{\lvert z\rvert = 25}$ or $zw = (48\sqrt{3}+14) + (14\sqrt{3}-48)\mathrm{i}$ or awrt 97.1-23.8i	B1
	$\lvert zw\rvert = \lvert z\rvert \times \lvert w\rvert = (25)(4)$	Applies $\lvert z\rvert \times \lvert w\rvert$ or $\lvert zw\rvert$	M1
	$= \underline{100}$	$\underline{100}$	A1 (3) [9]

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Question Number	Scheme		Marks
8.	$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$		
(a)	$\det \mathbf{A} = 2(3) - (-1)(-2) = 6 - 2 = \underline{4}$	$\underline{4}$	$\underline{\text{B1}}$ (1)
(b)	$\mathbf{A}^{-1} = \frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$	$\frac{1}{\det \mathbf{A}}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$	M1
		$\frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$	A1 (2)
(c)	$\text{Area}(R) = \frac{72}{4} = \underline{18}\ (\text{units})^2$	$\frac{72}{\text{their } \det \mathbf{A}}$ or $72\left(\text{their } \det \mathbf{A}\right)$	M1
		$\underline{18}$ or ft answer.	A1 $\sqrt{}$ (2)
(d)	$\mathbf{AR} = \mathbf{S} \Rightarrow \mathbf{A}^{-1}\mathbf{AR} = \mathbf{A}^{-1}\mathbf{S} \Rightarrow \mathbf{R} = \mathbf{A}^{-1}\mathbf{S}$		
	$\mathbf{R} = \frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 0 & 8 & 12 \\ 4 & 16 & 4 \end{pmatrix}$	At least one attempt to apply $\mathbf{A}^{-1}$ by any of the three vertices in $\mathbf{S}$.	M1
	$= \frac{1}{4}\begin{pmatrix} 8 & 56 & 44 \\ 8 & 40 & 20 \end{pmatrix}$		
	$= \begin{pmatrix} 2 & 14 & 11 \\ 2 & 10 & 5 \end{pmatrix}$	At least one correct **column** o.e.	A1 $\sqrt{}$
		At least two correct **columns** o.e.	A1
	Vertices are $(2, 2)$, $(14, 10)$ and $(11, 5)$.	All three **coordinates** correct.	A1 (4) [9]

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Question Number	Scheme		Marks
9.	$u_{n+1} = 4u_n + 2$, $u_1 = 2$ and $u_n = \frac{2}{3}(4^n - 1)$		
	$n=1$; $u_1 = \frac{2}{3}(4^1 - 1) = \frac{2}{3}(3) = 2$ So u_n is true when $n=1$.	Check that $u_n = \frac{2}{3}(4^n - 1)$ yields 2 when $n=1$.	B1
	Assume that for $n=k$ that, $u_k = \frac{2}{3}(4^k - 1)$ is true for $k \in \mathbb{Z}^+$.		
	Then $u_{k+1} = 4u_k + 2$		
	$= 4\left(\frac{2}{3}(4^k - 1)\right) + 2$	Substituting $u_k = \frac{2}{3}(4^k - 1)$ into $u_{n+1} = 4u_n + 2$.	M1
	$= \frac{8}{3}(4)^k - \frac{8}{3} + 2$	An attempt to multiply out the brackets by 4 or $\frac{8}{3}$	M1
	$= \frac{2}{3}(4)(4)^k - \frac{2}{3}$		
	$= \frac{2}{3}4^{k+1} - \frac{2}{3}$		
	$= \frac{2}{3}(4^{k+1} - 1)$	$\frac{2}{3}(4^{k+1} - 1)$	A1
	Therefore, the general statement, $u_n = \frac{2}{3}(4^n - 1)$ is true when $n = k+1$. (As u_n is true for $n=1$,) then u_n is true for all positive integers by mathematical induction	Require 'True when n=1', 'Assume true when *n*=*k*' and 'True when $n = k+1$ ' then true for all *n* o.e.	A1
			(5) [5]

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Question Number	Scheme		Marks
10.	$xy = 36$ at $\left(6t, \frac{6}{t}\right)$.		
(a)	$y = \dfrac{36}{x} = 36x^{-1} \Rightarrow \dfrac{dy}{dx} = -36x^{-2} = -\dfrac{36}{x^2}$	An attempt at $\dfrac{dy}{dx}$. or $\dfrac{dy}{dt}$ and $\dfrac{dx}{dt}$	M1
	At $\left(6t, \frac{6}{t}\right)$, $\dfrac{dy}{dx} = -\dfrac{36}{(6t)^2}$	An attempt at $\dfrac{dy}{dx}$. in terms of t	M1
	So, $m_T = \dfrac{dy}{dx} = -\dfrac{1}{t^2}$	$\dfrac{dy}{dx} = -\dfrac{1}{t^2}$ * **Must see working to award here**	A1
	T: $y - \dfrac{6}{t} = -\dfrac{1}{t^2}(x - 6t)$	Applies $y - \dfrac{6}{t} =$ their $m_T(x - 6t)$	M1
	T: $y - \dfrac{6}{t} = -\dfrac{1}{t^2}x + \dfrac{6}{t}$		
	T: $y = -\dfrac{1}{t^2}x + \dfrac{6}{t} + \dfrac{6}{t}$		
	T: $y = -\dfrac{1}{t^2}x + \dfrac{12}{t}$ *	Correct solution .	A1 cso
			(5)
(b)	Both **T** meet at $(-9, 12)$ gives		
	$12 = -\dfrac{1}{t^2}(-9) + \dfrac{12}{t}$	Substituting (-9,12) into **T**.	M1
	$12 = \dfrac{9}{t^2} + \dfrac{12}{t} \quad (\times t^2)$		
	$12t^2 = 9 + 12t$		
	$12t^2 - 12t - 9 = 0$	An attempt to form a "3 term quadratic"	M1
	$4t^2 - 4t - 3 = 0$		
	$(2t - 3)(2t + 1) = 0$	An attempt to factorise.	M1
	$t = \frac{3}{2}, -\frac{1}{2}$	$t = \frac{3}{2}, -\frac{1}{2}$	A1
	$t = \frac{3}{2} \Rightarrow x = 6\left(\frac{3}{2}\right) = 9$, $y = \dfrac{6}{\left(\frac{3}{2}\right)} = 4 \Rightarrow (9, 4)$	An attempt to substitute either their $t = \frac{3}{2}$ or their $t = -\frac{1}{2}$ into x and y.	M1
	$t = -\frac{1}{2} \Rightarrow x = 6\left(-\frac{1}{2}\right) = -3$,	At least one of $(9, 4)$ or $(-3, -12)$.	A1
	$y = \dfrac{6}{\left(-\frac{1}{2}\right)} = -12 \Rightarrow (-3, -12)$	Both $(9, 4)$ and $(-3, -12)$.	A1
			(7) [12]

Other Possible Solutions

Question Number	Scheme		Marks
4.	$z^2 + pz + q = 0, \quad z_1 = 2 - 4\text{i}$		
(a) (i)	$z_2 = 2 + 4\text{i}$	$2 + 4\text{i}$	B1
Aliter (ii) Way 2	Product of roots $= (2 - 4\text{i})(2 + 4\text{i})$	No i^2. Attempt Sum and Product of roots or Sum and discriminant	M1
	$= 4 + 16 = 20$		
	or $b^2 - 4ac = (8\text{i})^2$		
	Sum of roots $= (2 - 4\text{i}) + (2 + 4\text{i}) = 4$		
	$= z^2 - 4z + 20 = 0$	Any one of $p = -4, q = 20$.	A1
		Both $p = -4, q = 20$.	A1
			(4)
4.	$z^2 + pz + q = 0, \quad z_1 = 2 - 4\text{i}$		
(a) (i)	$z_2 = 2 + 4\text{i}$	$2 + 4\text{i}$	B1
Aliter (ii) Way 3	$(2 - 4\text{i})^2 + p(2 - 4\text{i}) + q = 0$	An attempt to substitute either z_1 or z_2 into $z^2 + pz + q = 0$ and no i^2.	M1
	$-12 - 16\text{i} + p(2 - 4\text{i}) + q = 0$		
	Imaginary part: $-16 - 4p = 0$		
	Real part: $-12 + 2p + q = 0$		
	$4p = -16 \Rightarrow p = -4$	Any one of $p = -4, q = 20$.	A1
	$q = 12 - 2p \Rightarrow q = 12 - 2(-4) = 20$	Both $p = -4, q = 20$.	A1
			(4)

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Question Number	Scheme		Marks
Aliter 7. (c) Way 2	$\lvert w \rvert = 4$, $\arg w = \frac{5\pi}{6}$ and $w = a + \mathrm{i}b$		
	$\lvert w \rvert = 4 \Rightarrow a^2 + b^2 = 16$	Attempts to write down an equation in terms of a and b for either the modulus or the argument of w.	M1
	$\arg w = \frac{5\pi}{6} \Rightarrow \arctan\left(\frac{b}{a}\right) = \frac{5\pi}{6} \Rightarrow \frac{b}{a} = -\frac{1}{\sqrt{3}}$	Either $a^2 + b^2 = 16$ or $\frac{b}{a} = -\frac{1}{\sqrt{3}}$	A1
	$a = -\sqrt{3}\,b \Rightarrow a^2 = 3b^2$		
	So, $3b^2 + b^2 = 16 \Rightarrow b^2 = 4$		
	$\Rightarrow b = \pm 2$ and $a = \mp 2\sqrt{3}$		
	As w is in the second quadrant		
	$w = -2\sqrt{3} + 2\mathrm{i}$	either $-2\sqrt{3} + 2\mathrm{i}$ or awrt $-3.5 + 2\mathrm{i}$	A1 (3)
	$a = -2\sqrt{3},\ b = 2$		

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Mark Scheme (Results)

June 2011

GCE Further Pure FP1 (6667) Paper 1

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June 2011
6667 Further Pure Mathematics FP1
Mark Scheme

Question Number	Scheme	Notes	Marks
1.	$\mathrm{f}(x) = 3^x + 3x - 7$		
(a)	$\mathrm{f}(1) = -1$ $\mathrm{f}(2) = 8$	Either any one of f(1) = -1 or f(2) = 8.	M1
	Sign change (positive, negative) (and $\mathrm{f}(x)$ is continuous) therefore (a **root**) α is between $x = 1$ and $x = 2$.	Both values correct, sign change and conclusion	A1
			(2)
(b)	$\mathrm{f}(1.5) = 2.696152423...$ $\{\Rightarrow 1 \leqslant \alpha \leqslant 1.5\}$	$\mathrm{f}(1.5) =$ awrt 2.7 (or truncated to 2.6)	B1
	$\mathrm{f}(1.25) = 0.698222038...$ $\Rightarrow 1 \leqslant \alpha \leqslant 1.25$	Attempt to find $\mathrm{f}(1.25)$.	M1
		$\mathrm{f}(1.25) =$ awrt 0.7 with $1 \leqslant \alpha \leqslant 1.25$ or $1 < \alpha < 1.25$ or $[1, 1.25]$ or $(1, 1.25)$. or equivalent in words.	A1
	In (b) there is no credit for linear interpolation and a correct answer with no working scores no marks.		(3)
			5

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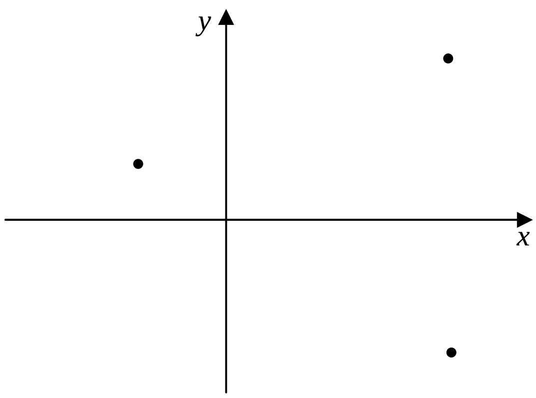
Question Number	Scheme	Notes	Marks
2. (a)	$\lvert z_1 \rvert = \sqrt{(-2)^2 + 1^2} = \sqrt{5} = 2.236...$	$\sqrt{5}$ or awrt 2.24	B1
			(1)
(b)	$\arg z = \pi - \tan^{-1}\left(\frac{1}{2}\right)$	$\tan^{-1}\left(\frac{1}{2}\right)$ or $\tan^{-1}\left(\frac{2}{1}\right)$ or $\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ or $\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ or $\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ or $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$	M1
	$= 2.677945045... \;=\; 2.68$ (2 dp)	awrt 2.68	A1 oe
	Can work in degrees for the method mark ($\arg z = 153.4349488^{\circ}$)		(2)
	$\arg z = \tan^{-1}\left(\frac{1}{-2}\right) = -0.46$ on its own is M0 but $\pi + \tan^{-1}\left(\frac{1}{-2}\right) = 2.68$ scores M1A1 $\pi - \tan^{-1}\left(\frac{1}{-2}\right) =$ is M0 as is $\pi - \tan\left(\frac{1}{2}\right)$ (2.60)		
(c)	$z^2 - 10z + 28 = 0$		
	$z = \dfrac{10 \pm \sqrt{100 - 4(1)(28)}}{2(1)}$	An attempt to use the quadratic formula (usual rules)	M1
	$= \dfrac{10 \pm \sqrt{100 - 112}}{2}$		
	$= \dfrac{10 \pm \sqrt{-12}}{2}$		
	$= \dfrac{10 \pm 2\sqrt{3}\,\mathrm{i}}{2}$	Attempt to simplify their $\sqrt{-12}$ in terms of i. E.g. $\mathrm{i}\sqrt{12}$ or $\mathrm{i}\sqrt{3 \times 4}$	M1
	If their b^2-4ac >0 then only the first M1 is available.		
	So, $z = 5 \pm \sqrt{3}\,\mathrm{i}$. $\{p = 5,\; q = 3\}$	$5 \pm \sqrt{3}\,\mathrm{i}$	A1 oe
	Correct answers with no working scores full marks. **See appendix for alternative solution by completing the square**		(3)
(d)		Note that the points are $(-2, 1)$, $(5, \sqrt{3})$ and $(5, -\sqrt{3})$.	
		The point $(-2, 1)$ plotted correctly on the Argand diagram with/without label.	B1
		The distinct points z_2 and z_3 plotted correctly and symmetrically about the x-axis on the Argand diagram with/without label.	B1 $\sqrt{}$
	The points must be correctly placed relative to each other. If you are in doubt about awarding the marks then consult your team leader or use review.		(2)
	NB the second B mark in (d) depends on having obtained complex numbers in (c)		
			8

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Question Number	Scheme	Notes	Marks
3. (a)	$\mathbf{A} = \begin{pmatrix} 1 & \text{Ö}\,2 \\ \text{Ö}\,2 & -1 \end{pmatrix}$		
(i)	$\mathbf{A}^2 = \begin{pmatrix} 1 & \text{Ö}\,2 \\ \text{Ö}\,2 & -1 \end{pmatrix}\begin{pmatrix} 1 & \text{Ö}\,2 \\ \text{Ö}\,2 & -1 \end{pmatrix}$		
	$= \begin{pmatrix} 1+2 & \text{Ö}\,2-\text{Ö}\,2 \\ \text{Ö}\,2-\text{Ö}\,2 & 2+1 \end{pmatrix}$	A correct method to multiply out two matrices. Can be implied by two out of four correct elements.	M1
	$= \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	Correct answer	A1
			(2)
(ii)	**Enlargement**; scale factor 3, centre $(0, 0)$.	**Enlargement**; scale factor **3**, centre **(0, 0)**	B1; B1
	Allow 'from' or 'about' for centre and 'O' or 'origin' for (0, 0)		
			(2)
(b)	$\mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$		
	Reflection; in the line $y = -x$.	**Reflection**; $y = -x$	B1; B1
	Allow 'in the axis' 'about the line' $y = -x$ etc.		
	The question does not specify a <u>single</u> transformation so we would need to accept any combinations that are correct e.g. Anticlockwise rotation of 90^o about the origin followed by a reflection in the *x*-axis is acceptable. In cases like these, the combination has to be <u>completely</u> correct and scored as B2 (no part marks). If in doubt consult your Team Leader.		(2)
(c)	$\mathbf{C} = \begin{pmatrix} k+1 & 12 \\ k & 9 \end{pmatrix}$, k is a constant.		
	$\mathbf{C}$ is singular $\Rightarrow \det \mathbf{C} = 0$. (Can be implied)	$\det \mathbf{C} = 0$	B1
	Special Case $\dfrac{1}{9(k+1)-12k} = 0$ **B1(implied)M0A0**		
	$9(k+1) - 12k \; (= 0)$	Applies $9(k+1) - 12k$	M1
	$9k + 9 = 12k$		
	$9 = 3k$		
	$k = 3$	$k = 3$	A1
	***k* = 3 with no working can score full marks**		(3)
			9

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Question Number	Scheme	Notes	Marks
4.	$\mathrm{f}(x) = x^2 + \dfrac{5}{2x} - 3x - 1, \quad x \neq 0$		
(a)	$\mathrm{f}(x) = x^2 + \dfrac{5}{2}x^{-1} - 3x - 1$		
	$\mathrm{f}'(x) = 2x - \dfrac{5}{2}x^{-2} - 3 \{+0\}$	At least two of the four terms differentiated correctly.	M1
		Correct differentiation. (Allow any correct unsimplified form)	A1
	$\left\{\mathrm{f}'(x) = 2x - \dfrac{5}{2x^2} - 3\right\}$		(2)
(b)	$\mathrm{f}(0.8) = 0.8^2 + \dfrac{5}{2(0.8)} - 3(0.8) - 1 (= 0.365)\left(= \dfrac{73}{200}\right)$	A correct numerical expression for f(0.8)	B1
	$\mathrm{f}'(0.8) = -5.30625\left(= \dfrac{-849}{160}\right)$	Attempt to insert $x = 0.8$ into their f'(x). Does not require an evaluation. (If $\mathrm{f}'(0.8)$ is incorrect for their derivative and there is no working score M0)	M1
	$\alpha_2 = 0.8 - \left(\dfrac{"0.365"}{"-5.30625"}\right)$	Correct application of Newton-Raphson using their values. Does not require an evaluation.	M1
	$= 0868786808...$		
	$= 0.869 \ (3\text{dp})$	0.869	A1 **cao**
	A correct answer only with no working scores no marks. N-R must be seen. Ignore any further applications of N-R		(4)
	A derivative of $2x - 5(2x)^{-2} - 3$ is quite common and leads to f'(0.8) = -3.353125 and a final answer of 0.909. This would normally score M1A0B1M1M1A0 (4/6) Similarly for a derivative of $2x - 10x^{-2} - 3$ where the corresponding values are f'(0.8) = -17.025 and answer 0.821		
			6

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Question Number	Scheme	Notes	Marks
5.	$\mathbf{A} = \begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}$, where a and b are constants.		
(a)	$\mathbf{A}\begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$		
	Therefore, $\begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}\begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$	Using the information in the question to form the matrix equation. Can be implied by both correct equations below.	M1
	Do not allow this mark for other incorrect statements unless interpreted correctly later e.g. $\begin{pmatrix} 4 \\ 6 \end{pmatrix}\begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$ would be M0 unless followed by correct equations or $\begin{pmatrix} -16+6a \\ 4b-12 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$		
	So, $-16 + 6a = 2$ and $4b - 12 = -8$ Allow $\begin{pmatrix} -16+6a \\ 4b-12 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$	Any one correct equation. Any correct horizontal line	M1
	giving $a = 3$ and $b = 1$.	Any one of $a = 3$ or $b = 1$.	A1
		Both $a = 3$ and $b = 1$.	A1
			(4)
(b)	$\det \mathbf{A} = 8 - (3)(1) = 5$	Finds determinant by applying $8 -$ their ab.	M1
		$\det \mathbf{A} = 5$	A1
	Special case: The equations -16 + 6b = 2 and 4a – 12 = -8 give a = 1 and b = 3. This comes from incorrect matrix multiplication. This will score nothing in (a) but allow all the marks in (b).		
	Note that $\det \mathbf{A} = \frac{1}{8-ab}$ scores M0 here but the following 2 marks are available. However, beware $\det \mathbf{A} = \frac{1}{8-ab} = \frac{1}{5} \Rightarrow area\, S = \frac{30}{\frac{1}{5}} = 150$ This scores M0A0 M1A0		
	Area $S = (\det \mathbf{A})(\text{Area } R)$		
	Area $S = 5 \times 30 = 150$ (units)^2	$\frac{30}{\text{their } \det \mathbf{A}}$ or $30 \times (\text{their } \det \mathbf{A})$	M1
		150 or ft answer	A1 $\sqrt{}$
	If their det A < 0 then allow ft provided final answer > 0		(4)
	In (b) Candidates may take a more laborious route for the area scale factor and find the area of the unit square, for example, after the transformation represented by A. This needs to be a complete method to score any marks. Correctly establishing the area scale factor M1. Correct answer 5 A1. Then mark as original scheme.		
			8

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Question Number	Scheme	Notes	Marks
6.	$z + 3\mathrm{i}z^* = -1 + 13\mathrm{i}$		
	$(x + \mathrm{i}y) + 3\mathrm{i}(x - \mathrm{i}y)$	$z^* = x - \mathrm{i}y$	B1
		Substituting $z = x + \mathrm{i}y$ and their z^* into $z + 3\mathrm{i}z^*$	M1
	$x + \mathrm{i}y + 3\mathrm{i}x + 3y = -1 + 13\mathrm{i}$	Correct equation in *x* and *y* with $\mathrm{i}^2 = -1$. Can be implied.	A1
	$(x + 3y) + \mathrm{i}(y + 3x) = -1 + 13\mathrm{i}$		
	Re part: $x + 3y = -1$ Im part: $y + 3x = 13$	An attempt to equate real **and** imaginary parts.	M1
		Correct equations.	A1
	$3x + 9y = -3$ $3x + y = 13$		
	$8y = -16 \Rightarrow y = -2$	Attempt to solve simultaneous equations to find one of *x* or *y*. **At least one of the equations must contain both *x* and *y* terms.**	M1
	$x + 3y = -1 \Rightarrow x - 6 = -1 \Rightarrow x = 5$	Both $x = 5$ and $y = -2$.	A1
	$\{z = 5 - 2\mathrm{i}\}$		(7)
			7

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Question Number	Scheme	Notes	Marks
7.	$\{S_n =\} \sum_{r=1}^{n} (2r-1)^2$		
(a)	$= \sum_{r=1}^{n} 4r^2 - 4r + 1$	Multiplying out brackets and an attempt to use at least one of the two standard formulae correctly.	M1
	$= \underline{4.\frac{1}{6}n(n+1)(2n+1) - 4.\frac{1}{2}n(n+1)} + n$	First two terms correct.	A1
		$+n$	B1
	$= \frac{2}{3}n(n+1)(2n+1) - 2n(n+1) + n$		
	$= \frac{1}{3}n\{2(n+1)(2n+1) - 6(n+1) + 3\}$	Attempt to factorise out $\frac{1}{3}n$	M1
		Correct expression with $\frac{1}{3}n$ factorised out with no errors seen.	A1
	$= \frac{1}{3}n\{2(2n^2+3n+1) - 6(n+1) + 3\}$ $= \frac{1}{3}n\{4n^2+6n+2 - 6n - 6 + 3\}$ $= \frac{1}{3}n(4n^2 - 1)$		
	$= \frac{1}{3}n(2n+1)(2n-1)$	Correct proof. No errors seen.	A1 *
			(6)
	Note that there are no marks for proof by induction.		
(b)	$\sum_{r=n+1}^{3n} (2r-1)^2 = S_{3n} - S_n$		
	$= \frac{1}{3}.3n(6n+1)(6n-1) - \frac{1}{3}n(2n+1)(2n-1)$	Use of $S_{3n} - S_n$ or $S_{3n} - S_{n+1}$ with the result from (a) used at least once.	M1
		Correct unsimplified expression. E.g. Allow $2(3n)$ for $6n$.	A1
	Note that (b) says **hence** so they have to be using the result from (a)		
	$= n(36n^2 - 1) - \frac{1}{3}n(4n^2 - 1)$		
	$= \frac{1}{3}n(108n^2 - 3 - 4n^2 + 1)$	Factorising out $\frac{1}{3}n$ (or $\frac{2}{3}n$)	dM1
	$= \frac{1}{3}n(104n^2 - 2)$		
	$= \frac{2}{3}n(52n^2 - 1)$	$\frac{2}{3}n(52n^2 - 1)$	A1
	$\{a = 52,\ b = -1\}$		(4)
			10

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Question Number	Scheme	Notes	Marks
8.	$C: y^2 = 48x$ with general point $P\left(12t^2, 24t\right)$.		
(a)	$y^2 = 4ax \Rightarrow \quad a = \frac{48}{4} = 12$	Using $y^2 = 4ax$ to find a.	M1
	So, directrix has the equation $x + 12 = 0$	$x + 12 = 0$	A1 oe
	Correct answer with no working allow full marks		(2)
(b)	$y = \sqrt{48}\,x^{\frac{1}{2}} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{2}\sqrt{48}\,x^{-\frac{1}{2}}\left(= 2\sqrt{3}\,x^{-\frac{1}{2}}\right)$ or (implicitly) $y^2 = 48x \Rightarrow 2y\dfrac{dy}{dx} = 48$ or (chain rule) $\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx} = 24 \times \dfrac{1}{24t}$	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \pm k\,x^{-\frac{1}{2}}$ $ky\dfrac{\mathrm{d}y}{\mathrm{d}x} = c$ their $\dfrac{dy}{dt} \times \left(\dfrac{1}{\text{their } \dfrac{dx}{dt}}\right)$	M1
	When $x = 12t^2$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\sqrt{48}}{2\sqrt{12t^2}} = \dfrac{\sqrt{4}}{2t} = \dfrac{1}{t}$ or $\dfrac{dy}{dx} = \dfrac{48}{2y} = \dfrac{48}{48t} = \dfrac{1}{t}$	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{t}$	A1
	T: $y - 24t = \dfrac{1}{t}\left(x - 12t^2\right)$	Applies $y - 24t =$ their $m_T\left(x - 12t^2\right)$ or $y = \left(\text{their } m_T\right)x + c$ using $x = 12t^2$ *and* $y = 24t$ in an attempt to find c. **Their m_T must be a function of t .**	M1
	T: $ty - 24t^2 = x - 12t^2$		
	T: $x - ty + 12t^2 = 0$	Correct solution.	A1 **cso ***
	Special case: If the gradient is <u>quoted</u> as 1/t, this can score M0A0M1A1		(4)
(c)	Compare $P\left(12t^2, 24t\right)$ with $(3, 12)$ gives $t = \frac{1}{2}$.	$t = \frac{1}{2}$	B1
	NB $x - ty + 12t^2 = 0$ with $x = 3$ and $y = 12$ gives $4t^2 - 4t + 1 = 0 = (2t - 1)^2 \Rightarrow t = \dfrac{1}{2}$		
	$t = \frac{1}{2}$ into **T** gives $x - \frac{1}{2}y + 3 = 0$	Substitutes their t into **T**.	M1
	See Appendix for an alternative approach to find the tangent		
	At X, $x = -12 \Rightarrow -12 - \frac{1}{2}y + 3 = 0$	Substitutes their x from (a) into **T**.	M1
	So, $-9 = \frac{1}{2}y \Rightarrow y = -18$		
	So the coordinates of X are $(-12, -18)$.	$(-12, -18)$	A1
	The coordinates must be together at the end for the final A1 e.g. as above or x = -12, y = -18		(4)
			10

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Question Number	Scheme	Notes	Marks
9. (a)	$n=1;\quad \text{LHS} = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^1 = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$ $\text{RHS} = \begin{pmatrix} 3^1 & 0 \\ 3(3^1-1) & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$ As LHS = RHS, the matrix result is true for $n = 1$.	Check to see that the result is true for $n = 1$.	B1
	Assume that the matrix equation is true for $n = k$, ie. $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^k = \begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}$		
	With $n = k+1$ the matrix equation becomes $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^k \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$		
	$= \begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$ or $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}\begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}$	$\begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}$ by $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$	M1
	$= \begin{pmatrix} 3^{k+1}+0 & 0+0 \\ 9(3^k-1)+6 & 0+1 \end{pmatrix}$ or $\begin{pmatrix} 3^{k+1}+0 & 0+0 \\ 6.3^k+3(3^k-1) & 0+1 \end{pmatrix}$	Correct unsimplified matrix with no errors seen.	A1
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 9(3^k)-3 & 1 \end{pmatrix}$		
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 3\left(3(3^k)-1\right) & 1 \end{pmatrix}$		
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 3(3^{k+1}-1) & 1 \end{pmatrix}$	Manipulates so that $k \to k+1$ on at least one term.	dM1
		Correct result with no errors seen with some working between this and the previous A1	A1
	If the result is true for $n = k$, (1) then it is now true for $n = k+1$. (2) As the result has shown to be true for $n = 1$, (3) then the result is true for all n. (4) **All 4 aspects need to be mentioned at some point for the last A1.**	Correct conclusion with all previous marks earned	A1 cso
			(6)

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Question Number	Scheme	Notes	Marks
9. (b)	$f(1) = 7^{2-1} + 5 = 7 + 5 = 12$,	Shows that $f(1) = 12$.	B1
	{which is divisible by 12}. $\{\therefore f(n)$ is divisible by 12 when $n = 1.\}$		
	Assume that for $n = k$, $f(k) = 7^{2k-1} + 5$ is divisible by 12 for $k \in$ ¢$^{+}$.		
	So, $f(k+1) = 7^{2(k+1)-1} + 5$	Correct unsimplified expression for $f(k+1)$.	B1
	giving, $f(k+1) = 7^{2k+1} + 5$		
	$\therefore f(k+1) - f(k) = \left(7^{2k+1} + 5\right) - \left(7^{2k-1} + 5\right)$	Applies $f(k+1) - f(k)$. No simplification is necessary and condone missing brackets.	M1
	$= 7^{2k+1} - 7^{2k-1}$		
	$= 7^{2k-1}\left(7^2 - 1\right)$	Attempting to isolate 7^{2k-1}	M1
	$= 48\left(7^{2k-1}\right)$	$48\left(7^{2k-1}\right)$	A1cso
	$\therefore f(k+1) = f(k) + 48\left(7^{2k-1}\right)$, which is divisible by 12 as both $f(k)$ and $48\left(7^{2k-1}\right)$ are both divisible by 12.(1) If the result is true for $n = k$, (2) then it is now true for $n = k+1$. (3) As the result has shown to be true for $n = 1$,(4) then the result is true for all *n*. (5). **All 5 aspects need to be mentioned at some point for the last A1.**	Correct conclusion with no incorrect work. Don't condone missing brackets.	A1 cso
	There are other ways of proving this by induction. See appendix for 3 alternatives. If you are in any doubt consult your team leader and/or use the review system.		(6)
			12

Appendix

Question Number	Scheme	Notes	Marks
Aliter **2.** (c) **Way 2**			
	$z^2 - 10z + 28 = 0$		
	$(z-5)^2 - 25 + 28 = 0$	$(z \pm 5)^2 \pm 25 + 28 = 0$	M1
	$(z-5)^2 = -3$		
	$z - 5 = \sqrt{-3}$		
	$z - 5 = \sqrt{3}\,\mathrm{i}$	Attempt to express their $\sqrt{-3}$ in terms of i.	M1
	So, $z = 5 \pm \sqrt{3}\,\mathrm{i}$. $\{p = 5,\ q = 3\}$	$5 \pm \sqrt{3}\,\mathrm{i}$	A1 oe
			(3)

Question Number	Scheme		Marks
Aliter **2.** (c) **Way 3**			
	$z^2 - 10z + 28 = 0$		
	$\left(z-\left(p+i\sqrt{q}\right)\right)\left(z-\left(p-i\sqrt{q}\right)\right) = z^2 - 2pz + p^2 + q$		
	$2p = \pm 10$ *and* $p^2 \pm q = 28$	Uses sum and product of roots	M1
	$2p = \pm 10 \Rightarrow p = 5$	Attempt to solve for p(or q)	M1
	$p = 5$ *and* $q = 3$		A1
			(3)

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Question Number	Scheme	Notes	Marks
Aliter **8.** (c) **Way 2**			
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2\sqrt{3}\,x^{-\frac{1}{2}} = \frac{2\sqrt{3}}{\sqrt{3}} = 2$		B1
	Gives $y - 12 = 2(x - 3)$	Uses (3, 12) and their "2" to find the equation of the tangent.	M1
	$x = -12 \Rightarrow y - 12 = 2(-12 - 3)$	Substitutes their x from (a) into their tangent	M1
	$y = -18$		
	So the coordinates of X are $(-12, -18)$.		A1
			(4)

Question Number	Scheme	Notes	Marks
Aliter **9.** (b) **Way 2**			
	$\mathrm{f}(1) = 7^{2-1} + 5 = 7 + 5 = 12$,	Shows that $\mathrm{f}(1) = 12$.	B1
	{which is divisible by 12}. {$\therefore \mathrm{f}(n)$ is divisible by 12 when $n = 1$.}		
	Assume that for $n = k$,		
	$\mathrm{f}(k) = 7^{2k-1} + 5$ is divisible by 12 for $k \in ¢^+$.		
	So, $\mathrm{f}(k+1) = 7^{2(k+1)-1} + 5$	Correct expression for $\mathrm{f}(k+1)$.	B1
	giving, $\mathrm{f}(k+1) = 7^{2k+1} + 5$		
	$7^{2k+1} + 5 = 49 \times 7^{2k-1} + 5$	Attempt to isolate 7^{2k-1}	M1
	$= 49 \times (7^{2k-1} + 5) - 240$	M1 Attempt to isolate $7^{2k-1} + 5$	M1
	$\mathrm{f}(k+1) = 49 \times \mathrm{f}(k) - 240$	Correct expression in terms of $\mathrm{f}(k)$	A1
	As both $\mathrm{f}(k)$ and 240 are divisible by 12 then so is $\mathrm{f}(k+1)$. If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all n.	Correct conclusion	A1
			(6)

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Question Number	Scheme	Notes	Marks
Aliter			
9. (b)	$\mathrm{f}(1) = 7^{2-1} + 5 = 7 + 5 = 12,$	Shows that $\mathrm{f}(1) = 12.$	B1
Way 3	{which is divisible by 12}. $\{\therefore \mathrm{f}(n)$ is divisible by 12 when $n = 1.\}$		
	Assume that for $n = k$, f(k) is divisible by 12		
	so $\mathrm{f}(k) = 7^{2k-1} + 5 = 12m$		
	So, $\mathrm{f}(k+1) = 7^{2(k+1)-1} + 5$	Correct expression for $\mathrm{f}(k+1)$.	B1
	giving, $\mathrm{f}(k+1) = 7^{2k+1} + 5$		
	$7^{2k+1} + 5 = 7^2.7^{2k-1} + 5 = 49 \times 7^{2k-1} + 5$	Attempt to isolate 7^{2k-1}	M1
	$= 49 \times (12m - 5) + 5$	Substitute for m	M1
	$\mathrm{f}(k+1) = 49 \times 12m - 240$	Correct expression in terms of m	A1
	As both $49 \times 12m$ and 240 are divisible by 12 then so is f(k + 1). If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all n.	Correct conclusion	A1
			(6)

Question Number	Scheme	Notes	Marks
Aliter **9.** (b) **Way 4**			
	$f(1) = 7^{2-1} + 5 = 7 + 5 = 12$,	Shows that $f(1) = 12$.	B1
	{which is divisible by 12}. {$\therefore$ f (n) is divisible by 12 when $n = 1$.}		
	Assume that for $n = k$, $f(k) = 7^{2k-1} + 5$ is divisible by 12 for $k \in ¢^{+}$.		
	$f(k+1) + 35f(k) = \underline{7^{2(k+1)-1} + 5} + 35(7^{2k-1} + 5)$	Correct expression for $f(k+1)$.	B1
	$f(k+1) + 35f(k) = 7^{2k+1} + 5 + 35(7^{2k-1} + 5)$	Add appropriate multiple of f(k) For 7^{2k} this is likely to be 35 (119, 203,.) For 7^{2k-1} 11 (23, 35, 47,..)	M1
	giving, $7.7^{2k} + 5 + 5.7^{2k} + 175$	Attempt to isolate 7^{2k}	M1
	$= 180 + 12 \times 7^{2k} = 12\left(15 + 7^{2k}\right)$	Correct expression	A1
	$\therefore f(k+1) = 12\left(7^{2k} + 15\right) - 35f(k)$. As both f($k$) and $12\left(7^{2k} + 15\right)$ are divisible by 12 then so is f(k + 1). If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all n.	Correct conclusion	A1
			(6)

Mark Scheme (Results)

January 2012

GCE Further Pure FP1 (6667) Paper 01

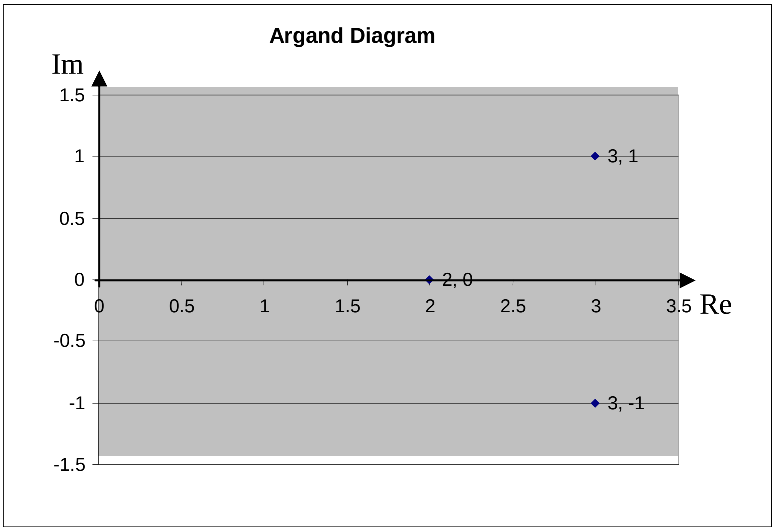
January 2012
6667 Further Pure Mathematics FP1
Mark Scheme

Question Number	Scheme	Notes	Marks
1(a)	$\arg z_1 = -\arctan(1)$	$-\arctan(1)$ or $\arctan(1)$ or $\arctan(-1)$	M1
	$= -\dfrac{\pi}{4}$	or -45 or awrt -0.785 (oe e.g $\dfrac{7\pi}{4}$)	A1
	Correct answer only 2/2		**(2)**
(b)	$z_1 z_2 = (1-i)(3+4i) = 3-3i+4i-4i^2$	At least 3 correct terms (Unsimplified)	M1
	$= 7+\mathrm{i}$	cao	A1
			(2)
(c)	$\dfrac{z_2}{z_1} = \dfrac{(3+4\mathrm{i})}{(1-\mathrm{i})} = \dfrac{(3+4\mathrm{i}).(1+\mathrm{i})}{(1-\mathrm{i}).(1+\mathrm{i})}$	Multiply top and bottom by (1 + i)	M1
	$= \dfrac{(3+4\mathrm{i}).(1+\mathrm{i})}{2}$	$(1+\mathrm{i})(1-\mathrm{i}) = 2$	A1
	$= -\dfrac{1}{2} + \dfrac{7}{2}\mathrm{i}$	or $\dfrac{-1+7\mathrm{i}}{2}$	A1
	Special case $\dfrac{z_1}{z_2} = \dfrac{(1-\mathrm{i})}{(3+4\mathrm{i})} = \dfrac{(1-i).(3-4i)}{(3+4i).(3-4i)}$ **Allow M1A0A0**		
			(3)
	Correct answers only in (b) and (c) scores no marks		**Total 7**

Question Number	Scheme	Notes	Marks
2	$\mathrm{f}(x) = x^4 + x - 1$		
(a)	$\mathrm{f}(0.5) = -0.4375 \quad (-\frac{7}{16})$ $\mathrm{f}(1) = 1$	Either any one of f(0.5) = awrt -0.4 or f(1) = 1	M1
	Sign change (positive, negative) (and $\mathrm{f}(x)$ is continuous) therefore (a **root**) α is between $x = 0.5$ and $x = 1.0$	f(0.5) = awrt -0.4 **and** f(1) = 1, sign change and conclusion	A1
			(2)
(b)	$\mathrm{f}(0.75) = 0.06640625(\frac{17}{256})$	Attempt f(0.75)	M1
	$\mathrm{f}(0.625) = -0.222412109375(-\frac{911}{4096})$	f(0.75) = awrt 0.07 **and** f(0.625) = awrt -0.2	A1
	$0.625 „ \alpha „ 0.75$	$0.625 „ \alpha „ 0.75$ or $0.625 < \alpha < 0.75$ or $[0.625, 0.75]$ or $(0.625, 0.75)$. or equivalent in words.	A1
	In (b) there is no credit for linear interpolation and a correct answer with no working scores no marks.		**(3)**
(c)	$\mathrm{f}'(x) = 4x^3 + 1$	Correct derivative (May be implied later by e.g. $4(0.75)^3 + 1$)	B1
	$x_1 = 0.75$		
	$x_2 = 0.75 - \frac{f(0.75)}{f'(0.75)} = 0.75 - \frac{0.06640625}{2.6875\left(43/16\right)}$	Attempt Newton-Raphson	M1
	$x_2 = 0.72529(06976...) = \frac{499}{688}$	Correct first application – a correct numerical expression e.g. $0.75 - \frac{17/256}{43/16}$ or awrt 0.725 (may be implied)	A1
	$x_3 = 0.724493\left(\frac{499}{688} - \frac{0.002015718978}{2.562146811}\right)$	Awrt 0.724	A1
	$(\alpha) = 0.724$	cao	A1
	A final answer of 0.724 with evidence of NR applied twice with no incorrect work should score 5/5		**(5)**
			Total 10

Question Number	Scheme	Notes	Marks
3(a)	Focus $(4,0)$		B1
	Directrix $x+4=0$	x + "4" = 0 or x = - "4"	M1
		x + 4 = 0 or x = - 4	A1
			(3)
(b)	$y=4x^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=2x^{-\frac{1}{2}}$ $y^2=16x \Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x}=16$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{dy}{dt}.\frac{dt}{dx}=8.\frac{1}{8t}$	$\frac{\mathrm{d}y}{\mathrm{d}x}=k\,x^{-\frac{1}{2}}$ $ky\frac{\mathrm{d}y}{\mathrm{d}x}=c$ their $\frac{dy}{dt}\times\left(\frac{1}{\text{their}\,\frac{dx}{dt}}\right)$	M1
	$\frac{dy}{dx}=2x^{-\frac{1}{2}}$ *or* $2y\frac{dy}{dx}=16$ *or* $\frac{dy}{dx}=8.\frac{1}{8t}$	Correct differentiation	A1
	At P, gradient of normal = $-t$	Correct normal gradient with no errors seen.	A1
	$y-8t=-t(x-4t^2)$	Applies $y-8t$ = their $m_N\left(x-4t^2\right)$ or $y=\left(\text{their } m_N\right)x+c$ using $x=4t^2$ *and* $y=8t$ in an attempt to find c. **Their m_N must be different from their m_T and must be a function of t.**	M1
	$y+tx=8t+4t^3$ *	cso **given answer**	A1
	Special case – if the correct gradient is <u>quoted</u> could score M0A0A0M1A1		**(5)**
			Total 8

Question Number	Scheme	Notes	Marks
4(a)	$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}1 & 2 & 2\\1 & 1 & 4\end{pmatrix}=\begin{pmatrix}1 & 1 & 4\\1 & 2 & 2\end{pmatrix}$	Attempt to multiply the right way round with at least 4 correct elements	M1
	T' has coordinates (1,1), (1,2) and (4,2) or $\begin{pmatrix}1\\1\end{pmatrix},\begin{pmatrix}1\\2\end{pmatrix},\begin{pmatrix}4\\2\end{pmatrix}$ **NOT just** $\begin{pmatrix}1 & 1 & 4\\1 & 2 & 2\end{pmatrix}$	Correct coordinates or vectors	A1
			(2)
(b)	**Reflection** in the line $\boldsymbol{y = x}$	Reflection	B1
		$y = x$	B1
	Allow 'in the axis' 'about the line' $y = x$ etc. Provided both features are mentioned ignore any reference to the origin unless there is a clear contradiction.		
			(2)
(c)	$\mathbf{QR} = \begin{pmatrix}4 & -2\\3 & -1\end{pmatrix}\begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}=\begin{pmatrix}-2 & 0\\0 & 2\end{pmatrix}$	2 correct elements	M1
		Correct matrix	A1
	Note that $\mathbf{RQ} = \begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}\begin{pmatrix}4 & -2\\3 & -1\end{pmatrix}=\begin{pmatrix}10 & -4\\24 & -10\end{pmatrix}$ **scores M0A0 in (c) but allow all the marks in (d) and (e)**		
			(2)
(d)	$\det(\mathbf{QR}) = -2\times 2 - 0 = -4$	"-2"x"2" – "0"x"0"	M1
		-4	A1
	Answer only scores 2/2 $\dfrac{1}{\det(\mathbf{QR})}$ **scores M0**		**(2)**
(e)	Area of $T = \frac{1}{2}\times 1\times 3 = \frac{3}{2}$	Correct area for T	B1
	Area of $T'' = \frac{3}{2}\times 4 = 6$	Attempt at $"\frac{3}{2}"\times \pm"4"$	M1
		6 or follow through their det(QR) x Their triangle area provided area > 0	A1ft
			(3)
			Total 11

Question Number	Scheme	Notes	Marks
5(a)	$(z_2) = 3 - i$		B1
	$(z-(3+i))(z-(3-i)) = z^2 - 6z + 10$	Attempt to expand $(z-(3+i))(z-(3-i))$ or any valid method to establish the quadratic factor e.g. $z = 3 \pm i \Rightarrow z - 3 = \pm i \Rightarrow z^2 - 6z + 9 = -1$ $z = 3 \pm \sqrt{-1} = \dfrac{6 \pm \sqrt{-4}}{2} \Rightarrow b = -6, c = 10$ Sum of roots 6, product of roots 10 $\therefore z^2 - 6z + 10$	M1
	$(z^2 - 6z + 10)(z - 2) = 0$	Attempt at linear factor with their *cd* in $(z^2 + az + c)(z + d) = \pm 20$ Or $(z^2 - 6z + 10)(z + a) \Rightarrow 10a = -20$ Or attempts f(2)	M1
	$(z_3) = 2$		A1
	Showing that $f(2) = 0$ **is equivalent to scoring both M's so it is possible to gain all 4 marks quite easily e.g.** $z_2 = 3 - i$ **B1, shows** $f(2) = 0$ **M2,** $z_3 = 2$ **A1.** **Answers only can score 4/4**		**(4)**
5(b)	 First B1 for plotting (3, 1) and (3, -1) correctly with an indication of scale or labelled with coordinates (allow points/lines/crosses/vectors etc.) Allow *i*/-*i* for 1/-1 marked on imaginary axis. Second B1 for plotting (2, 0) correctly relative to the conjugate pair with an indication of scale or labelled with coordinates or just 2		B1 B1 **(2)**
			Total **6**

Question Number	Scheme	Notes	Marks
6(a)	$n=1, \text{LHS}=1^3=1, \text{RHS}=\frac{1}{4}\times1^2\times2^2=1$	Shows **both** LHS = 1 **and** RHS = 1	B1
	Assume true for n = k		
	When n = k + 1 $\sum_{r=1}^{k+1} r^3 = \frac{1}{4}k^2(k+1)^2+(k+1)^3$	Adds $(k+1)^3$ to the given result	M1
	$=\frac{1}{4}(k+1)^2[k^2+4(k+1)]$	Attempt to factorise out $\frac{1}{4}(k+1)^2$	dM1
		Correct expression with $\frac{1}{4}(k+1)^2$ factorised out.	A1
	$=\frac{1}{4}(k+1)^2(k+2)^2$ Must see 4 things: true for n = 1, assumption true for n = k, said true for n = k + 1 and therefore true for all n	Fully complete proof with no errors and comment. All the previous marks must have been scored.	A1cso
	See extra notes for alternative approaches		(5)
(b)	$\sum(r^3-2)=\sum r^3-\sum 2$	Attempt two sums	M1
	$\sum r^3-\sum 2n$ **is M0**		
	$=\frac{1}{4}n^2(n+1)^2-2n$	Correct expression	A1
	$=\frac{n}{4}(n^3+2n^2+n-8)$ *	Completion to printed answer with no errors seen.	A1
			(3)
(c)	$\sum_{r=20}^{r=50}(r^3-2)=\frac{50}{4}\times130042-\frac{19}{4}\times7592$	Attempt $S_{50}-S_{20}$ or $S_{50}-S_{19}$ and substitutes into a correct expression at least once.	M1
	$(=1625525-36062)$	Correct numerical expression (unsimplified)	A1
	= 1 589 463	cao	A1
			(3)
(c) Way 2	$\sum_{r=20}^{r=50}(r^3-2)=\sum_{r=20}^{r=50}r^3-\sum_{r=20}^{r=50}(2)=\frac{50^2}{4}\times51^2-\frac{19^2}{4}\times20^2-2\times31$	M1 for ($S_{50}-S_{20}$ or $S_{50}-S_{19}$ for cubes) – (2x30 or 2x31)	**Total 11**
		A1 correct numerical expression	
	=1 589 463	A1	

Question Number	Scheme	Notes	Marks
7(a)	$u_2 = 3,\ u_3 = 7$		B1, B1
			(2)
(b)	At $n = 1,\ u_1 = 2^1 - 1 = 1$ and so result true for $n = 1$		B1
	Assume true for $n = k$; $u_k = 2^k - 1$		
	and so $u_{k+1} \left(= 2u_k + 1\right) = 2(2^k - 1) + 1$	Substitutes u_k into u_{k+1} (**must see this line**)	M1
		Correct expression	A1
	$u_{k+1} \left(= 2^{k+1} - 2 + 1\right) = 2^{k+1} - 1$	Correct completion to $u_{k+1} = 2^{k+1} - 1$	A1
	Must see 4 things: <u>true for n = 1</u>, <u>assumption true for n = k</u>, <u>said true for n = k + 1</u> and therefore <u>true for all n</u>	Fully complete proof with no errors and comment. All the previous marks in (b) must have been scored.	A1cso
	Ignore any subsequent attempts e.g. $u_{k+2} = 2u_{k+1} + 1 = 2(2^{k+1} - 1) + 1$ **etc.**		(5)
			Total 7

Question Number	Scheme	Notes	Marks
8(a)	$\det(\mathbf{A}) = 3\times 0 - 2\times 1 (= -2)$	Correct attempt at the determinant	M1
	$\det(\mathbf{A}) \neq 0$ (so **A** is non singular)	det(A) = -2 **and** some reference to zero	A1
	$\frac{1}{\det(\mathbf{A})}$ **scores M0**		**(2)**
(b)	$\mathbf{BA}^2 = \mathbf{A} \Rightarrow \mathbf{BA} = \mathbf{I} \Rightarrow \mathbf{B} = \mathbf{A}^{-1}$	Recognising that $\mathbf{A}^{-1}$ is required	M1
	$\mathbf{B} = -\frac{1}{2}\begin{pmatrix} 3 & -1 \\ -2 & 0 \end{pmatrix}$	At least 3 correct terms in $\begin{pmatrix} 3 & -1 \\ -2 & 0 \end{pmatrix}$	M1
		$\frac{1}{\text{their} \det(\text{A})}\begin{pmatrix} * & * \\ * & * \end{pmatrix}$	B1ft
		Fully correct answer	A1
	Correct answer only score 4/4		**(4)**
	Ignore poor matrix algebra notation if the intention is clear		**Total 6**
(b) Way 2	$\mathbf{A}^2 = \begin{pmatrix} 2 & 3 \\ 6 & 11 \end{pmatrix}$	Correct matrix	B1
	$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 6 & 11 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} \Rightarrow \begin{matrix} 2a+6b=0 \\ 3a+11b=1 \end{matrix} \text{ or } \begin{matrix} 2c+6d=2 \\ 3c+11d=3 \end{matrix}$	2 equations in a and b **or** 2 equations in c and d	M1
	$a = -\frac{3}{2}, b = \frac{1}{2}, c = 1, d = 0$	M1 Solves for a and b or c and d A1 All 4 values correct	M1A1
(b) Way 3	$\mathbf{A}^2 = \begin{pmatrix} 2 & 3 \\ 6 & 11 \end{pmatrix}$	Correct matrix	B1
	$\left(\mathbf{A}^2\right)^{-1} = \frac{1}{\text{"2"}\times\text{"11"}-\text{"3"}\times\text{"6"}}\begin{pmatrix} \text{"11"} & \text{"}-3\text{"} \\ \text{"}-6\text{"} & \text{"2"} \end{pmatrix}$ **see note**	Attempt inverse of $\mathbf{A}^2$	M1
	$\mathbf{A}\left(\mathbf{A}^2\right)^{-1} = \frac{1}{4}\begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 11 & -3 \\ -6 & 2 \end{pmatrix} \text{ or } \frac{1}{4}\begin{pmatrix} 11 & -3 \\ -6 & 2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$	Attempts $\mathbf{A}\left(\mathbf{A}^2\right)^{-1}$ *or* $\left(\mathbf{A}^2\right)^{-1}\mathbf{A}$	M1
	$\mathbf{B} = -\frac{1}{2}\begin{pmatrix} 3 & -1 \\ -2 & 0 \end{pmatrix}$	Fully correct answer	A1
(b) Way 4	**BA** = **I**	Recognising that **BA** = **I**	B1
	$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \begin{matrix} 2b=1 \\ a+3b=0 \end{matrix} \text{ or } \begin{matrix} 2d=0 \\ c+3d=1 \end{matrix}$	2 equations in a and b **or** 2 equations in c and d	M1
	$a = -\frac{3}{2}, b = \frac{1}{2}, c = 1, d = 0$	M1 Solves for a and b or c and d	M1A1
		A1 All 4 values correct	

Question Number	Scheme	Notes	Marks
9 (a)	$y=9x^{-1}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-9x^{-2}$ $xy=9\Rightarrow x\frac{\mathrm{d}y}{\mathrm{d}x}+y=0$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{dy}{dt}.\frac{dt}{dx}=\frac{-3}{p^2}.\frac{1}{3}$	$\frac{\mathrm{d}y}{\mathrm{d}x}=kx^{-2}$ Correct use of product rule. The sum of two terms, one of which is correct. their $\frac{dy}{dt}\times\left(\frac{1}{\text{their}\frac{dx}{dt}}\right)$	M1
	$\frac{dy}{dx}=-9x^{-2}$ *or* $x\frac{dy}{dx}+y=0$ *or* $\frac{dy}{dx}=\frac{-3}{p^2}.\frac{1}{3}$	Correct differentiation.	A1
	$y-\frac{3}{p}=-\frac{1}{p^2}(x-3p)$	Applies $y-\frac{3}{p}=(\text{their } m)(x-3p)$ or $y=(\text{their } m)x+c$ using $x=3p$ *and* $y=\frac{3}{p}$ in an attempt to find c. **Their *m* must be a function of *p* and come from their dy/dx.**	M1
	$x+p^2y=6p$ *	Cso **given answer**	A1
	Special case – if the correct gradient is <u>quoted</u> could score M0A0M1A1		**(4)**
(b)	$x+q^2y=6q$	Allow this to score here or in (c)	B1
			(1)
(c)	$6p-p^2y=6q-q^2y$	Attempt to obtain an **equation** in one variable *x* or *y*	M1
	$y(q^2-p^2)=6(q-p)\Rightarrow y=\frac{6(q-p)}{q^2-p^2}$ $x(q^2-p^2)=6pq(q-p)\Rightarrow x=\frac{6pq(q-p)}{q^2-p^2}$	Attempt to isolate *x* or *y* – must reach *x* or *y* = f(*p*, *q*) or f(*p*) or f(*q*)	M1
	$y=\frac{6}{p+q}$	One correct **simplified** coordinate	A1
	$x=\frac{6pq}{p+q}$	Both coordinates correct and **simplified**	A1
			(4)
			Total 9

<u>Extra Notes</u>

6(a) **To show equivalence between** $\frac{1}{4}k^2(k+1)^2+(k+1)^3$ **and** $\frac{1}{4}(k+1)^2(k+2)^2$

$$\frac{1}{4}k^2(k+1)^2+(k+1)^3=\frac{1}{4}k^4+\frac{3}{2}k^3+\frac{13}{4}k^2+3k+1$$

Attempt to expand one correct expression up to a quartic **M1**

$$\frac{1}{4}(k+1)^2(k+2)^2=\frac{1}{4}k^4+\frac{3}{2}k^3+\frac{13}{4}k^2+3k+1$$

Attempt to expand both correct expressions up to a quartic **M1**

One expansion completely correct (dependent on both M's) **A1**

Both expansions correct and conclusion **A1**

Or

To show $\frac{1}{4}(k+1)^2(k+2)^2-\frac{1}{4}k^2(k+1)^2=(k+1)^3$

$\frac{1}{4}(k+1)^2(k+2)^2-\frac{1}{4}k^2(k+1)^2$ **Attempt to subtract** **M1**

$\frac{1}{4}(k+1)^2(k+2)^2-\frac{1}{4}k^2(k+1)^2=k^3+3k^2+3k+1$ **Obtains a cubic expression** **M1**

Correct expression **A1**

$\frac{1}{4}(k+1)^2(k+2)^2-\frac{1}{4}k^2(k+1)^2=(k+1)^3$ **Correct completion and comment** **A1**

8(b) Way 3

Attempting inverse of $\mathbf{A}^2$ needs to be recognisable as an attempt at an inverse

E.g $\left(\mathbf{A}^2\right)^{-1}=\frac{1}{Their\ Det(\mathbf{A}^2)}\left(\text{A}\ changed\ \mathbf{A}^2\right)$

PMT

edexcel

Mark Scheme (Results)

Summer 2012

GCE Mathematics
6667 Further Pure 1

Summer 2012
6667 Further Pure FP1
Mark Scheme

Question Number	Scheme	Notes	Marks
	$\text{f}(x) = 2x^3 - 6x^2 - 7x - 4$		
1. (a)	$\text{f}(4) = \underline{128 - 96 - 28 - 4 = 0}$	$\underline{128 - 96 - 28 - 4 = 0}$	B1
	<u>Just</u> $2(4)^3 - 6(4)^2 - 7(4) - 4 = 0$ or $2(64) - 6(16) - 7(4) - 4 = 0$ is B0 But $2(64) - 6(16) - 7(4) - 4 = 128 - 128 = 0$ or $2(4)^3 - 6(4)^2 - 7(4) - 4 = 4 - 4 = 0$ is B1		
	There must be sufficient working to <u>show</u> that f(4) = 0		
			[1]
(b)	$\text{f}(4) = 0 \Rightarrow (x - 4)$ is a factor.		
	$\text{f}(x) = (x - 4)(2x^2 + 2x + 1)$	M1: $(2x^2 + kx + 1)$ Uses inspection or long division or compares coefficients **and** $(x - 4)$ (**not** $(x + 4)$) to obtain a quadratic factor of this form. A1: $(2x^2 + 2x + 1)$ cao	M1A1
	So, $x = \dfrac{-2 \pm \sqrt{4 - 4(2)(1)}}{2(2)}$ $(2)\left(x^2 + x + \dfrac{1}{2}\right) = 0 \Rightarrow (2)\left(\left(x \pm \dfrac{1}{2}\right)^2 \pm k \pm \dfrac{1}{2}\right)\ k \neq 0 \Rightarrow x =$	Use of correct quadratic formula for their <u>3TQ</u> or completes the square.	M1
	Allow an attempt at factorisation provided the usual conditions are satisfied and proceeds as far as x = ..		
	$\Rightarrow x = \dfrac{-2 \pm \sqrt{-4}}{2(2)}$		
	$\Rightarrow x = 4,\ \dfrac{-2 \pm 2\text{i}}{4}$	**All <u>three</u> roots stated somewhere in (b).** Complex roots must be at least as given but apply isw if necessary.	A1
			[4]
			5 marks

Question Number	Scheme	Notes	Marks
2. (a)	$\mathbf{A} = \begin{pmatrix} 3 & 1 & 3 \\ 4 & 5 & 5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix}$		
	$\mathbf{AB} = \begin{pmatrix} 3 & 1 & 3 \\ 4 & 5 & 5 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix}$		
	$= \begin{pmatrix} 3+1+0 & 3+2-3 \\ 4+5+0 & 4+10-5 \end{pmatrix}$	A correct method to multiply out two matrices. Can be implied by two out of four correct (unsimplified) elements in a <u>dimensionally correct</u> matrix. A 2x2 matrix with a number or a calculation at each corner.	M1
	$= \begin{pmatrix} 4 & 2 \\ 9 & 9 \end{pmatrix}$	Correct answer	A1
	A correct answer with no working can score both marks		
			[2]
(b)	$\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 8 & 6 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} 5 & 2k \\ 4 & k \end{pmatrix}$, where k is a constant,		
	$\mathbf{C} + \mathbf{D} = \begin{pmatrix} 3 & 2 \\ 8 & 6 \end{pmatrix} + \begin{pmatrix} 5 & 2k \\ 4 & k \end{pmatrix} = \begin{pmatrix} 8 & 2k+2 \\ 12 & 6+k \end{pmatrix}$	An attempt to add C to D. Can be implied by two out of four correct (unsimplified) elements in a <u>dimensionally correct</u> matrix.	M1
	E does not have an inverse $\Rightarrow \det \mathbf{E} = 0$.		
	$8(6+k) - 12(2k+2)$	Applies "$ad - bc$" to **E** where **E** is a 2x2 matrix.	M1
	$8(6+k) - 12(2k+2) = 0$	States or applies $\det(\mathbf{E}) = 0$ where $\det(\mathbf{E}) = ad - bc$ or $ad + bc$ only and **E** is a 2x2 matrix.	M1
	Note $8(6+k) - 12(2k+2) = 0$ or $8(6+k) = 12(2k+2)$ could score both M's		
	$48 + 8k = 24k + 24$ $24 = 16k$		
	$k = \frac{3}{2}$		A1 oe
			[4]
			6 marks

Question Number	Scheme	Notes	Marks
3.	$\mathrm{f}(x) = x^2 + \dfrac{3}{4\sqrt{x}} - 3x - 7\,, \quad x > 0$		
	$\mathrm{f}(x) = x^2 + \dfrac{3}{4}x^{-\frac{1}{2}} - 3x - 7$		
	$\mathrm{f}'(x) = 2x - \dfrac{3}{8}x^{-\frac{3}{2}} - 3\left\{+0\right\}$	M1: $x^n \to x^{n-1}$ on at least one term A1: Correct differentiation.	M1A1
	$\mathrm{f}(4) = -2.625 = -\dfrac{21}{8} = -2\dfrac{5}{8}$ *or* $4^2 + \dfrac{3}{4\sqrt{4}} - 3\times 4 - 7$	$\mathrm{f}(4) = -2.625$ A correct evaluation of f(4) or a correct numerical expression for f(4). This can be implied by a correct answer below but in all other cases, f(4) must be seen explicitly evaluated or as an expression.	B1
	$\mathrm{f}'(4) = 4.953125 = \dfrac{317}{64} = 4\dfrac{61}{64}$	Attempt to insert $x = 4$ into their $\mathrm{f}'(x)$. Not dependent on the first M but must be what they think is $\mathrm{f}'(x)$.	M1
	$\alpha_2 = 4 - \left(\dfrac{"-2.625"}{"4.953125"}\right)$	Correct application of Newton-Raphson using their values.	M1
	$= 4.529968454... \quad \left(= \dfrac{1436}{317} = 4\tfrac{168}{317}\right)$		
	$= 4.53 \;\; (2\,\mathrm{dp})$	4.53 **cso**	A1 **cao**
	Note that the kind of errors that are being made in differentiating are sometimes giving 4.53 but the final mark is cso and the final A1 should not be awarded in these cases.		
	Ignore any further iterations		
	A correct derivative followed by $\alpha_2 = 4 - \dfrac{f(4)}{f'(4)} = 4.53$ can score full marks.		
			[6]
			6 marks

Question Number	Scheme	Notes	Marks
4. (a)	$\sum_{r=1}^{n}\left(r^3+6r-3\right)$		
	$=\underline{\frac{1}{4}n^2(n+1)^2+6.\frac{1}{2}n(n+1)}-3n$	M1; An attempt to use at least one of the standard formulae correctly in summing at least 2 terms of r^3+6r-3 A1: Correct underlined expression. B1: $-3\rightarrow -3n$	M1A1B1
	$=\frac{1}{4}n^2(n+1)^2+3n^2+3n-3n$		
	If any marks have been lost, no further marks are available in part (a)		
	$=\frac{1}{4}n^2(n+1)^2+3n^2$ $=\frac{1}{4}n^2\left((n+1)^2+12\right)$	Cancels out the $3n$ and attempts to factorise out at least $\frac{1}{4}n$.	dM1
	$=\frac{1}{4}n^2\left(n^2+2n+13\right)$ **(AG)**	Correct answer **with no errors seen**.	A1 *
	Provided the first 3 marks are scored, allow the next two marks for correctly showing the algebraic equivalence. E.g. showing that both $\frac{1}{4}n^2(n+1)^2+6.\frac{1}{2}n(n+1)-3n$ and $\frac{1}{4}n^2\left(n^2+2n+13\right)=\frac{1}{4}n^4+\frac{1}{2}n^3+\frac{13}{4}n^2$		
	There are no marks for proof by induction but apply the scheme if necessary.		
			[5]
(b)	$S_n=\sum_{r=16}^{30}\left(r^3+6r-3\right)=S_{30}-S_{15}$		
	$=\frac{1}{4}(30)^2\left(30^2+2(30)+13\right)-\frac{1}{4}(15)^2\left(15^2+2(15)+13\right)$	<u>Use</u> of $S_{30}-S_{15}$ or $S_{30}-S_{16}$	M1
	NB They must be using $S_n=\frac{1}{4}n^2\left(n^2+2n+13\right)$ **not** $S_n=n^3+6n-3$		
	$=218925-15075$		
	$=203850$	203850	A1 **cao**
	NB $S_{30}-S_{16}$ =218925 – 19264 = 199661 (Scores M1 A0)		
			[2]
			7 marks

Question Number	Scheme	Notes	Marks
5.	$C: y^2 = 8x \Rightarrow a = \frac{8}{4} = 2$		
(a)	$PQ = 12 \Rightarrow$ By symmetry $y_P = \frac{12}{2} = \underline{6}$	$y = \underline{6}$	B1
			[1]
(b)	$y^2 = 8x \Rightarrow 6^2 = 8x$	Substitutes their y-coordinate into $y^2 = 8x$.	M1
	$\Rightarrow x = \underline{\frac{36}{8}} = \underline{\frac{9}{2}}$ (So P has coordinates $\left(\frac{9}{2}, 6\right)$)	$\Rightarrow x = \underline{\frac{36}{8}}$ or $\underline{\frac{9}{2}}$	A1 oe
			[2]
(c)	Focus $S(2, 0)$	Focus has coordinates $(2, 0)$. Seen or implied. Can score anywhere.	B1
	Gradient $PS = \dfrac{6-0}{\frac{9}{2}-2}$ $\left\{= \dfrac{6}{\left(\frac{5}{2}\right)} = \dfrac{12}{5}\right\}$	Correct method for finding the gradient of the line segment PS. If no gradient formula is quoted and the gradient is incorrect, score M0 but allow this mark if there is a clear use of $\dfrac{y_2 - y_1}{x_2 - x_1}$ even if their coordinates are 'confused'.	M1
	Either $y - 0 = \frac{12}{5}(x-2)$ or $y - 6 = \frac{12}{5}(x - \frac{9}{2})$; - - - - - - - - - - - **or** $y = \frac{12}{5}x + c$ and $0 = \frac{12}{5}(2) + c \Rightarrow c = -\frac{24}{5}$;	$y - y_1 = m(x - x_1)$ with 'their PS gradient' and their (x_1, y_1) **Their PS gradient must have come from using P and S (not calculus) and they must use their P or S as (x_1, y_1).** - - - - - - - - - - - or uses $y = mx + c$ with 'their gradient' in an attempt to find c. **Their PS gradient must have come from using P and S (not calculus) and they must use their P or S as (x_1, y_1).**	M1
	l: $\underline{12x - 5y - 24 = 0}$	$\underline{12x - 5y - 24 = 0}$	A1
	Allow any equivalent form e.g. $k(12x - 5y - 24) = 0$ where k is an integer		
			[4]
			7 marks

 PMT

Question Number	Scheme	Notes	Marks
6.	$\mathrm{f}(x) = \tan\left(\frac{x}{2}\right) + 3x - 6, \quad -\pi < x < \pi$		
(a)	$\mathrm{f}(1) = -2.45369751...$ $\mathrm{f}(2) = 1.557407725...$	Attempts to evaluate both $\mathrm{f}(1)$ and $\mathrm{f}(2)$ and evaluates at least one of them correctly to awrt (or trunc.) 2 sf. Nm	M1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore a root α is between $x = 1$ and $x = 2$.	Both values correct to awrt (or trunc.) 2 sf, sign change (or a statement which implies this e.g. -2.453.. < 0 < 1.5574..) and conclusion.	A1
			[2]
(b)	$\frac{\alpha - 1}{"2.45369751..."} = \frac{2 - \alpha}{"1.557407725..."}$ **or** $\frac{"2.45369751..." + "1.557407725"}{1} = \frac{"2.45369751..."}{\alpha - 1}$	Correct linear interpolation method. It must be a <u>correct statement</u> using their f(2) and f(1). Can be implied by working below.	M1
	If any "negative lengths" are used, score M0		
	$\alpha = 1 + \left(\frac{"2.45369751..."}{"1.557407725..." + "2.45369751..."}\right)1$ $= \frac{6.464802745}{4.011105235}$	Correct follow through expression to find α .Method can be implied here. (Can be implied by awrt 1.61.)	A1 $\sqrt{}$
	$= 1.611726037...$	awrt 1.61	A1
			[3]
			5 marks
	Special Case – Use of Degrees		
	$\mathrm{f}(1) = -2.991273132...$ $\mathrm{f}(2) = 0.017455064...$	Attempts to evaluate both $\mathrm{f}(1)$ and $\mathrm{f}(2)$ and evaluates at least one of them correctly to awrt (or trunc.) 2 sf.	M1A0
	$\frac{\alpha - 1}{"2.991273132..."} = \frac{2 - \alpha}{"0.017455064..."}$	Correct linear interpolation method. It must be a <u>correct statement</u> using their f(2) and f(1). Can be implied by working below.	M1
	If any "negative lengths" are used, score M0		
	$\alpha = 1 + \left(\frac{"2.99127123..."}{"0.017455064..." + "2.99127123..."}\right)1$	Correct follow through expression to find α .Method can be implied here. (Can be implied by awrt 1.99.)	A1 $\sqrt{}$
	$= 1.994198523...$		A0

Question Number	Scheme	Notes	Marks
7. (a)	$\arg z = -\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$	$\tan^{-1}\left(\pm\frac{\sqrt{3}}{2}\right)$ or $\tan^{-1}\left(\pm\frac{2}{\sqrt{3}}\right)$ seen or evaluated	M1
	Awrt ±0.71 or awrt ±0.86 can be taken as evidence for the method mark. Or ±40.89 or ±49.10 if working in degrees		
	$= -0.7137243789.. = -0.71$ (2 dp)	awrt -0.71 or awrt 5.57	A1
	NB $\tan\left(\frac{\sqrt{3}}{2}\right) = 1.18$ and $\tan\left(\frac{2}{\sqrt{3}}\right) = 2.26$ **and both score M0**		
			[2]
(b)	$z^2 = \left(2 - \mathrm{i}\sqrt{3}\right)\left(2 - \mathrm{i}\sqrt{3}\right)$ $= 4 - 2\mathrm{i}\sqrt{3} - 2\mathrm{i}\sqrt{3} + 3\mathrm{i}^2$	An attempt to multiply out the brackets to give four terms (or four terms implied).	M1
	$= 2 - \mathrm{i}\sqrt{3} + \left(4 - 4\mathrm{i}\sqrt{3} - 3\right)$ $= 2 - \mathrm{i}\sqrt{3} + \left(1 - 4\mathrm{i}\sqrt{3}\right)$ $= 3 - 5\mathrm{i}\sqrt{3}$ (Note: $a = 3, b = -5$.)	M1: An understanding that $\mathrm{i}^2 = -1$ and an attempt to add z and put in the form $a + b\mathrm{i}\sqrt{3}$ A1: $3 - 5\mathrm{i}\sqrt{3}$	M1A1
	$z + z^2 = 2 - \mathrm{i}\sqrt{3} + \left(4 - 4\mathrm{i}\sqrt{3} + 3\right) = 9 - 5\mathrm{i}\sqrt{3}$ scores M1M0A0 (No evidence of $\mathrm{i}^2 = -1$)		
			[3]
(c)	$\frac{z+7}{z-1} = \frac{2 - \mathrm{i}\sqrt{3} + 7}{2 - \mathrm{i}\sqrt{3} - 1}$	Substitutes $z = 2 - \mathrm{i}\sqrt{3}$ into both numerator and denominator.	M1
	$= \frac{\left(9 - \mathrm{i}\sqrt{3}\right)}{\left(1 - \mathrm{i}\sqrt{3}\right)} \times \frac{\left(1 + \mathrm{i}\sqrt{3}\right)}{\left(1 + \mathrm{i}\sqrt{3}\right)}$	Simplifies $\frac{z+7}{z-1}$ **and** multiplies by $\frac{\textbf{their}\left(1 + \mathrm{i}\sqrt{3}\right)}{\textbf{their}\left(1 + \mathrm{i}\sqrt{3}\right)}$	**d**M1
	$= \frac{9 + 9\mathrm{i}\sqrt{3} - \mathrm{i}\sqrt{3} + 3}{1+3}$ $= \frac{12 + 8\mathrm{i}\sqrt{3}}{4}$	Simplifies realising that a real number is needed in the denominator and applies $\mathrm{i}^2 = -1$ in their numerator expression and denominator expression.	M1
	$= 3 + 2\mathrm{i}\sqrt{3}$ (Note: $c = 3, d = 2$.)	$3 + 2\mathrm{i}\sqrt{3}$	A1
			[4]
(d)	$w = \lambda - 3\mathrm{i}$, and $\arg(4 - 5\mathrm{i} + 3w) = -\frac{\pi}{2}$		
	$\left(4 - 5i + 3w = 4 + 3\lambda - 14\mathrm{i}\right)$		
	So real part of $(4 - 5\mathrm{i} + 3w) = 0$ or $4 + 3\lambda = 0$	States real part of $(4 - 5\mathrm{i} + 3w) = 0$ or $4 + 3\lambda = 0$	M1
	So, $\lambda = -\frac{4}{3}$	$-\frac{4}{3}$	A1
			[2]
	Allow $\pm\left(\frac{14}{3\lambda + 4}\right) = \pm\infty \Rightarrow 3\lambda + 4 = 0$ M1 $\Rightarrow \lambda = -\frac{4}{3}$ A1		
			11 marks

Question Number	Scheme	Notes	Marks
8.	$xy = c^2$ at $\left(ct, \frac{c}{t}\right)$.		
(a)	$y = \frac{c^2}{x} = c^2x^{-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -c^2x^{-2} = -\frac{c^2}{x^2}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = kx^{-2}$	
	$xy = c^2 \Rightarrow x\frac{\mathrm{d}y}{\mathrm{d}x} + y = 0$	Correct use of product rule. The sum of two terms, one of which is correct **and** rhs = 0	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{dy}{dt}.\frac{dt}{dx} = -\frac{c}{t^2}.\frac{1}{c}$	their $\frac{dy}{dt} \times \left(\frac{1}{\text{their } \frac{dx}{dt}}\right)$	
	$\frac{dy}{dx} = -c^2x^{-2}$ *or* $x\frac{dy}{dx} + y = 0$ *or* $\frac{dy}{dx} = \frac{-c}{t^2}.\frac{1}{c}$ or equivalent expressions	Correct differentiation	A1
	So, $m_T = \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{t^2}$	$-\frac{1}{t^2}$	
	$y - \frac{c}{t} = -\frac{1}{t^2}(x - ct) \qquad (\times t^2)$	$y - \frac{c}{t} =$ their $m_T(x - ct)$ or $y = mx + c$ with their m_T and $(ct, \frac{c}{t})$ in an attempt to find 'c'. **Their m_T must have come from calculus and should be a function of t or c or both c and t.**	M1
	$x + t^2y = 2ct$ (Allow $t^2y + x = 2ct$)	Correct solution.	A1 *
	(a) Candidates who derive $x + t^2y = 2ct$, by stating that $m_T = -\frac{1}{t^2}$, with no justification score <u>no</u> marks in (a).		
			[4]
(b)	$y = 0 \Rightarrow x = 2ct \Rightarrow A(2ct, 0)$.	$x = 2ct$, seen or implied.	B1
	$x = 0 \Rightarrow y = \frac{2ct}{t^2} \Rightarrow B\left(0, \frac{2c}{t}\right)$.	$y = \frac{2ct}{t^2}$ or $\frac{2c}{t}$, seen or implied.	B1
	Area $OAB = 36 \Rightarrow \frac{1}{2}(2ct)\left(\frac{2c}{t}\right) = 36$	Applies $\frac{1}{2}(\text{their } x)(\text{their } y) = 36$ where x and y are functions of c or t or both (not x or y) and some attempt was made to substitute both $x = 0$ **and** $y = 0$ in the tangent to find A and B.	M1
	Do not allow the x and y coordinates of P to be used for the dimensions of the triangle.		
	$\Rightarrow 2c^2 = 36 \Rightarrow c^2 = 18 \Rightarrow c = 3\sqrt{2}$	$c = 3\sqrt{2}$	A1
		Do <u>not</u> allow $c = \pm 3\sqrt{2}$	**[4]**
			8 marks

Question Number	Scheme	Notes	Marks
9. (a)	$\det \mathbf{M} = 3(-5)-(4)(2) = -15-8 = \underline{-23}$	$\underline{-23}$	B1
			[1]
(b)	Therefore, $\begin{pmatrix}3 & 4\\ 2 & -5\end{pmatrix}\begin{pmatrix}2a-7\\ a-1\end{pmatrix} = \begin{pmatrix}25\\ -14\end{pmatrix}$	Using the information in the question to form the matrix equation. Can be implied by any of the correct equations below.	M1
	Either, $3(2a-7)+4(a-1)=25$ or $2(2a-7)-5(a-1)=-14$ or $\begin{pmatrix}3(2a-7)+4(a-1)\\ 2(2a-7)-5(a-1)\end{pmatrix} = \begin{pmatrix}25\\ -14\end{pmatrix}$	Any one correct equation (unsimplified) inside or outside matrices	A1
	giving $a = 5$	$a = 5$	A1
			[3]
(c)	$\text{Area}(ORS) = \frac{1}{2}(6)(4); = \underline{12}$ (units)^2	M1: $\frac{1}{2}(6)(\text{Their } a-1)$ A1: 12 **cao and cso**	M1A1
	Note A(6, 0) is sometimes misinterpreted as (0, 6) – this is the wrong triangle and scores M0 e.g.1/2x6x3 = 9		
			[2]
(d)	$\text{Area}(OR'S') = \pm 23\times(12)$	$\pm\det\mathbf{M}\times$ (their part (c) answer)	M1
		$\underline{276}$ (follow through provided area > 0)	A1$\sqrt{}$
	A method not involving the determinant requires the coordinates of R$'$ to be calculated ((18, 12)) and then a <u>correct</u> method for the area e.g. (26x25 – 7x13 – 9x12 – 7x25) M1 = 276 A1		
			[2]
(e)	Rotation; 90° anti-clockwise (or 270° clockwise) about $(0, 0)$.	B1: Rotation, Rotates, Rotate, Rotating (not **turn**) B1: 90° anti-clockwise (or 270° clockwise) about (around/from etc.) $(0, 0)$	B1;B1
			[2]
(f)	$\mathbf{M} = \mathbf{BA}$	$\mathbf{M} = \mathbf{BA}$, seen or implied.	M1
	$\mathbf{A}^{-1} = \dfrac{1}{(0)(0)-(1)(-1)}\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}; = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$	$\mathbf{A}^{-1} = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$	A1
	$\mathbf{B} = \begin{pmatrix}3 & 4\\ 2 & -5\end{pmatrix}\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$	Applies $\mathbf{M}(\text{their } \mathbf{A}^{-1})$	M1
	$\mathbf{B} = \begin{pmatrix}-4 & 3\\ 5 & 2\end{pmatrix}$		A1
	NB some candidates state $\mathbf{M} = \mathbf{AB}$ and then calculate $\mathbf{MA}^{-1}$ or state $\mathbf{M} = \mathbf{BA}$ and then calculate $\mathbf{A}^{-1}\mathbf{M}$. These could score M0A0 M1A1ft and M1A1M0A0 respectively.		**[4]**
			14 marks
	Special case		
(f)	$\mathbf{M} = \mathbf{AB}$	$\mathbf{M} = \mathbf{AB}$, seen or implied.	M0
		$\mathbf{A}^{-1} = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$	A0
	$\mathbf{B} = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}3 & 4\\ 2 & -5\end{pmatrix} = \begin{pmatrix}2 & -5\\ -3 & -4\end{pmatrix}$	Applies $(\text{their } \mathbf{A}^{-1})\mathbf{M}$	M1A1ft

Question Number	Scheme	Notes	Marks
10.	$f(n) = 2^{2n-1} + 3^{2n-1}$ is divisible by 5.		
	$f(1) = 2^1 + 3^1 = 5,$	Shows that $f(1) = 5$.	B1
	Assume that for $n = k$, $f(k) = 2^{2k-1} + 3^{2k-1}$ is divisible by 5 for $k \in ¢^+$.		
	$f(k+1) - f(k) = 2^{2(k+1)-1} + 3^{2(k+1)-1} - \left(2^{2k-1} + 3^{2k-1}\right)$	M1: Attempts $f(k+1) - f(k)$. A1: Correct expression for <u>f(k + 1)</u> (Can be unsimplified)	M1A1
	$= 2^{2k+1} + 3^{2k+1} - 2^{2k-1} - 3^{2k-1}$		
	$= 2^{2k-1+2} + 3^{2k-1+2} - 2^{2k-1} - 3^{2k-1}$		
	$= 4\left(2^{2k-1}\right) + 9\left(3^{2k-1}\right) - 2^{2k-1} - 3^{2k-1}$	Achieves an expression in 2^{2k-1} and 3^{2k-1}	M1
	$= 3\left(2^{2k-1}\right) + 8\left(3^{2k-1}\right)$		
	$= 3\left(2^{2k-1}\right) + 3\left(3^{2k-1}\right) + 5\left(3^{2k-1}\right)$		
	$= 3f(k) + 5\left(3^{2k-1}\right)$		
	$\therefore f(k+1) = 4f(k) + 5\left(3^{2k-1}\right)$ or $4(2^{2k-1} + 3^{2k-1}) + 5\left(3^{2k-1}\right)$	Where f(k + 1) is correct and is clearly a multiple of 5.	A1
	If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all *n*.	Correct conclusion **at the end,** at least as given, and all previous marks scored.	A1 cso
			[6]
			6 marks
	All methods should complete to f(k + 1) = ... where f(k +1) is clearly shown to be divisible by 5 to enable the final 2 marks to be available.		
Note that there are many different ways of proving this result by induction.			

Appendix

- dM1 denotes a method mark which is dependent upon the award of the previous method mark.
- ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.
- depM1$*$ denotes a method mark which is dependent upon the award of M1$*$.

- ft denotes "follow through"
- cao denotes "correct answer only"
- aef denotes "any equivalent form"

Other Possible Solutions

Question Number	Scheme	Notes	Marks
***Aliter* 4.(a) Way 2**	$\sum_{r=1}^{n}\left(r^3 + 6r - 3\right)$		
	$= \underline{\frac{1}{4}n^2(n+1)^2 + 6.\frac{1}{2}n(n+1)} - 3n$	An attempt to use at least one of the standard formulae correctly. Correct underlined expression. $-3 \rightarrow -3n$	M1 A1 B1
	If any marks have been lost, no further marks are available in part (a).		
	$= \frac{1}{4}n\left(n(n+1)^2 + 12(n+1) - 12\right)$ $= \frac{1}{4}n\left(n(n+1)^2 + 12n + 12 - 12\right)$ $= \frac{1}{4}n\left(n(n+1)^2 + 12n\right)$	Attempts to factorise out at least $\frac{1}{4}n$ from a correct expression and cancels the constant inside the brackets.	dM1
	$= \frac{1}{4}n^2\left(n^2 + 2n + 13\right)$ **(AG)**	Correct answer	A1 $*$ **[5]**
			5 marks

Question Number	Scheme	Notes	Marks
***Aliter* 6.(b) Way 2**	$y - f(2) = \frac{f(2) - f(1)}{2 - 1}(x - 2)$ or $y - f(1) = \frac{f(2) - f(1)}{2 - 1}(x - 1)$ or $y = \frac{f(2) - f(1)}{2 - 1}x + c$ *with an attempt to find* c	Correct straight line method. It must be a correct statement using their f(2) and f(1). Can be implied by working below.	M1
	NB 'm' = 4.011105235		
	$y = 0 \Rightarrow \alpha = \frac{f(2)}{f(1) - f(2)} + 2$ or $\alpha = \frac{f(1)}{f(1) - f(2)} + 1$	Correct follow through expression to find α .Method can be implied here. (Can be implied by awrt 1.61.)	A1$\sqrt{}$
	$= 1.611726037...$	awrt 1.61	A1
			[3]

Question Number	Scheme	Notes	Marks
***Aliter* 7. (b) Way 2**	$z + z^2 = z(1 + z)$		
	$= \left(2 - i\sqrt{3}\right)\left(1 + \left(2 - i\sqrt{3}\right)\right)$ $= \left(2 - i\sqrt{3}\right)\left(3 - i\sqrt{3}\right)$ $= 6 - 2i\sqrt{3} - 3i\sqrt{3} + 3i^2$	An attempt to multiply out the brackets to give four terms (or four terms implied).	M1
	$= 6 - 2i\sqrt{3} - 3i\sqrt{3} - 3$	M1: An understanding that $i^2 = -1$ and an attempt to put in the form $a + bi\sqrt{3}$	M1
	$= 3 - 5i\sqrt{3}$ (Note: $a = 3, b = -5$.)	$3 - 5i\sqrt{3}$	A1
			[3]

Question Number	Scheme	Notes	Marks
***Aliter* 9. (b) Way 2**	$\mathbf{M}: \begin{pmatrix} 2a-7 \\ a-1 \end{pmatrix} \to \begin{pmatrix} 25 \\ -14 \end{pmatrix}$		
	Therefore, $\begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} 2a-7 \\ a-1 \end{pmatrix} = \begin{pmatrix} 25 \\ -14 \end{pmatrix}$ or $\begin{pmatrix} 2a-7 \\ a-1 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}^{-1}\begin{pmatrix} 25 \\ -14 \end{pmatrix}$	Using the information in the question to form the matrix equation. Can be implied by any of the correct equations below.	M1
	$\begin{pmatrix} 2a-7 \\ a-1 \end{pmatrix} = \frac{1}{(-23)}\begin{pmatrix} -5 & -4 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 25 \\ -14 \end{pmatrix} = \frac{1}{(-23)}\begin{pmatrix} -125+56 \\ -50-42 \end{pmatrix}$		
	Either, $(2a-7)=3$ or $(a-1)=4$	Any one correct equation.	A1
	giving $a=5$	$a=5$	A1
			[3]

Question Number	Scheme	Notes	Marks
***Aliter* 9. (c)** **Way 2 Determinant**	$Area\ ORS = \frac{1}{2}\begin{vmatrix}6 & 3 & 0 & 6\\ 0 & 4 & 0 & 0\end{vmatrix}$ $= \frac{1}{2}\left\|(6\times4 - 3\times0 + 0 - 0 + 0 - 0)\right\|$	Correct calculation	M1
	$= 12$		A1
			[2]

Question Number	Scheme	Notes	Marks
***Aliter* 9. (d)** **Way 2 Determinant**	$Area\ ORS = \frac{1}{2}\begin{vmatrix}18 & 25 & 0 & 18\\ 12 & -14 & 0 & 12\end{vmatrix}$ $= \frac{1}{2}\left\|(18\times -14 - 12\times25 + 0 - 0 + 0 - 0)\right\|$	Correct calculation	M1
	$= 276$		A1 $\sqrt{}$
			[2]

Question Number	Scheme	Notes	Marks
***Aliter* 9. (f) Way 2**	$\mathbf{M} = \mathbf{BA}$	$\mathbf{M} = \mathbf{BA}$, seen or implied.	M1
	$\begin{pmatrix}3 & 4\\ 2 & -5\end{pmatrix} = \begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$	$\begin{pmatrix}3 & 4\\ 2 & -5\end{pmatrix} = \begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$, with constants to be found.	A1
	$\begin{pmatrix}3 & 4\\ 2 & -5\end{pmatrix} = \begin{pmatrix}b & -a\\ d & -c\end{pmatrix}$	$\begin{pmatrix}3 & 4\\ 2 & -5\end{pmatrix} =$ their $\begin{pmatrix}b & -a\\ d & -c\end{pmatrix}$ with at least two elements correct on RHS.	M1
	$\mathbf{B} = \begin{pmatrix}-4 & 3\\ 5 & 2\end{pmatrix}$	Correct matrix for **B** of $\begin{pmatrix}-4 & 3\\ 5 & 2\end{pmatrix}$ or $a = -4$, $b = 3$, $c = 5$, $d = 2$	A1
			[4]

 PMT

Question Number	Scheme	Notes	Marks
Aliter **10.** **Way 2**	$\mathrm{f}(n) = 2^{2n-1} + 3^{2n-1}$ is divisible by 5.		
	$\mathrm{f}(1) = 2^{1} + 3^{1} = 5$	Shows that $\mathrm{f}(1) = 5$.	B1
	Assume that for $n = k$, $\mathrm{f}(k) = 2^{2k-1} + 3^{2k-1}$ is divisible by 5 for $k \in$ ¢$^{+}$.		
	$\mathrm{f}(k+1) = 2^{2(k+1)-1} + 3^{2(k+1)-1}$	M1: Attempts $\mathrm{f}(k+1)$. A1: Correct expression for <u>f(k + 1)</u> (Can be unsimplified)	M1A1
	$= 2^{2k+1} + 3^{2k+1}$		
	$= 4\left(2^{2k-1}\right) + 9\left(3^{2k-1}\right)$	Achieves an expression in 2^{2k-1} and 3^{2k-1}	M1
	$\mathrm{f}(k+1) = 4\left(2^{2k-1} + 3^{2k-1}\right) + 5\left(3^{2k-1}\right)$ or $\mathrm{f}(k+1) = 4\mathrm{f}(k) + 5\left(3^{2k-1}\right)$ or $\mathrm{f}(k+1) = 9\mathrm{f}(k) - 5\left(2^{2k-1}\right)$ or $\mathrm{f}(k+1) = 9\left(2^{2k-1} + 3^{2k-1}\right) - 5\left(2^{2k-1}\right)$	Where f(k + 1) is correct and is clearly a multiple of 5.	A1
	If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all *n*.	Correct conclusion **at the end**, at least as given, and all previous marks scored.	A1 cso
			[6]

Question Number	Scheme	Notes	Marks
Aliter **10.** **Way 3**	$\mathrm{f}(n) = 2^{2n-1} + 3^{2n-1}$ is divisible by 5.		
	$\mathrm{f}(1) = 2^1 + 3^1 = 5,$	Shows that $\mathrm{f}(1) = 5.$	B1
	Assume that for $n = k$, $\mathrm{f}(k) = 2^{2k-1} + 3^{2k-1}$ is divisible by 5 for $k \in ¢^{+}$.		
	$\mathrm{f}(k+1) + \mathrm{f}(k) = 2^{2(k+1)-1} + 3^{2(k+1)-1} + 2^{2k-1} + 3^{2k-1}$	M1: Attempts $\mathrm{f}(k+1) + \mathrm{f}(k)$. A1: Correct expression for <u>f(*k* + 1)</u> (Can be unsimplified)	M1A1
	$= 2^{2k+1} + 3^{2k+1} + 2^{2k-1} + 3^{2k-1}$		
	$= 2^{2k-1+2} + 3^{2k-1+2} + 2^{2k-1} + 3^{2k-1}$		
	$= 4\left(2^{2k-1}\right) + 2^{2k-1} + 9\left(3^{2k-1}\right) + 3^{2k-1}$	Achieves an expression in 2^{2k-1} and 3^{2k-1}	M1
	$= 5\left(2^{2k-1}\right) + 10\left(3^{2k-1}\right)$		
	$= 5\left(2^{2k-1}\right) + 5\left(3^{2k-1}\right) + 5\left(3^{2k-1}\right)$		
	$= 5\mathrm{f}(k) + 5\left(3^{2k-1}\right)$		
	$\therefore\ \mathrm{f}(k+1) = 4\mathrm{f}(k) + 5\left(3^{2k-1}\right)$ or $4\left(2^{2k-1} + 3^{2k-1}\right) + 5\left(3^{2k-1}\right)$	Where f(*k* + 1) is correct and is clearly a multiple of 5.	A1
	If the result is true for $n = k$**, then it is now true for** $n = k+1$**. As the result has shown to be true for** $n = 1$**, then the result is true for all *n*.**	Correct conclusion **at the end**, at least as given, and all previous marks scored.	A1 cso
			[6]
			6 marks

Question Number	Scheme	Notes	Marks
Aliter **10. Way 4**	$\mathrm{f}(n) = 2^{2n-1} + 3^{2n-1}$ is divisible by 5.		
	$\mathrm{f}(1) = 2^1 + 3^1 = 5,$	Shows that $\mathrm{f}(1) = 5$.	B1
	Assume that for $n = k$, $\mathrm{f}(k) = 2^{2k-1} + 3^{2k-1}$ is divisible by 5 for $k \in ¢^+$.		
	$\mathrm{f}(k+1) = \mathrm{f}(k+1) + \mathrm{f}(k) - \mathrm{f}(k)$		
	$\mathrm{f}(k+1) = 2^{2(k+1)-1} + 3^{2(k+1)-1} + 2^{2k-1} + 3^{2k-1} - (2^{2k-1} + 3^{2k-1})$	M1: Attempts $\mathrm{f}(k+1) + \mathrm{f}(k) - \mathrm{f}(k)$ A1: Correct expression for <u>f(*k* + 1)</u> (Can be unsimplified)	M1A1
	$= 4\left(2^{2k-1}\right) + 9\left(3^{2k-1}\right) + 2^{2k-1} + 3^{2k-1} - \left(2^{2k-1} + 3^{2k-1}\right)$	Achieves an expression in 2^{2k-1} and 3^{2k-1}	M1
	$= 5\left(2^{2k-1}\right) + 10\left(3^{2k-1}\right) - \left(2^{2k-1} + 3^{2k-1}\right)$		
	$= 5\left(\left(2^{2k-1}\right) + 2\left(3^{2k-1}\right)\right) - \left(2^{2k-1} + 3^{2k-1}\right)$		
	$= 5\left(\left(2^{2k-1}\right) + 2\left(3^{2k-1}\right)\right) - \mathrm{f}(k)$ *or* $5\left(\left(2^{2k-1}\right) + 2\left(3^{2k-1}\right)\right) - (2^{2k-1} + 3^{2k-1})$	Where f(*k* + 1) is correct and is clearly a multiple of 5.	A1
	If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all *n*.	Correct conclusion **at the end**, at least as given, and all previous marks scored.	A1 cso
			[6]
			6 marks

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Mark Scheme (Results)

January 2013

GCE Further Pure Mathematics FP1 (6667/01)

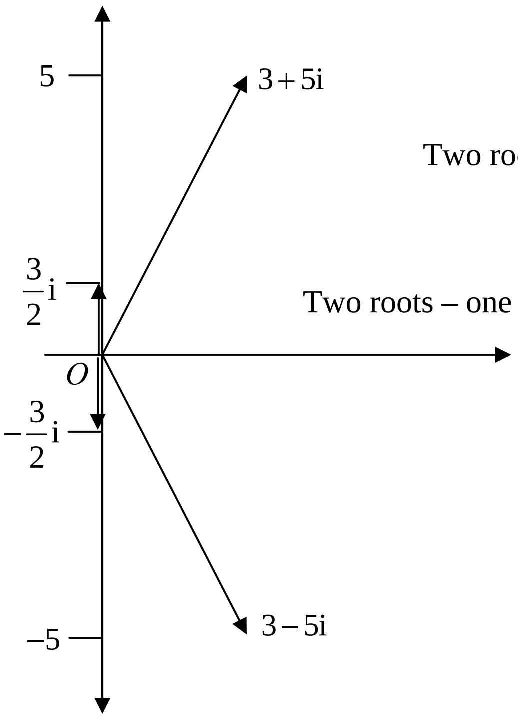
Jan 2013
Further Pure Mathematics FP1 6667
Mark Scheme

Question Number	Scheme	Marks
1.	$\sum_{r=1}^{n} 3(4r^2 - 4r + 1) = 12\sum_{r=1}^{n} r^2 - 12\sum_{r=1}^{n} r + \sum_{r=1}^{n} 3$	M1
	$= \frac{12}{6}n(n+1)(2n+1) - \frac{12}{2}n(n+1), \quad +3n$	A1, B1
	$= n\left[2(n+1)(2n+1) - 6(n+1) + 3\right]$	M1
	$= n\left[4n^2 - 1\right] = n(2n+1)(2n-1)$	A1 cso
		[5]
Notes:	Induction is not acceptable here First M for expanding given expression to give a 3 term quadratic and attempt to substitute. First A for first two terms correct or equivalent. B for $+3n$ appearing Second M for factorising by n Final A for completely correct solution	

Question Number	Scheme	Marks
2.	(a) $\frac{50}{3+4i}=\frac{50(3-4i)}{(3+4i)(3-4i)}=\frac{50(3-4i)}{25}=6-8i$	M1 A1cao **(2)**
	(b) $z^2=\left(6-8i\right)^2=36-64-96\text{i}=-28-96\text{i}$	M1 A1 **(2)**
	(c) $\lvert z\rvert=\sqrt{6^2+\left(-8\right)^2}=10$	M1 A1ft **(2)**
	(d) $\tan\alpha=\frac{-96}{-28}$ so $\alpha=-106.3^{\text{o}}$ or 253.7^{o}	M1 A1cao **(2)** **[8]**
	<u>Alternatives</u> (c) $\lvert z\rvert=\frac{50}{\lvert 3+4\text{i}\rvert}=10$	M1 A1
	(d) arg (3+4i) = 53.13.. so $\arg\left(\frac{50}{3+4\text{i}}\right)^2=-2\times 53.13\ldots$ = -106.3	M1 A1
Notes:	(a) M for $\times\frac{3-4i}{3-4i}$ (accept use of -3+4i) and attempt to expand using $i^2=-1$, A for 6-8i only (b) M for attempting to expand their z^2 using $i^2=-1$, A for -28-96i only. If using original z then must attempt to multiply top and bottom by conjugate and use $i^2=-1$. (c) M for $\sqrt{a^2+b^2}$, A for 'their 10' (d) M for use of tan or $\tan^{-1}$ and values from their z^2 either way up ignoring signs. Radians score A0.	

Question Number	Scheme	Marks
3.	(a) $f'(x) = x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$	M1 A1
		(2)
	(b) f(5) = - 0.0807	B1
	$f'(5) = 0.4025$	M1
	$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 5 - \dfrac{-0.0807}{0.4025}$	M1
	=5.2(0)	A1
		(4)
		[6]
Notes	The B and M marks are implied by a correct answer only with no working or by $\frac{5}{9}(10\sqrt{5} - 13)$ (a) M for at least one of $\pm ax^{-\frac{1}{2}}$ or $\pm bx^{-\frac{3}{2}}$, A for correct (equivalent) answer only (b) B for awrt -0.0807, first M for attempting their f'(5), M for correct formula and attempt to substitute, A for awrt 5.20, but accept 5.2	

Question Number	Scheme	Marks
4.	(a) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	B1 **(1)**
	(b) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1 **(1)**
	(c) **R = QP**	B1 **(1)**
	(d) $R = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	M1 A1 cao **(2)**
	(e) Reflection in the y axis	B1 B1 **(2)** **[7]**
Notes	(a) and (b) Signs must be clear for B marks. (c) Accept **QP** or their 2x2 matrices in the correct order only for B1. (d) M for their **QP** where answer involves ±1 and 0 in a 2x2 matrix, A for correct answer only. (e) First B for Reflection, Second B for 'y axis' or 'x=0'. Must be single transformation. Ignore any superfluous information.	

Question Number	Scheme	Marks
5.	(a) $4x^2+9=0 \quad \Rightarrow \quad x=k\text{i}\,, \quad x=\pm\frac{3}{2}\text{i}$ or equivalent	M1, A1
	Solving 3-term quadratic by formula or completion of the square	M1
	$x=\frac{6\pm\sqrt{36-136}}{2}$ or $(x-3)^2-9+34=0$	
	$=3+5\text{i}$ and $3-5\text{i}$	A1 A1ft **(5)**
	(b) 	
	Two roots on imaginary axis	B1ft
	Two roots – one the conjugate of the other	B1ft
	Accept points or vectors	**(2)** **[7]**
Notes	(a) Final A follow through conjugate of their first root. (b) First B award only for first pair imaginary, Second B award only if second pair complex. Complex numbers labelled , scales or coordinates or vectors required for B marks.	

Question Number	Scheme	Marks
6.	(a) Determinant: $2 - 3a = 0$ and solve for $a =$	M1
	So $a = \frac{2}{3}$ or equivalent	A1 **(2)**
	(b) Determinant: $(1\times 2) - (3\times -1) = 5 \quad (\Delta)$	
	$Y^{-1} = \frac{1}{5}\begin{pmatrix} 2 & 1 \\ -3 & 1 \end{pmatrix} \qquad \left[= \begin{pmatrix} 0.4 & 0.2 \\ -0.6 & 0.2 \end{pmatrix} \right]$	M1A1 **(2)**
	(c) $\frac{1}{5}\begin{pmatrix} 2 & 1 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 1-\lambda \\ 7\lambda - 2 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 2-2\lambda+7\lambda-2 \\ -3+3\lambda+7\lambda-2 \end{pmatrix} = \begin{pmatrix} \lambda \\ 2\lambda - 1 \end{pmatrix}$	M1depM1A1 A1 **(4)** **[8]**
	<u>Alternative method for (c)</u>	
	$\begin{pmatrix} 1 & -1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1-\lambda \\ 7\lambda-2 \end{pmatrix}$ so $x - y = 1 - \lambda$ and $3x + 2y = 7\lambda - 2$	M1M1
	Solve to give $x = \lambda$ and $y = 2\lambda - 1$	A1A1
Notes	(b) M for $\frac{1}{\text{their det}}\begin{pmatrix} 2 & 1 \\ -3 & 1 \end{pmatrix}$	
	(c) First M for their $\mathbf{Y}^{-1}\mathbf{B}$ in correct order with **B** written as a 2x1 matrix**,** second M dependent on first for attempt at multiplying their matrices resulting in a 2x1 matrix, first A for λ , second A for $2\lambda - 1$	
	Alternative for (c) First M to obtain two linear equations in x, y, λ Second M for attempting to solve for x or y in terms of λ	

Question Number	Scheme	Marks
7.	(a) $y=\frac{25}{x}$ so $\frac{\mathrm{d}y}{\mathrm{d}x}=-25x^{-2}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{25}{(5p)^2}=-\frac{1}{p^2}$	A1
	$y-\frac{5}{p}=-\frac{1}{p^2}(x-5p) \Rightarrow \quad p^2y+x=10p \qquad (*)$	M1 A1 **(4)**
	(b) $q^2y+x=10q$ only	B1 **(1)**
	(c) $(p^2-q^2)y=10(p-q)$ so $y=\frac{10(p-q)}{(p^2-q^2)}=\frac{10}{p+q}$	M1 A1cso
	$x=10p-p^2\frac{10}{p+q}=\frac{10pq}{p+q}$	M1 A1 cso **(4)**
	(d) Line *PQ* has gradient $\frac{\frac{5}{p}-\frac{5}{q}}{5p-5q}\left(=-\frac{1}{pq}\right)$	M1 A1
	ON has gradient $\frac{\frac{10}{p+q}}{\frac{10pq}{p+q}}\left(=\frac{1}{pq}\right)$ or $\frac{-1}{\frac{-1}{pq}}(=pq)$ could be as unsimplified equivalents seen anywhere	B1
	As these lines are perpendicular $\frac{1}{pq}\times-\frac{1}{pq}=-1$ so $p^2q^2=1$	
	OR for *ON* $y-y_1=m(x-x_1)$ with gradient (equivalent to) pq and sub in points *O* AND *N* to give $p^2q^2=1$	
	OR for *PQ* $y-y_1=m(x-x_1)$ with gradient (equivalent to) -pq and sub in points *P* AND *Q* to give $p^2q^2=1$. NB –pq used as gradient of *PQ* implies first M1A1	M1 A1
		(5) **[14]**

Question Number	Scheme	Marks
	<u>Alternatives for first M1 A1 in part (a)</u> $x\frac{dy}{dx}+y=0\Rightarrow\frac{dy}{dx}=-\frac{y}{x}$ So at P gradient = $\frac{-\frac{5}{p}}{5p}=-\frac{1}{p^2}$ Or $x=5t,\ y=\frac{5}{t}\ \Rightarrow\ \frac{dx}{dt}=5,\ \frac{dy}{dt}=-\frac{5}{t^2}$ so $\frac{dy}{dx}=$ $\frac{-\frac{5}{t^2}}{5}=-\frac{1}{t^2}$ so at P gradient = $-\frac{1}{p^2}$	M1 A1 M1 A1
Notes	(a) First M for attempt at explicit, implicit or parametric differentiation not using p or q as an initial parameter, first A for $\frac{-1}{p^2}$ or equivalent. Quoting gradient award first M0A0. Second M for using $y-y_1=m(x-x_1)$ and attempt to substitute or $y=mx+c$ and attempt to find c; gradient in terms of p only and using $\left(5p,\frac{5}{p}\right)$, second A for correct solution only. (c) First M for eliminating x and reaching $y=\mathrm{f}(p,q)$, second M for eliminating y and reaching $x=\mathrm{f}(p,q)$, both As for given answers. Minimum amount of working given in the main scheme above for 4/4, but do not award accuracy if any errors are made. (d) First M for use of $\frac{y_2-y_1}{x_2-x_1}$ and substituting, first A for $\frac{-1}{pq}$ or unsimplified equivalent . Second M for their product of gradients=-1 (or equating equivalent gradients of ON or equating equivalent gradients of PQ), second A for correct answer only.	

Question Number	Scheme	Marks
8.	(a) If $n=1$, $\sum_{r=1}^{n} r(r+3) = 1\times 4 = 4$ and $\frac{1}{3}n(n+1)(n+5) = \frac{1}{3}\times 1\times 2\times 6 = 4$,	B1
	(so true for $n = 1$. Assume true for $n = k$)	
	So $\sum_{r=1}^{k+1} r(r+3) = \frac{1}{3}k(k+1)(k+5) + (k+1)(k+4)$	M1
	$= \frac{1}{3}(k+1)\left[k(k+5)+3(k+4)\right] = \frac{1}{3}(k+1)\left[k^2+8k+12\right]$	A1
	$= \frac{1}{3}(k+1)(k+2)(k+6)$ which implies is true for $n = k+1$	dA1
	As result is true for $n=1$ this implies true for all positive integers and so result is true by induction	dM1A1cso
		(6)
	(b) $u_1 = 1^2(1-1)+1 = 1$	B1
	(so true for $n=1$. Assume true for $n = k$)	
	$u_{k+1} = k^2(k-1)+1+k(3k+1)$	M1,
	$= k(k^2-k+3k+1)+1 = k(k+1)^2+1$ which implies is true for $n = k + 1$	A1
	As result is true for $n = 1$ this implies true for all positive integers and so result is true by induction	M1A1cso **(5)**
		[11]
Notes	(a) First B for LHS=4 and RHS =4 First M for attempt to use $\sum_{1}^{k} r(r+3) + u_{k+1}$ First A for $\frac{1}{3}(k+1)$, $\frac{1}{3}(k+2)$ or $\frac{1}{3}(k+6)$ as a factor before the final line Second A dependent on first for $\frac{1}{3}(k+1)(k+2)(k+6)$ with no errors seen Second M dependent on first M and for any 3 of 'true for n=1' 'assume true for n=k' 'true for n=k+1', 'true for all n' (or 'true for all positive integers') seen anywhere Third A for correct solution only with all statements and no errors	

	(b) First B for both some working and 1. First M for $u_{k+1} = u_k + k(3k+1)$ and attempt to substitute for u_k First A for $k(k+1)^2 + 1$ with some correct intermediate working and no errors seen Second M dependent on first M and for any 3 of 'true for n=1' 'assume true for n=k' 'true for n=k+1', 'true for all n' (or 'true for all positive integers') seen anywhere Second A for correct solution only with all statements and no errors	

Question Number	Scheme	Marks
9.	(a) $y = 6x^{\frac{1}{2}}$ so $\frac{dy}{dx} = 3x^{-\frac{1}{2}}$ Gradient when x = 4 is $\frac{3}{2}$ and gradient of normal is $-\frac{2}{3}$ So equation of normal is $(y-12) = -\frac{2}{3}(x-4)$ (or $3y+2x=44$) (b) S is at point (9,0) N is at (22,0), found by substituting y=0 into their part (a) Both B marks can be implied or on diagram. So area is $\frac{1}{2}\times 12\times(22-9) = 78$ Alternatives: First M1 for $ky\frac{dy}{dx} = 36$ **or** for $x = 9t^2, y = 18t \rightarrow \frac{dx}{dt} = 18t, \frac{dy}{dt} = 18 \rightarrow \frac{dy}{dx} = \frac{1}{t}$	M1 M1 A1 M1 A1 **(5)** B1 B1ft M1 A1 cao **(4)** **[9]**
Notes	(a) First M for $\frac{\mathrm{d}y}{\mathrm{d}x} = ax^{-\frac{1}{2}}$, Second M for substituting x=4 (or y=12 or t=2/3 if alternative used) into their gradient and applying negative reciprocal. First A for $-\frac{2}{3}$ Third M for $y - y_1 = m(x - x_1)$ or $y = mx + c$ and attempt to substitute a changed gradient AND (4,12) Second A for $3y+2x=44$ or any equivalent equation (b) M for Area=$\frac{1}{2}$ base x height and attempt to substitute including their numerical ‘(22-9)’ or equivalent complete method to find area of triangle *PSN*.	

Mark Scheme (Results)

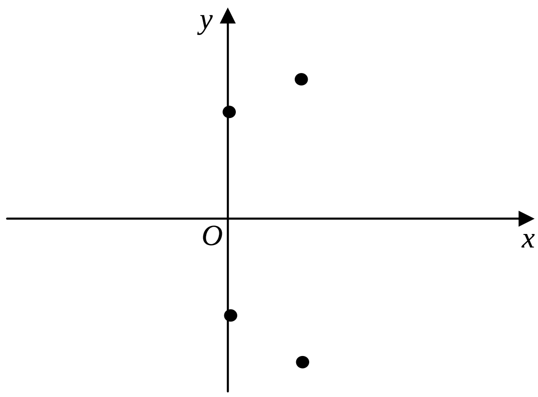
Summer 2013

GCE Further Pure Mathematics 1 (6667/01R)

Question Number	Scheme		Marks
1.	$z = 8 + 3\text{i}, \quad w = -2\text{i}$		
(a)	$z - w \left\{ = (8 + 3\text{i}) - (-2\text{i}) \right\} = 8 + 5\text{i}$	$8 + 5\text{i}$	B1
			[1]
(b)	$zw \left\{ = (8 + 3\text{i})(-2\text{i}) \right\} = 6 - 16\text{i}$	Either the real or imaginary part is correct.	M1
		$6 - 16\text{i}$	A1
			[2]
			3

Question Number	Scheme		Marks
2.	$\mathbf{A} = \begin{pmatrix} 2k+1 & k \\ -3 & -5 \end{pmatrix}$, $\mathbf{B} = \mathbf{A} + 3\mathbf{I}$		
(i)(a)	$\mathbf{B} = \mathbf{A} + 3\mathbf{I} = \begin{pmatrix} 2k+1 & k \\ -3 & -5 \end{pmatrix} + 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	For applying $\mathbf{A} + 3\mathbf{I}$. Can be implied by three out of four correct elements in candidate's final answer. Solution must come from addition.	M1
	$= \begin{pmatrix} 2k+4 & k \\ -3 & -2 \end{pmatrix}$	Correct answer.	A1
			[2]
(b)	$\mathbf{B}$ is singular $\Rightarrow$ $\det \mathbf{B} = 0$.		
	$-2(2k+4) - (-3k) = 0$	Applies "$ad - bc$" to $\mathbf{B}$ and equates to 0	M1
	$-4k - 8 + 3k = 0$		
	$k = -8$	$k = -8$	A1cao
			[2]
(ii)	$\mathbf{C} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} 2 & -1 & 5 \end{pmatrix}$, $\mathbf{E} = \mathbf{C}\mathbf{D}$		
	$\mathbf{E} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \begin{pmatrix} 2 & -1 & 5 \end{pmatrix} = \begin{pmatrix} 4 & -2 & 10 \\ -6 & 3 & -15 \\ 8 & -4 & 20 \end{pmatrix}$	Candidate writes down a 3×3 matrix.	M1
		Correct answer.	A1
			[2] 6

Question Number	Scheme		Marks
3.	$\mathrm{f}(x) = \dfrac{1}{2}x^4 - x^3 + x - 3$		
(a)	$\mathrm{f}(2) = -1$ $\mathrm{f}(2.5) = 3.40625$	Either any one of $\mathrm{f}(2) = -1$ or $\mathrm{f}(2.5) =$ awrt 3.4	M1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore a root α exists between $x = 2$ and $x = 2.5$	both values correct, sign change and conclusion	A1
			[2]
(b)	$\mathrm{f}(2.25) = 0.673828125 \left\{= \frac{345}{512}\right\}$ $\{\Rightarrow 2 \leqslant \alpha \leqslant 2.25\}$	$\mathrm{f}(2.25) =$ awrt 0.7	B1
		Attempt to find $\mathrm{f}(2.125)$	M1
	$\mathrm{f}(2.125) = -0.2752685547...$ $\Rightarrow 2.125 \leqslant \alpha \leqslant 2.25$	$\mathrm{f}(2.125) =$ awrt -0.3 with $2.125 \leqslant \alpha \leqslant 2.25$ or $2.125 < \alpha < 2.25$ or $[2.125, 2.25]$ or $(2.125, 2.25)$.	A1
			[3]
(c)	$\mathrm{f}'(x) = 2x^3 - 3x^2 + 1 \{+0\}$	At least two of the four terms differentiated correctly.	M1
		Correct derivative.	A1
	$\mathrm{f}(-1.5) = 1.40625 \left(= 1\frac{13}{32}\right)$ $\{\mathrm{f}'(-1.5) = -12.5\}$	$\mathrm{f}(-1.5) =$ awrt 1.41	B1
	$\beta_2 = -1.5 - \left(\dfrac{"1.40625"}{"-12.5"}\right)$	Correct application of Newton-Raphson using their values.	M1
	$= -1.3875 \quad \left(= -1\frac{31}{80}\right)$	-1.3875 seen as answer to first iteration, award M1A1B1M1	
	$= -1.39 \; (2\,\mathrm{dp})$	-1.39	A1 **cao**
			[5] **10**

Question Number	Scheme		Marks
4.	$\mathrm{f}(x) = (4x^2 + 9)(x^2 - 2x + 5) = 0$		
(a)	$(4x^2 + 9) = 0 \Rightarrow x = \frac{3\mathrm{i}}{2}, -\frac{3\mathrm{i}}{2}$	An attempt to solve $(4x^2 + 9) = 0$ which involves i.	M1
		$\frac{3\mathrm{i}}{2}, -\frac{3\mathrm{i}}{2}$	A1
	$(x^2 - 2x + 5) = 0 \Rightarrow x = \frac{2 \pm \sqrt{4 - 4(1)(5)}}{2(1)}$	Solves the 3TQ	M1
	$\Rightarrow x = \frac{2 \pm \sqrt{-16}}{2}$		
	$\Rightarrow x = 1 \pm 2\mathrm{i}$	$1 \pm 2\mathrm{i}$	A1 **[4]**
(b)		Any two of their roots plotted correctly on a single diagram, which have been found in part (a).	B1ft
		Both sets of their roots plotted correctly on a single diagram with symmetry about $y = 0$.	B1ft
			[2] 6
	Method mark for solving 3 term quadratic: 1. <u>Factorisation</u> $(x^2 + bx + c) = (x + p)(x + q)$, where $\lvert pq \rvert = \lvert c \rvert$, leading to x = $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $\lvert pq \rvert = \lvert c \rvert$ and $\lvert mn \rvert = \lvert a \rvert$, leading to x = 2. <u>Formula</u> Attempt to use <u>correct</u> formula (with values for *a*, *b* and *c*). 3. <u>Completing the square</u> Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c$, $q \neq 0$, leading to x =...		

Question Number	Scheme		Marks
	Ignore part labels and mark part (a) and part (b) together		
5.	$H: x = 3t,\ y = \frac{3}{t},\quad L: 6y = 4x - 15$		
(a)	$H = L \Rightarrow 6\left(\frac{3}{t}\right) = 4(3t) - 15$	An attempt to substitute $x = 3t$ and $y = \frac{3}{t}$ into L Correct equation in t.	M1 A1
	$\Rightarrow 18 = 12t^2 - 15t \Rightarrow 12t^2 - 15t - 18 = 0$		
	$\Rightarrow 4t^2 - 5t - 6 = 0$ *	Correct solution only, involving at least one intermediate step to given answer.	A1 **cso**
			[3]
(b)	$(t-2)(4t+3)\ \{= 0\}$	A valid attempt at solving the quadratic.	M1
	$\Rightarrow t = 2, -\frac{3}{4}$	Both $t = 2$ and $t = -\frac{3}{4}$	A1
	When $t = 2$, $x = 3(2) = 6,\ y = \frac{3}{2} \Rightarrow \left(6, \frac{3}{2}\right)$	An attempt to use one of their t-values to find one of either x or y.	M1
	When $t = -\frac{3}{4}$,	One set of coordinates correct or both x-values are correct.	A1
	$x = 3\left(-\frac{3}{4}\right) = -\frac{9}{4},\ y = \frac{3}{\left(-\frac{3}{4}\right)} = -4 \Rightarrow \left(-\frac{9}{4}, -4\right)$	Both sets of values correct.	A1
			[5] **8**
(b)	**<u>Alt Method:</u>** An attempt to eliminate either x or y from $xy = 9$ and $6y = 4x - 15$ 1st M1: A full method to obtain a quadratic equation in either x or y. 1st A1: For either $4x^2 - 15x - 54 = 0$ or $6y^2 + 15y - 36 = 0$ or equivalent. 2nd M1: A valid attempt at solving the quadratic. 2nd A1: For either $x = 6, -\frac{9}{4}$ or $y = \frac{3}{2}, -4$ 3rd A1: Both $\left(6, \frac{3}{2}\right)$ and $\left(-\frac{9}{4}, -4\right)$.		

Question Number	Scheme		Marks
6.	$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$		
(a)	$\mathbf{P} = \mathbf{AB} \left\{ = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} \right\}$	$\mathbf{P} = \mathbf{AB}$, seen or implied.	M1
	$\mathbf{P} = \begin{pmatrix} 1 & 4 \\ -2 & -3 \end{pmatrix}$	Correct answer.	A1
			[2]
(b)	$\det \mathbf{P} = 1(-3) - (4)(-2)\ \{= -3 + 8 = 5\}$	Applies "$ad - bc$".	M1
	$\text{Area}(T) = \underline{\dfrac{24}{5}}$ (units)^2	$\dfrac{24}{\text{their } \det \mathbf{P}}$, dependent on previous M	dM1
		$\underline{\dfrac{24}{5}}$ or $\underline{4.8}$	A1ft
			[3]
(c)	$\mathbf{QP} = \mathbf{I} \Rightarrow \mathbf{QPP}^{-1} = \mathbf{IP}^{-1} \Rightarrow \mathbf{Q} = \mathbf{P}^{-1}$		
	$\mathbf{Q} = \mathbf{P}^{-1} = \dfrac{1}{5}\begin{pmatrix} -3 & -4 \\ 2 & 1 \end{pmatrix}$	$\mathbf{Q} = \mathbf{P}^{-1}$ stated or an attempt to find $\mathbf{P}^{-1}$.	M1
		Correct ft inverse matrix.	A1ft
			[2]
			7
	Using **BA,** area is the same in (b) and inverse is $\dfrac{1}{5}\begin{pmatrix} 1 & -2 \\ 4 & -3 \end{pmatrix}$ in (c) and could gain ft marks.		

Question Number	Scheme		Marks
7.	$y^2 = 4ax$, at $P(at^2, 2at)$.		
(a)	$y = 2\sqrt{a}\,x^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{a}\,x^{-\frac{1}{2}}$ or (implicitly) $2y\frac{\mathrm{d}y}{\mathrm{d}x} = 4a$ or (chain rule) $\frac{\mathrm{d}y}{\mathrm{d}x} = 2a \times \frac{1}{2at}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = \pm k\,x^{-\frac{1}{2}}$ or $k\,y\frac{\mathrm{d}y}{\mathrm{d}x} = c$ or $\frac{\text{their } \frac{\mathrm{d}y}{\mathrm{d}t}}{\text{their } \frac{\mathrm{d}x}{\mathrm{d}t}}$	M1
	When $x = at^2$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{a}}{\sqrt{at^2}} = \frac{\sqrt{a}}{\sqrt{a}\,t} = \frac{1}{t}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4a}{2(2at)} = \frac{1}{t}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{t}$	A1
	T: $y - 2at = \frac{1}{t}(x - at^2)$	Applies $y - 2at =$ their $m_T\,(x - at^2)$ **Their m_T must be a function of t from calculus.**	M1
	T: $ty - 2at^2 = x - at^2$		
	T: $ty = x + at^2$	Correct solution.	A1 **cso** ***** **[4]**
(b)	At Q, $x = 0 \Rightarrow y = \frac{at^2}{t} = at \Rightarrow Q(0, at)$	$y = at$ or $Q(0, at)$	B1 **[1]**
(c)	$S(a, 0)$		
	$\mathrm{m}(PQ) = \frac{at - 2at}{0 - at^2} = \frac{-at}{-at^2} = \frac{1}{t}$	A correct method for finding either $\mathrm{m}(PQ)$ or $\mathrm{m}(SQ)$ for their Q or S.	M1
	$\mathrm{m}(SQ) = \frac{at - 0}{0 - a} = \frac{at}{-a} = -t$	$\mathrm{m}(PQ) = \frac{1}{t}$ and $\mathrm{m}(SQ) = -t$	A1
	$\mathrm{m}(PQ) \times \mathrm{m}(SQ) = \frac{1}{t} \times -t = -1 \Rightarrow PQ \perp SQ$	Shows $\mathrm{m}(PQ) \times \mathrm{m}(SQ) = -1$ and conclusion.	A1 **cso** **[3]** **8**

Question Number	Scheme		Marks
8. (a)	$\sum_{r=1}^{n} r(2r-1) = \frac{1}{6}n(n+1)(4n-1)$		
	$n=1$; LHS $= \sum_{r=1}^{1} r(2r-1) = 1$ RHS $= \frac{1}{6}(1)(2)(3) = 1$ As LHS = RHS, the summation formula is true for $n = 1$.	$\frac{1}{6}(1)(2)(3) = 1$ seen	B1
	Assume that the summation formula is true for $n = k$.		
	ie. $\sum_{r=1}^{k} r(2r-1) = \frac{1}{6}k(k+1)(4k-1)$.		
	With $n = k+1$ terms the summation formula becomes:		
	$\sum_{r=1}^{k+1} r(2r-1) = \underline{\frac{1}{6}k(k+1)(4k-1) + (k+1)(2(k+1)-1)}$	$S_{k+1} = S_k + u_{k+1}$ with $S_k = \frac{1}{6}k(k+1)(4k-1)$.	M1
	$= \frac{1}{6}k(k+1)(4k-1) + (k+1)(2k+1)$		
	$= \frac{1}{6}(k+1)\left(k(4k-1) + 6(2k+1)\right)$	Factorise by $\frac{1}{6}(k+1)$	dM1
	$= \frac{1}{6}(k+1)\left(4k^2 + 11k + 6\right)$	$\left(4k^2 + 11k + 6\right)$ or equivalent quadratic seen	A1
	$= \frac{1}{6}(k+1)(k+2)(4k+3)$		
	$= \frac{1}{6}(k+1)(k+1+1)(4(k+1)-1)$	Correct completion to S_{k+1} in terms of $k+1$ dependent on both Ms.	dM1
	If the summation formula is <u>true for $n = k$</u>, then it is shown to be <u>true for $n = k+1$</u>. As the result is <u>true for $n = 1$</u>, it is now also <u>true for all n</u> and $n \in \mathbb{Z}^+$ by mathematical induction.	Conclusion with all 4 underlined elements that can be seen anywhere in the solution	A1 cso
			[6]

Question Number	Scheme		Marks
8. (b)	$\sum_{r=n+1}^{3n} r(2r-1) = S_{3n} - S_n$		
	$= \frac{1}{6}.3n(3n+1)(12n-1) - \frac{1}{6}n(n+1)(4n-1)$	Use of $S_{3n} - S_n$ or $S_{3n} - S_{n+1}$ with the result from (a) used at least once.	M1
		Correct un-simplified expression.	A1
	$= \frac{1}{6}n\{3(3n+1)(12n-1) - (n+1)(4n-1)\}$		
	$= \frac{1}{6}n\{3(36n^2 + 9n - 1) - (4n^2 + 3n - 1)\}$	Factorises out $\frac{1}{6}n$ or $\frac{1}{3}n$ and an attempt to open up the brackets.	dM1
	$= \frac{1}{6}n\{108n^2 + 27n - 3 - 4n^2 - 3n + 1\}$		
	$= \frac{1}{6}n\{104n^2 + 24n - 2\}$		
	$= \frac{1}{3}n(52n^2 + 12n - 1)$	$= \frac{1}{3}n(52n^2 + 12n - 1)$	A1
	$\{a = 52, b = 12, c = -1\}$		**[4]**
			10

Question Number	Scheme		Marks
9.	$w = 10 - 5\mathrm{i}$		
(a)	$\lvert w\rvert = \left\{\sqrt{10^2 + (-5)^2}\right\} = \sqrt{125}$ or $5\sqrt{5}$ or $11.1803...$	$\underline{\sqrt{125}}$ or $\underline{5\sqrt{5}}$ or <u>awrt 11.2</u>	B1
			[1]
(b)	$\arg w = -\tan^{-1}\left(\frac{5}{10}\right)$	Use of $\tan^{-1}$ or $\tan$	M1
	$= -0.463647609... = -0.46$ (2 dp)	awrt -0.46 or awrt 5.82	A1 oe
			[2]
(c)	$(2 + \mathrm{i})(z + 3\mathrm{i}) = w$		
	$z + 3\mathrm{i} = \dfrac{10 - 5\mathrm{i}}{(2 + \mathrm{i})}$	Simplifies to give $* = \dfrac{\text{complex no.}}{(2 + \mathrm{i})}$	B1
	$z + 3\mathrm{i} = \dfrac{(10 - 5\mathrm{i})}{(2 + \mathrm{i})} \times \dfrac{(2 - \mathrm{i})}{(2 - \mathrm{i})}$	Multiplies by $\dfrac{\text{their } (2 - \mathrm{i})}{\text{their } (2 - \mathrm{i})}$	M1
	$z + 3\mathrm{i} = \dfrac{20 - 10\mathrm{i} - 10\mathrm{i} - 5}{1 + 4}$	Simplifies realising that a real number is needed on the denominator and applies $\mathrm{i}^2 = -1$ on their numerator expression and denominator expression.	M1
	$z + 3\mathrm{i} = \dfrac{15 - 20\mathrm{i}}{5}$		
	$z + 3\mathrm{i} = 3 - 4\mathrm{i}$ $z = 3 - 7\mathrm{i}$ (Note: $a = 3, b = -7$.)	$z = 3 - 7\mathrm{i}$	A1
			[4]
(d)	$\arg(\lambda + 9\mathrm{i} + w) = \dfrac{\pi}{4}$ $\lambda + 9\mathrm{i} + w = \lambda + 9\mathrm{i} + 10 - 5\mathrm{i} = (\lambda + 10) + 4\mathrm{i}$		
	$\arg(\lambda + 9\mathrm{i} + w) = \dfrac{\pi}{4} \Rightarrow \lambda + 10 = 4$	Combines real and imaginary parts and puts "Real part = Imaginary part" i.e. $\dfrac{\lambda + 10}{4} = 1$ or $\dfrac{4}{\lambda + 10} = 1$ o.e.	M1
	So, $\lambda = -6$	-6	A1
			[2] **9**
(c)	**<u>Alt 1: Scheme as above:</u>** $(2 + \mathrm{i})z + 6\mathrm{i} + 3\mathrm{i}^2 = 10 - 5\mathrm{i} \Rightarrow (2 + \mathrm{i})z = 13 - 11\mathrm{i}$ B1 for $z = \dfrac{13 - 11\mathrm{i}}{2 + \mathrm{i}}$; M1 for $z = \dfrac{(13 - 11\mathrm{i})}{(2 + \mathrm{i})} \times \dfrac{(2 - \mathrm{i})}{(2 - \mathrm{i})}$; M1 for $z = \dfrac{26 - 13\mathrm{i} - 22\mathrm{i} - 11}{4 + 1}$; A1 for $z = 3 - 7\mathrm{i}$		
(c)	**<u>Alt 2:</u> Let** $z = a + \mathrm{i}b$ gives $(2 + i)(a + \mathrm{i}b + 3\mathrm{i}) = 10 - 5\mathrm{i}$ for B1 Equating real and imaginary parts to form two equations both involving a and b for M1 Solves simultaneous equations as far as a= or b= for M1 a=3, b=-7 or $z = 3 - 7\mathrm{i}$ for A1		

Question Number	Scheme		Marks
10.	$\sum_{r=1}^{24}\left(r^3 - 4r\right)$		
(i)	$= \frac{1}{4}24^2(24+1)^2 - 4.\frac{1}{2}24(24+1)$	An attempt to use at least one of the standard formulae correctly and substitute 24.	M1
	$\{= 90000 - 1200\}$		
	$= 88800$	88800	A1 **cao**
			[2]
(ii)	$\sum_{r=0}^{n}\left(r^2 - 2r + 2n + 1\right)$		
		An attempt to use at least one of the standard formulae correctly.	M1
	$= \underline{\frac{1}{6}n(n+1)(2n+1) - 2.\frac{1}{2}n(n+1)} + 2n(n+1) + (n+1)$	<u>Correct underlined expression.</u>	A1
		$2n \rightarrow 2n(n+1)$	B1
		$1 \rightarrow (n+1)$	B1
	$= \frac{1}{6}(n+1)\left\{2n^2 + n - 6n + 12n + 6\right\}$	An attempt to factorise out $\frac{1}{6}(n+1)$ or $\frac{1}{6}n$.	M1
	$= \frac{1}{6}(n+1)\left\{2n^2 + 7n + 6\right\}$		
	$= \frac{1}{6}(n+1)(n+2)(2n+3)$	Correct answer. (Note: $a = 2, b = 2, c = 3$.)	A1
			[6] **8**

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Mark Scheme (Results)

Summer 2013

GCE Further Pure Mathematics 1 (6667/01)

Question Number	Scheme	Notes	Marks
1.	$\mathbf{M}=\begin{pmatrix} x & x-2 \\ 3x-6 & 4x-11 \end{pmatrix}$		
	det$\mathbf{M}$ = $x(4x-11)-(3x-6)(x-2)$	Correct attempt at determinant	M1
	x^2+x-12 (=0)	Correct 3 term quadratic	A1
	$(x+4)(x-3)$ (= 0) $\rightarrow$ x = ...	Their 3TQ = 0 and attempts to solve relevant quadratic using factorisation or completing the square or correct quadratic formula leading to x =	M1
	$x=-4,\ x=3$	Both values correct	A1
			(4)
			Total 4
Notes			
	$x(4x-11)=(3x-6)(x-2)$ award first M1		
	$\pm(x^2+x-12)$ seen award first M1A1		
	Method mark for solving 3 term quadratic: 1. <u>Factorisation</u> $(x^2+bx+c)=(x+p)(x+q)$, where $\lvert pq\rvert=\lvert c\rvert$, leading to x = $(ax^2+bx+c)=(mx+p)(nx+q)$, where $\lvert pq\rvert=\lvert c\rvert$ and $\lvert mn\rvert=\lvert a\rvert$, leading to x = 2. <u>Formula</u> Attempt to use <u>correct</u> formula (with values for *a*, *b* and *c*). 3. <u>Completing the square</u> Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c,\quad q\neq 0,$ leading to x =...		
	Both correct with no working 4/4, only one correct 0/4		

Question Number	Scheme	Notes	Marks
2	$\mathrm{f}(x) = \cos\left(x^2\right) - x + 3$		
(a)	f(2.5) = 1.499..... f(3) = -0.9111.....	Either any one of f(2.5) = awrt 1.5 or f(3) = awrt -0.91	M1
	Sign change (positive, negative) (and $\mathrm{f}(x)$ is continuous) therefore root or equivalent.	Both f(2.5) = awrt 1.5 and f(3) = awrt -0.91, sign change and conclusion.	A1
	Use of degrees gives f(2.5) = 1.494 and f(3) = 0.988 which is awarded M1A0		**(2)**
(b)	$\dfrac{3-\alpha}{\text{"}0.91113026188\text{"}} = \dfrac{\alpha - 2.5}{\text{"}1.4994494182\text{"}}$	Correct linear interpolation method – accept equivalent equation - ensure signs are correct.	M1 A1ft
	$\alpha = \dfrac{3 \times 1.499... + 2.5 \times 0.9111....}{1.499... + 0.9111....}$		
	$\alpha = 2.81\,(\text{2d.p.})$	cao	A1
			(3)
			Total 5
Notes	Alternative (b)		
	Gradient of line is $-\dfrac{'1.499...' + '0.9111...'}{0.5} (= -4.82)\ (3\text{sf})$. Attempt to find equation of straight line and equate y to 0 award M1 and A1ft for their gradient awrt 3sf.		

Question Number	Scheme	Notes	Marks
3(a)	**Ignore part labels and mark part (a) and part (b) together.**		
	$f\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3-9\left(\frac{1}{2}\right)^2+k\left(\frac{1}{2}\right)-13$	Attempts f(0.5)	M1
	$\left(\frac{1}{4}\right)-\left(\frac{9}{4}\right)+\left(\frac{k}{2}\right)-13=0\Rightarrow k=......$	Sets f(0.5) = 0 **and** leading to k=	dM1
	k = 30	cao	A1
	Alternative using long division:		
	$2x^3-9x^2+kx-13\div(2x-1)$ $=x^2-4x+\frac{1}{2}k-2\text{(Quotient)}$ $\text{Remainder}\ \frac{1}{2}k-15$	Full method to obtain a remainder as a function of k	M1
	$\frac{1}{2}k-15=0$	Their remainder = 0	dM1
	$k=30$		A1
	Alternative by inspection:		
	$(2x-1)(x^2-4x+13)=2x^3-9x^2+30x-13$	First M for $(2x-1)(x^2+bx+c)$ or $(x-\frac{1}{2})(2x^2+bx+c)$ Second M1 for ax^2+bx+c where ($b=-4$ or $c=13$)or ($b=-8$ or $c=26$)	M1dM1
	k = 30		A1
			(3)
(b)	$f(x)=(2x-1)(x^2-4x+13)$ *or* $\left(x-\frac{1}{2}\right)(2x^2-8x+26)$	M1: $(x^2+bx\pm 13)$ or $(2x^2+bx\pm 26)$ Uses inspection or long division or compares coefficients **and** $(2x-1)$ or $\left(x-\frac{1}{2}\right)$ to obtain a quadratic factor of this form.	M1
	$x^2-4x+13$ or $2x^2-8x+26$	A1 $(x^2-4x+13)$ or $(2x^2-8x+26)$ seen	A1
	$x=\frac{4\pm\sqrt{4^2-4\times 13}}{2}$ or equivalent	Use of correct quadratic formula for their 3TQ or completes the square.	M1
	$x=\frac{4\pm 6i}{2}=2\pm 3i$	oe	A1
			(4)
			Total 7

Question Number	Scheme	Notes	Marks
4(a)	$y = \frac{4}{x} = 4x^{-1} \Rightarrow \frac{dy}{dx} = -4x^{-2} = -\frac{4}{x^2}$	$\frac{dy}{dx} = kx^{-2}$	
	$xy = 4 \Rightarrow x\frac{dy}{dx} + y = 0$	Use of the product rule. The sum of two terms including dy/dx, one of which is correct.	M1
	$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = -\frac{2}{t^2}.\frac{1}{2}$	their $\frac{dy}{dt} \times \left(\frac{1}{\text{their } \frac{dx}{dt}}\right)$	
	$\frac{dy}{dx} = -4x^{-2}$ *or* $x\frac{dy}{dx} + y = 0$ or $\frac{dy}{dx} = -\frac{2}{t^2}.\frac{1}{2}$ or equivalent expressions	Correct derivative $-4x^{-2}$, $-\frac{y}{x}$ or $\frac{-1}{t^2}$	A1
	So, $m_N = t^2$	Perpendicular gradient rule $m_N m_T = -1$	M1
	$y - \frac{2}{t} = t^2(x - 2t)$	$y - \frac{2}{t} =$ their $m_N (x - 2t)$ or $y = mx + c$ with their m_N and $(2t, \frac{2}{t})$ in an attempt to find 'c'. **Their gradient of the normal must be different from their gradient of the tangent and have come from calculus and should be a function of *t*.**	M1
	$ty - t^3x = 2 - 2t^4$ *		A1* cso
			(5)
(b)	$t = -\frac{1}{2} \Rightarrow -\frac{1}{2}y - \left(-\frac{1}{2}\right)^3 x = 2 - 2\left(-\frac{1}{2}\right)^4$	Substitutes the given value of t into the normal	M1
	$4y - x + 15 = 0$		
	$y = \frac{4}{x} \Rightarrow x^2 - 15x - 16 = 0$ or $\left(2t, \frac{2}{t}\right) \rightarrow \frac{8}{t} - 2t + 15 = 0 \Rightarrow 2t^2 - 15t - 8 = 0$ or $x = \frac{4}{y} \Rightarrow 4y^2 + 15y - 4 = 0$.	Substitutes to give a quadratic	M1
	$(x+1)(x-16) = 0 \Rightarrow x =$ or $(2t+1)(t-8) = 0 \Rightarrow t =$ or $(4y-1)(y+4) = 0 \Rightarrow y =$	Solves their 3TQ	M1
	$(P: x = -1, y = -4)(Q:) x = 16, y = \frac{1}{4}$	Correct values for x and y	A1
			(4)
			Total 9

Question Number	Scheme	Notes	Marks
5(a)	$(r+2)(r+3)=r^2+5r+6$		B1
	$\sum(r^2+5r+6)=\frac{1}{6}n(n+1)(2n+1)+5\times\frac{1}{2}n(n+1),+6n$	M1: Use of correct expressions for $\sum r^2$ **and** $\sum r$ B1ft: $\sum k=nk$	M1,B1ft
	$=\frac{1}{3}n\left[\frac{1}{2}(n+1)(2n+1)+\frac{15}{2}(n+1)+18\right]$	M1:Factors out n ignoring treatment of constant. A1: Correct expression with $\frac{1}{3}n$ or $\frac{1}{6}n$ factored out, allow recovery.	M1 A1
	$\left(=\frac{1}{3}n\left[n^2+\frac{3}{2}n+\frac{1}{2}+\frac{15}{2}n+\frac{15}{2}+18\right]\right)$ $=\frac{1}{3}n\left[n^2+9n+26\right]$ *	Correct completion to printed answer	A1*cso
			(6)
5(b)	$\sum_{r=n+1}^{3n}=\frac{1}{3}3n\left((3n)^2+9(3n)+26\right)-\frac{1}{3}n\left(n^2+9n+26\right)$	M1: f(**3*n***) – f(*n* or *n*+1) and attempt to use part (a). A1: Equivalent correct expression	M1A1
	3f(***n***) – f(*n* or *n*+1) is M0		
	$(=n\left(9n^2+27n+26\right)-\frac{1}{3}n\left(n^2+9n+26\right))$		
	$=\frac{2}{3}n\left(\frac{27}{2}n^2+\frac{81}{2}n+39-\frac{1}{2}n^2-\frac{9}{2}n-13\right)$	Factors out $=\frac{2}{3}n$ dependent on previous M1	dM1
	$=\frac{2}{3}n\left(13n^2+36n+26\right)$	Accept correct expression.	A1
	$(a=13,b=36,c=26)$		
			(4)
			Total 10

Question Number	Scheme	Notes	Marks
6(a)	$y = 2a^{\frac{1}{2}}x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = a^{\frac{1}{2}}x^{-\frac{1}{2}}$	$x^{\frac{1}{2}} \to x^{-\frac{1}{2}}$	M1
	$y^2 = 4ax \Rightarrow 2y\frac{dy}{dx} = 4a$	$ky\frac{dy}{dx} = c$	
	or $\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = 2a.\frac{1}{2ap}$	$\frac{dy}{dt} \times \frac{1}{\frac{dx}{dt}}$. Can be a function of *p or t*.	
	$\frac{dy}{dx} = a^{\frac{1}{2}}x^{-\frac{1}{2}}$ or $2y\frac{dy}{dx} = 4a$ or $\frac{dy}{dx} = 2a.\frac{1}{2ap}$	Differentiation is accurate.	A1
	$y - 2ap = \frac{1}{p}(x - ap^2)$	Applies $y - 2ap =$ their $m\left(x - ap^2\right)$ or $y = (\text{their } m)x + c$ using $x = ap^2$ *and* $y = 2ap$ in an attempt to find c. **Their *m* must be a function of *p* from calculus.**	M1
	$py - x = ap^2$ *	Correct completion to printed answer*	A1 cso
			(4)
(b)	$qy - x = aq^2$		B1
			(1)
(c)	$qy - aq^2 = py - ap^2$	Attempt to obtain an **equation** in one variable *x* or *y*	M1
	$y\left(q - p\right) = aq^2 - ap^2$ $y = \frac{aq^2 - ap^2}{q - p}$	Attempt to isolate *x* or *y*	M1
	$y = a(p+q)$ *or* $ap + aq$ $x = apq$	A1: Either one correct simplified coordinate A1: Both correct simplified coordinates	A1,A1
	$(R(apq, ap + aq))$		
			(4)
(d)	$'apq' = -a$	Their *x* coordinate of $R = -a$	M1
	$pq = -1$	**Answer only**: Scores 2/2 if *x* coordinate of *R* is *apq* otherwise 0/2.	A1
			(2)
			Total 11

Question Number	Scheme	Notes	Marks
7	$z_1 = 2+3\text{i}, \quad z_2 = 3+2\text{i}$		
(a)	$z_1 + z_2 = 5+5\text{i} \Rightarrow \lvert z_1+z_2 \rvert = \sqrt{5^2+5^2}$	Adds z_1 and z_2 and correct use of Pythagoras. i under square root award M0.	M1
	$\sqrt{50}\ (=5\sqrt{2})$		A1 cao
			(2)
(b)	$\dfrac{z_1 z_3}{z_2} = \dfrac{(2+3\text{i})(a+b\text{i})}{3+2\text{i}}$ $= \dfrac{(2+3\text{i})(a+b\text{i})(3-2i)}{(3+2\text{i})(3-2\text{i})}$	Substitutes for z_1, z_2 and z_3 and multiplies by $\dfrac{3-2\text{i}}{3-2\text{i}}$	M1
	$(3+2\text{i})(3-2\text{i}) = 13$	13 seen.	B1
	$\dfrac{z_1 z_3}{z_2} = \dfrac{(12a-5b)+(5a+12b)\text{i}}{13}$	M1: Obtains a numerator with 2 real and 2 imaginary parts. A1: As stated or $\dfrac{(12a-5b)}{13} + \dfrac{(5a+12b)}{13}\text{i}$ ONLY.	dM1A1
			(4)
(c)	$12a - 5b = 17$ $5a + 12b = -7$	Compares real and imaginary parts to obtain 2 equations which both involve *a* and *b*. Condone sign errors only.	M1
	$\begin{array}{l} 60a - 25b = 85 \\ 60a + 144b = -84 \end{array} \Rightarrow b = -1$	Solves as far as $a =$ or $b =$	dM1
	$a = 1,\ b = -1$	Both correct	A1
		Correct answers with no working award 3/3.	
			(3)
(d)	$\arg(w) = -\tan^{-1}\left(\dfrac{7}{17}\right)$	Accept use of $\pm\tan^{-1}$ or $\pm\tan$. awrt ± 0.391 or ± 5.89 implies M1.	M1
	=awrt -0.391 or awrt 5.89		A1
			(2)
			Total 11

Question Number	Scheme	Notes	Marks
8(a)	$\mathbf{A}^2 = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix} = \begin{pmatrix} 44 & -14 \\ -28 & 9 \end{pmatrix}$	M1:Attempt both $\mathbf{A}^2$ and $7\mathbf{A} + 2\mathbf{I}$	M1A1
	$7\mathbf{A} + 2\mathbf{I} = \begin{pmatrix} 42 & -14 \\ -28 & 7 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 44 & -14 \\ -28 & 9 \end{pmatrix}$	A1: Both matrices correct	
	OR $\mathbf{A}^2 - 7\mathbf{A} = \mathbf{A}(\mathbf{A} - 7\mathbf{I})$	M1 for expression and attempt to substitute and multiply (2x2)(2x2)=2x2	
	$\mathbf{A}(\mathbf{A} - 7\mathbf{I}) = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} -1 & -2 \\ -4 & -6 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2\mathbf{I}$	A1 cso	
			(2)
(b)	$\mathbf{A}^2 = 7\mathbf{A} + 2\mathbf{I} \Rightarrow \mathbf{A} = 7\mathbf{I} + 2\mathbf{A}^{-1}$	Require one correct line using accurate expressions involving $\mathbf{A}^{-1}$ and identity matrix to be clearly stated as $\mathbf{I}$.	M1
	$\mathbf{A}^{-1} = \frac{1}{2}(\mathbf{A} - 7\mathbf{I})$*		A1* cso
	Numerical approach award 0/2.		
			(2)
(c)	$\mathbf{A}^{-1} = \frac{1}{2}\begin{pmatrix} -1 & -2 \\ -4 & -6 \end{pmatrix}$	Correct inverse matrix or equivalent	B1
	$\frac{1}{2}\begin{pmatrix} -1 & -2 \\ -4 & -6 \end{pmatrix}\begin{pmatrix} 2k+8 \\ -2k-5 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -2k-8+4k+10 \\ -8k-32+12k+30 \end{pmatrix}$	Matrix multiplication involving their inverse and k: (2x2)(2x1)=2x1. N.B. $\begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} 2k+8 \\ -2k-5 \end{pmatrix}$ is M0	M1
	$\begin{pmatrix} k+1 \\ 2k-1 \end{pmatrix}$ or $(k+1,\ 2k-1)$	$(k+1)$ first A1, $(2k-1)$ second A1	A1,A1
	Or:		
	$\begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2k+8 \\ -2k-5 \end{pmatrix}$	Correct matrix equation.	B1
	$6x - 2y = 2k + 8$ $-4x + y = -2k - 5 \Rightarrow x = ... or\ y = ...$	Multiply out and attempt to solve simultaneous equations for x or y in terms of k.	M1
	$\begin{pmatrix} k+1 \\ 2k-1 \end{pmatrix}$ or $(k+1,\ 2k-1)$	$(k+1)$ first A1, $(2k-1)$ second A1	A1,A1
			(4)
			Total 8

Question Number	Scheme	Notes	Marks
9(a)	$u_1 = 8$ given $n = 1 \Rightarrow u_1 = 4^1 + 3(1) + 1 = 8 \quad (\therefore \text{ true for } n = 1)$	$4^1 + 3(1) + 1 = 8$ seen	B1
	Assume true for $n = k$ so that $u_k = 4^k + 3k + 1$		
	$u_{k+1} = 4\left(4^k + 3k + 1\right) - 9k$	Substitute u_k into u_{k+1} as $u_{k+1} = 4u_k - 9k$	M1
	$= 4^{k+1} + 12k + 4 - 9k = 4^{k+1} + 3k + 4$	Expression of the form $4^{k+1} + ak + b$	A1
	$= 4^{k+1} + 3(k+1) + 1$	Correct completion to an expression in terms of $k + 1$	A1
	If <u>true for $n = k$</u> then <u>true for $n = k + 1$</u> and as <u>true for $n = 1$</u> <u>true for all n</u>	Conclusion with all 4 underlined elements that can be seen anywhere in the solution; n defined incorrectly award A0.	A1 cso
			(5)
(b)	**Condone use of *n* here.**		
	$lhs = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^1 = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$ $rhs = \begin{pmatrix} 2(1)+1 & -4(1) \\ 1 & 1-2(1) \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$	Shows true for $m = 1$	B1
	Assume $\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^k = \begin{pmatrix} 2k+1 & -4k \\ k & 1-2k \end{pmatrix}$		
	$\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k+1 & -4k \\ k & 1-2k \end{pmatrix}\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$	$\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 2k+1 & -4k \\ k & 1-2k \end{pmatrix}$ award M1	M1
	$= \begin{pmatrix} 6k+3-4k & -8k-4+4k \\ 3k+1-2k & -4k-1+2k \end{pmatrix}$	Or equivalent 2x2 matrix. $\begin{pmatrix} 6k+3-4k & -12k-4+8k \\ 2k+1-k & -4k-1+2k \end{pmatrix}$ award A1from above.	A1
	$= \left(\begin{pmatrix} 2k+3 & -4k-4 \\ k+1 & -2k-1 \end{pmatrix}\right)$		
	$= \begin{pmatrix} 2(k+1)+1 & -4(k+1) \\ k+1 & 1-2(k+1) \end{pmatrix}$	Correct completion to a matrix in terms of $k + 1$	A1
	If <u>true for $m = k$</u> then <u>true for $m = k + 1$</u> and as <u>true for $m = 1$</u> true <u>for all m</u>	Conclusion with all 4 underlined elements that can be seen anywhere in the solution; m defined incorrectly award A0.	A1 cso
			(5)
			Total 10

Mark Scheme (Results)

January 2014

Pearson Edexcel International Advanced Level

Further Pure Mathematics 1 (6667A/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2+bx+c)=(x+p)(x+q),\text{ where } |pq|=|c|,\text{ leading to x = }$$

$$(ax^2+bx+c)=(mx+p)(nx+q),\text{ where } |pq|=|c| \text{ and } |mn|=|a|,\text{ leading to x = }$$

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c,\quad q\neq 0,$ leading to x =...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Notes	Marks
1.	$\mathrm{f}(x) = 2x - 5\cos x$, x measured in radians		
(a)	$\mathrm{f}(1) = -0.7015115293...$ $\mathrm{f}(1.4) = 1.950164285...$	Either any one of $\mathrm{f}(1) =$ awrt -0.7 or $\mathrm{f}(1.4) = 1.9$ or awrt 2.0	M1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore a root α exists between $x = 1$ and $x = 1.4$	both values correct, sign change and conclusion	A1
			[2]
(b)	$\mathrm{f}(1.2) = 0.5882112276...$ $\{\Rightarrow 1 \le \alpha \le 1.2\}$	$\mathrm{f}(1.2) =$ awrt 0.6	B1
		Attempt to find $\mathrm{f}(1.1)$	M1
	$\mathrm{f}(1.1) = -0.06798060713...$ $\Rightarrow 1.1 \le \alpha \le 1.2$	$\mathrm{f}(1.1) = -0.06$ or awrt -0.07 with $1.1 \le \alpha \le 1.2$ or $1.1 < \alpha < 1.2$ or $[1.1, 1.2]$ or $(1.1, 1.2)$.	A1
			[3]
			5

Question Number	Scheme	Notes	Marks
2.	$\mathbf{A} = \begin{pmatrix} -4 & 10 \\ -3 & k \end{pmatrix}$, where k is a constant		
(i)	$\det \mathbf{A} = (-4)(k) - (-3)(10)$	Applies "$ad \pm bc$" to $\mathbf{A}$	M1
	$\Rightarrow -4k + 30 = 2$ or $-4k + 30 = -2$	Equates their $\det \mathbf{A}$ to either 2 or -2	dM1
	$\Rightarrow k = 7$ or $k = 8$	Either $k = 8$ or $k = 7$	A1
		Both $k = 8$ and $k = 7$	A1
			[4]
(ii)	$\mathbf{B} = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 5 & 1 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 2 & 8 \\ 0 & 2 \\ 1 & -2 \end{pmatrix}$		
	$\mathbf{BC} = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 5 & 1 \end{pmatrix}\begin{pmatrix} 2 & 8 \\ 0 & 2 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 5 & -2 \\ -3 & -8 \end{pmatrix}$	Writes down a complete 2×2 matrix.	M1
		Any 3 out of 4 elements correct	A1
		Correct answer.	A1
			[3]
			7

Question Number	Scheme	Notes	Marks
3.	$x = 2t,\ y = \frac{2}{t},\ t \neq 0$		
	$t = \frac{1}{2} \Rightarrow P(1, 4), \quad t = 4 \Rightarrow Q\left(8, \frac{1}{2}\right)$	Coordinates for either P or Q are correctly stated. (Can be implied).	B1
	$m(PQ) = \frac{\frac{1}{2} - 4}{8 - 1}\ \left\{= -\frac{1}{2}\right\}$	An attempt to find the gradient of the chord PQ.	M1
	$m(L) = 2$	Applying $m(L) = \frac{-1}{\text{their } m(PQ)}$	M1
	So, $L : y = 2x$	$y = 2x$	A1 oe
			[4] **4**

Question Number	Scheme	Notes	Marks
4.	$\mathrm{f}(x) = 2\sqrt{x} - \dfrac{6}{x^2} - 3, \quad x > 0$		
	$\mathrm{f}'(x) = x^{-\frac{1}{2}} + 12x^{-3}\ \{+0\}$	$\pm\lambda x^{-\frac{1}{2}}$ or $\pm\mu x^{-3}$	M1
		Correct differentiation	A1
	$\mathrm{f}(3.5) = 0.2518614684...$	$\mathrm{f}(3.5) = $ awrt 0.25	B1
	$\{\mathrm{f}'(3.5) = 0.8144058657...\}$		
	$\beta = 3.5 - \left(\dfrac{"0.2518614684..."}{"0.8144058657..."}\right)$	Correct application of Newton-Raphson using their values.	M1
	$= 3.190742075...$		
	$= 3.191\ (3\text{dp})$	3.191	A1 **cao**
			[5]
			5

Question Number	Scheme	Notes	Marks
5.	$z = 5 + \mathrm{i}\sqrt{3}, \quad w = \sqrt{3} - \mathrm{i}$		
(a)	$\lvert w \rvert = \left\{\sqrt{\left(\sqrt{3}\right)^2 + (-1)^2}\right\} = 2$	2	B1
			[1]
(b)	$zw = \left(5 + \mathrm{i}\sqrt{3}\right)\left(\sqrt{3} - \mathrm{i}\right)$		
	$= 5\sqrt{3} - 5\mathrm{i} + 3\mathrm{i} + \sqrt{3}$		
	$= 6\sqrt{3} - 2\mathrm{i}$	Either the real or imaginary part is correct.	M1
		$6\sqrt{3} - 2\mathrm{i}$	A1
			[2]
(c)	$\frac{z}{w} = \frac{\left(5 + \mathrm{i}\sqrt{3}\right)}{\left(\sqrt{3} - \mathrm{i}\right)} \times \frac{\left(\sqrt{3} + \mathrm{i}\right)}{\left(\sqrt{3} + \mathrm{i}\right)}$	Multiplies by $\frac{\left(\sqrt{3} + \mathrm{i}\right)}{\left(\sqrt{3} + \mathrm{i}\right)}$	M1
	$= \frac{5\sqrt{3} + 5\mathrm{i} + 3\mathrm{i} - \sqrt{3}}{3 + 1}$	Simplifies realising that a real number is needed on the denominator and applies $\mathrm{i}^2 = -1$ on their numerator expression and denominator expression.	M1
	$\left\{= \frac{4\sqrt{3} + 8\mathrm{i}}{4}\right\} = \sqrt{3} + 2\mathrm{i}$	$\sqrt{3} + 2\mathrm{i}$	A1
			[3]
(d)	$z + \lambda = 5 + \mathrm{i}\sqrt{3} + \lambda = (5 + \lambda) + \mathrm{i}\sqrt{3}$		
	$\left\{\arg(z + \lambda) = \frac{\pi}{3} \Rightarrow\right\} \quad \frac{\sqrt{3}}{5 + \lambda} = \tan\left(\frac{\pi}{3}\right)$	$\frac{\sqrt{3}}{\text{their combined real part}} = \tan\left(\frac{\pi}{3}\right)$	M1 oe
	$\left\{\frac{\sqrt{3}}{5 + \lambda} = \frac{\sqrt{3}}{1} \Rightarrow 5 + \lambda = 1 \Rightarrow\right\} \lambda = -4$	-4	A1
			[2] **8**

Question Number	Scheme	Notes	Marks
6. (a)	$\sum_{r=1}^{n} r(r+1)(r-1) = \sum_{r=1}^{n} (r^3 - r)$		
	$= \frac{1}{4}n^2(n+1)^2 - \frac{1}{2}n(n+1)$	An attempt to use at least one of the standard formulae correctly.	M1
		Correct expression.	A1
	$= \frac{1}{4}n(n+1)\left(n(n+1) - 2\right)$	An attempt to factorise out at least $n(n+1)$.	M1
	$= \frac{1}{4}n(n+1)\left(n^2 + n - 2\right)$		
	$= \frac{1}{4}n(n+1)(n-1)(n+2)$	Achieves the correct answer. (Note: $a = 2$).	A1
			[4]
	$\sum_{r=1}^{n} r(r+1)(r-1) = 10\sum_{r=1}^{n} r^2$		
(b)	$\frac{1}{4}n(n+1)(n-1)(n+2) = \frac{10}{6}n(n+1)(2n+1)$	Sets their part (a) $= \frac{10}{6}n(n+1)(2n+1)$	M1
	$\frac{1}{4}(n-1)(n+2) = \frac{5}{3}(2n+1)$		
	$3(n^2 + n - 2) = 20(2n+1)$		
	$3n^2 - 37n - 26 = 0$	Manipulates to a "3TQ = 0".	M1
		$3n^2 - 37n - 26 = 0$	A1
	$(3n+2)(n-13) = 0$	A valid method for factorising a 3TQ.	M1
	$n = 13$	Only one solution of $n = 13$	A1
			[5]
			9

Question Number	Scheme	Notes	Marks
7.	$\mathbf{P} = \begin{pmatrix} 3a & -2a \\ -b & 2b \end{pmatrix}$, $\mathbf{M} = \begin{pmatrix} -6a & 7a \\ 2b & -b \end{pmatrix}$		
(a)	$\mathbf{P}^{-1} = \frac{1}{4ab};\begin{pmatrix} 2b & 2a \\ b & 3a \end{pmatrix}$	$\frac{1}{4ab}$	B1;
		Two out of four elements correct.	M1
		Correct matrix.	A1
			[3]
	$\mathbf{M} = \mathbf{PQ}$		
(b)	$\Rightarrow \mathbf{P}^{-1}\mathbf{M} = \mathbf{P}^{-1}\mathbf{PQ} \Rightarrow \mathbf{Q} = \mathbf{P}^{-1}\mathbf{M}$		
	$\mathbf{Q} = \frac{1}{4ab}\begin{pmatrix} 2b & 2a \\ b & 3a \end{pmatrix}\begin{pmatrix} -6a & 7a \\ 2b & -b \end{pmatrix}$	Multiples their $\mathbf{P}^{-1}$ by $\mathbf{M}$	M1
	$= \frac{1}{4ab}\begin{pmatrix} -8ab & 12ab \\ 0 & 4ab \end{pmatrix}$		
	$= \begin{pmatrix} -2 & 3 \\ 0 & 1 \end{pmatrix}$	Two out of four elements correct.	A1
		Correct matrix.	A1
			[3]
			6

Question Number	Scheme	Notes	Marks
8.	$y^2 = 4ax$, at $P\left(ap^2, 2ap\right)$.		
(a)	$y = 2\sqrt{a}\,x^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{a}\,x^{-\frac{1}{2}}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = \pm k x^{-\frac{1}{2}}$	
	or (implicitly) $2y\frac{\mathrm{d}y}{\mathrm{d}x} = 4a$	or $ky\frac{\mathrm{d}y}{\mathrm{d}x} = c$	M1
	or (chain rule) $\frac{\mathrm{d}y}{\mathrm{d}x} = 2a \times \frac{1}{2ap}$	or $\frac{\text{their } \frac{\mathrm{d}y}{\mathrm{d}t}}{\text{their } \frac{\mathrm{d}x}{\mathrm{d}t}}$	
	When $x = ap^2$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{a}}{\sqrt{ap^2}} = \frac{\sqrt{a}}{\sqrt{a}\,p} = \frac{1}{p}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4a}{2(2ap)} = \frac{1}{p}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{p}$	A1
	So $m_N = -p$	Applies $m_N = \frac{-1}{\textit{their } m_T}$	M1
	N: $y - 2ap = -p\left(x - ap^2\right)$	Applies $y - 2ap = \left(\text{their } m_N\right)\left(x - ap^2\right)$	M1
	N: $y - 2ap = -px + ap^3$		
	N: $y + px = ap^3 + 2ap$	Correct solution.	A1 **cso** *
			[5]
(b)	$(6a, 0) \Rightarrow 0 + p(6a) = ap^3 + 2ap$	Substitutes $x = 6a$, $y = 0$ into **N**	M1
	$\Rightarrow 4ap = ap^3 \Rightarrow p = 2$	$p = 2$	A1
	$x = -a$, $p = 2 \Rightarrow y + 2(-a) = a(2)^3 + 2a(2)$	Substitutes $x = -a$ and their p into **N**	dM1
	$\Rightarrow y = 8a + 4a + 2a = 14a \Rightarrow D(-a, 14a)$	$D(-a, 14a)$	A1
			[4]
(c)	When $p = 2$, $x = a(2)^2 = 4a$	Substitutes their p into $x = ap^2$	M1
	$\text{Area}(XPD) = \frac{1}{2}(14a)(5a) = 35a^2$	Applies $\frac{1}{2}(\text{their } 14a)(\text{their "}4a\text{"} + a)$	M1
		$35a^2$	A1
			[3]
			12

Question Number	Scheme	Notes	Marks
9.	$(3-\mathrm{i})z^* + 2\mathrm{i}z = 9 - \mathrm{i}$		
	$(3-\mathrm{i})(x-\mathrm{i}y) + 2\mathrm{i}(x+\mathrm{i}y) = 9 - \mathrm{i}$	Substituting $z = x + \mathrm{i}y$ and $z^* = x - \mathrm{i}y$ into $(3-\mathrm{i})z^* + 2\mathrm{i}z = 9 - \mathrm{i}$	M1
	$\underline{3x - 3\mathrm{i}y - \mathrm{i}x - y} + 2\mathrm{i}x - 2y \; \{= 9 - \mathrm{i}\}$	Multiplies out $\underline{(3-\mathrm{i})(x-\mathrm{i}y)}$ correctly. This mark can be implied by correct later working.	A1
	Re part: $3x - y - 2y = 9$	Equating either real or imaginary parts.	M1
	Im part: $-3y - x + 2x = -1$	One set of correct equations.	A1
		Correct equations.	A1
	$3x - 3y = 9$ $x - 3y = -1$		
	$2x = 10 \Rightarrow x = 5$	Attempt to solve simultaneous equations to find one of x or y.	ddM1
	$x - 3y = -1 \Rightarrow 5 - 3y = -1 \Rightarrow y = 2$	Either $x = 5$ or $y = 2$.	A1
		Both $x = 5$ and $y = 2$.	A1
	$\{z = 5 + 2\mathrm{i}\}$		**[8]**
			8

Question Number	Scheme	Notes	Marks
10. (i)	$u_{n+1} = 5u_n + 3$, $u_1 = 3$ and $u_n = \frac{3}{4}(5^n - 1)$		
	$n = 1$; $u_1 = \frac{3}{4}(5^1 - 1) = \frac{3}{4}(4) = 3$ So u_n is true when $n = 1$.	Check that $u_n = \frac{3}{4}(5^n - 1)$ yields 3 when $n = 1$.	B1
	Assume that for $n = k$ that, $u_k = \frac{3}{4}(5^k - 1)$ is true for $k \in \mathbb{Z}^+$.		
	Then $u_{k+1} = 5u_k + 3$		
	$= 5\left(\frac{3}{4}(5^k - 1)\right) + 3$	Substituting $u_k = \frac{3}{4}(5^k - 1)$ into $u_{k+1} = 5u_k + 3$	M1
	$= \frac{3}{4}(5)^{k+1} - \frac{15}{4} + 3$	An attempt to multiply out in order to achieve $\pm\lambda(5^{k+1}) \pm$ constant	M1
	$= \frac{3}{4}(5)^{k+1} - \frac{3}{4}$		
	$= \frac{3}{4}(5^{k+1} - 1)$	$\frac{3}{4}(5^{k+1} - 1)$	A1
	Therefore, the general statement, $u_n = \frac{3}{4}(5^n - 1)$ is true when $n = k+1$. (As u_n is true for $n = 1$,) then u_n is true for all positive integers by mathematical induction	True when $n = k+1$, then by induction the result is true for all positive integers.	A1
			[5]

Question Number	Scheme	Notes	Marks
	$f(n) = 5\left(5^n\right) - 4n - 5$ is divisible by 16		
10. (ii)	$f(1) = 5(5^1) - 4(1) - 5 = 16$, {which is divisible by 16}. {$\therefore$ f (n) is divisible by 16 when $n = 1$.}	Shows that $f(1) = 16$	B1
	Assume that for $n = k$, $f(k) = 5\left(5^k\right) - 4k - 5$ is divisible by 16 for $k \in \mathbb{Z}^+$.		
	$f(k+1) - f(k) = 5\left(5^{k+1}\right) - 4(k+1) - 5 - \left(5\left(5^k\right) - 4k - 5\right)$	Applies $f(k+1) - f(k)$.	M1
		Correct expression for $f(k+1) - f(k)$.	A1
	$= 5\left(5^{k+1}\right) - 4k - 4 - 5 - 5\left(5^k\right) + 4k + 5$		
	$= 25\left(5^k\right) - 4k - 4 - 5 - 5\left(5^k\right) + 4k + 5$	Achieves an expression in 5^k.	M1
	$= 20\left(5^k\right) - 4$		
	$= 4\left(5\left(5^k\right) - 4k - 5\right) + 16k + 20 - 4$		
	$= 4\left(5\left(5^k\right) - 4k - 5\right) + 16k + 16$		
	$= 4f(k) + 16(k+1)$		
	$\therefore f(k+1) = 5f(k) + 16(k+1)$	$f(k+1) = 5f(k) + 16(k+1)$	A1
	{$\therefore f(k+1) = 5f(k) + 16(k+1)$, which is divisible by 16 as both $5f(k)$ and $16(k+1)$ are both divisible by 16.}		
	If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all n.	Correct conclusion	A1 cso
			[6] **11**

Appendix

- dM1 denotes a method mark which is dependent upon the award of the previous method mark.
- ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.
- ft denotes "follow through"
- cao denotes "correct answer only"
- oe denotes "or equivalent"

Other Possible Solutions

Question Number	Scheme	Notes	Marks
7. ***Aliter*** (b) **Way 2**	$\mathbf{P} = \begin{pmatrix} 3a & -2a \\ -b & 2b \end{pmatrix}$, $\mathbf{M} = \begin{pmatrix} -6a & 7a \\ 2b & -b \end{pmatrix}$ $\mathbf{M} = \mathbf{PQ}$ $\begin{pmatrix} -6a & 7a \\ 2b & -b \end{pmatrix} = \begin{pmatrix} 3a & -2a \\ -b & 2b \end{pmatrix}\begin{pmatrix} q_1 & q_2 \\ q_3 & q_4 \end{pmatrix}$ $-6 = 3q_1 - 2q_3 \quad 7 = 3q_2 - 2q_4$ $2 = -q_1 + 2q_3 \quad , \quad -1 = -q_2 + 2q_4$ $= \begin{pmatrix} -2 & 3 \\ 0 & 1 \end{pmatrix}$	Writes down a relevant pair of simultaneous equations. Can be implied by later working. Two out of four elements correct. Correct matrix.	M1 A1 A1 **[3]**

Question Number	Scheme	Notes	Marks
Aliter **10.** (ii) **Way 2**	$\text{f}(n) = 5\left(5^n\right) - 4n - 5$ is divisible by 16		
	$\text{f}(1) = 5(5^1) - 4(1) - 5 = 16$, {which is divisible by 16}. $\{\therefore \text{f}(n)$ is divisible by 16 when $n = 1.\}$	Shows that $\text{f}(1) = 16$	B1
	Assume that for $n = k$, $\text{f}(k) = 5\left(5^k\right) - 4k - 5$ is divisible by 16 for $k \in \mathbb{Z}^+$.		
	$\text{f}(k+1) = 5\left(5^{k+1}\right) - 4(k+1) - 5$	Applies $\text{f}(k+1)$. Correct expression for $\text{f}(k+1)$.	M1 A1
	$= 25\left(5^k\right) - 4k - 9$	Achieves an expression in 5^k.	M1
	$= 5\left(5\left(5^k\right) - 4k - 5\right) + 20k + 25 - 4k - 9$		
	$= 5\left(5\left(5^k\right) - 4k - 5\right) + 16(k+1)$		
	$\therefore \text{f}(k+1) = 5\text{f}(k) + 16\left(k+1\right)$	$\text{f}(k+1) = 5\text{f}(k) + 16\left(k+1\right)$	A1
	$\{\therefore \text{f}(k+1) = 5\text{f}(k) + 16\left(k+1\right)$, which is divisible by 16 as both $5\text{f}(k)$ and $16\left(k+1\right)$ are both divisible by 16.}		
	If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all n.	Correct conclusion	A1 cso
			[6]

Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Further Pure Mathematics FP1
(6667/01)

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Summer 2014
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q),\ \text{where}\ |pq|=|c|$, leading to x = ...

$(ax^2+bx+c)=(mx+p)(nx+q),\ \text{where}\ |pq|=|c|\ \text{and}\ |mn|=|a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2 \pm q \pm c = 0,\quad q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

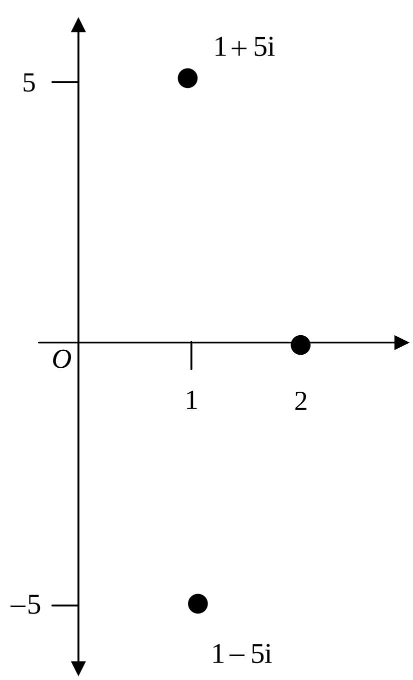
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Notes	Marks
1.(a)	$\frac{z_1}{z_2}=\frac{p+2i}{1-2i}.\frac{1+2i}{1+2i}$	Multiplying top and bottom by conjugate	M1
	$=\frac{p+2pi+2i-4}{5}$	At least 3 correct terms in the numerator, evidence that i^2 = -1 and denominator real.	M1
	$=\frac{p-4}{5}, \quad +\frac{2p+2}{5}\mathrm{i}$	Real + imaginary with i factored out. Accept single denominator with numerator in correct form. Accept 'a= ' and 'b='.	A1, A1
			(4)
(b)	$\left\|\frac{z_1}{z_2}\right\|^2=\left(\frac{p-4}{5}\right)^2+\left(\frac{2p+2}{5}\right)^2$	Accept their answers to part (a). Any erroneous i or i^2 award M0	M1
	$\left(\frac{p-4}{5}\right)^2+\left(\frac{2p+2}{5}\right)^2=13^2$ *or* $\sqrt{\left(\frac{p-4}{5}\right)^2+\left(\frac{2p+2}{5}\right)^2}=13$	$\left\|\frac{z_1}{z_2}\right\|^2=13^2$ or $\left\|\frac{z_1}{z_2}\right\|=13$	dM1
	$\frac{p^2-8p+16}{25}+\frac{4p^2+8p+4}{25}=169 \text{ or } 13^2$		
	$5p^2+20=4225$		
	$p^2=841\Rightarrow p=\pm 29$	dM1:Attempt to solve their quadratic in *p*, dependent on both previous Ms. A1:**both** 29 **and** -29	dM1A1
	OR		
	$\frac{\|z_1\|}{\|z_2\|}=\frac{\sqrt{p^2+4}}{\sqrt{5}}$	Finding moduli Any erroneous i or i^2 award M0	M1
	$\frac{\sqrt{p^2+4}}{\sqrt{5}}=13$ oe	Equating to 13	dM1
	$\frac{p^2+4}{5}=169 \text{ or } 13^2$ oe		
	$p^2=841\Rightarrow p=\pm 29$	dM1:Attempt to solve their quadratic in *p*, dependent on both previous Ms	dM1A1
		A1:**both** 29 **and** -29	
			(4)
			Total 8

Question Number	Scheme	Notes	Marks
2.	$\mathrm{f}(x) = x^3 - \dfrac{5}{2x^{\frac{3}{2}}} + 2x - 3$		
(a)	$\mathrm{f}(1.1) = -1.6359604,$ $\mathrm{f}(1.5) = 2.0141723$	Attempts to evaluate both $\mathrm{f}(1.1)$ and $\mathrm{f}(1.5)$ and evaluates at least one of them correctly to awrt (or trunc.) 2 sf.	M1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore **a root / α** is between $x = 1.1$ and $x = 1.5$	Both values correct to awrt (or trunc.) 2 sf, sign change (or a statement which implies this e.g. -1.63.. < 0 < 2.014..) and conclusion.	A1
			(2)
(b)	$\mathrm{f}(x) = x^3 - \frac{5}{2}x^{-\frac{3}{2}} + 2x - 3$ $\Rightarrow \mathrm{f}'(x) = 3x^2 + \frac{15}{4}x^{-\frac{5}{2}} + 2$	M1: $x^n \to x^{n-1}$ for at least one term A1:Correct derivative oe	M1A1
			(2)
(c)	$\mathrm{f}'(1.1) = 3(1.1)^2 + \frac{15}{4}(1.1)^{-\frac{5}{2}} + 2(=8.585)$	Attempt to find $\mathrm{f}'(1.1)$. Accept $\mathrm{f}'(1.1)$ seen and their value.	M1
	$\alpha_2 = 1.1 - \left(\dfrac{\text{"}-1.6359604\text{"}}{\text{"}8.585\text{"}}\right)$	Correct application of N-R	M1
	$\alpha_2 = 1.291$	cao	A1
			(3)
			Total 7

Question Number	Scheme	Notes	Marks
3.	$x^3 + px^2 + 30x + q = 0$		
(a)	$1+5i$		B1
			(1)
(b)	$\left(\left(x-(1+5i)\right)\left(x-(1-5i)\right)\right) = x^2 - 2x + 26$ $\left((x-2)\left(x-(1\pm 5i)\right)\right) = x^2 - (3\pm 5i)x + 2(1\pm 5i)$	M1: 1. Attempt to expand or 2. Use sum and product of the complex roots. A1: Correct expression	M1A1
	$\left(x^2 - 2x + 26\right)\left(x-2\right) = x^3 + px^2 + 30x + q$	Uses their third factor with their quadratic with at least 4 terms in the expansion	M1
	$p = -4, \qquad q = -52$	May be seen in cubic	A1, A1
OR	f(1+5i)=0 or f(1-5i)=0	Substitute one complex root to achieve 2 equations in *p* and / or q	M1
	$q - 24p - 44 = 0$ and $10p + 40 = 0$	Both equations correct oe	A1
		Solving for *p* and *q*	M1
	$p = -4, \quad q = -52$	May be seen in cubic	A1, A1
			(5)
(c)	 	B1: Conjugate pair correctly positioned and labelled with 1+5i, 1-5i or (1,5),(1,-5) or axes labelled 1 and 5. B1: The 2 correctly positioned relative to conjugate pair and labelled.	B1 B1
			(2) **Total 8**

Question Number	Scheme	Notes	Marks
4.	$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix}$		
(i)(a)	$\begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 5 & 6 \\ 5 & -6 & 11 \\ 13 & 11 & 21 \end{pmatrix}$	M1: 3x3 matrix with a number or numerical expression for each element A2:cao (-1 each error) Only 1 error award A1A0	M1A2
(b)	$\mathbf{BA} = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 15 & 25 \\ 14 & 4 \end{pmatrix}$	Allow any convincing argument. E.g.s BA is a 2x2 matrix (so AB ≠ BA) or dimensionally different. Attempt to evaluate product not required. NB ‘Not commutative’ only is B0	B1
			(4)
(ii)	$(\det \mathbf{C} =) 2k \times k - 3 \times (-2)$	Correct attempt at determinant	M1
	$\mathbf{C}^{-1} = \dfrac{1}{2k^2+6}\begin{pmatrix} k & 2 \\ -3 & 2k \end{pmatrix}$	M1: $\dfrac{\mathbf{1}}{\text{their}\det\mathbf{C}}\begin{pmatrix} k & 2 \\ -3 & 2k \end{pmatrix}$ A1:cao oe	M1A1
			(3)
			Total 7

Question Number	Scheme	Notes	Marks
5.(a)	$\left(\left(2r-1\right)^2=\right)4r^2-4r+1$		B1
	Proof by induction will usually score no more marks without use of standard results		
	$\sum_{r=1}^{n}\left(2r-1\right)^2=\sum_{r=1}^{n}\left(4r^2-4r+1\right)$		
	$=4\sum r^2-4\sum r+\sum 1$		
	$=\underline{4.\frac{1}{6}n\left(n+1\right)\left(2n+1\right)-4.\frac{1}{2}n\left(n+1\right)},+n$	M1: An attempt to use at least one of the standard results correctly in summing at least 2 terms of their expansion of $\left(2r-1\right)^2$ A1: Correct underlined expression oe B1: $\sum 1=n$	M1A1B1
	$=\frac{1}{3}n\left[4n^2+6n+2-6n-6+3\right]$	Attempt to factor out $\frac{1}{3}n$ before given answer	M1
	$=\frac{1}{3}n\left[4n^2-1\right]$	cso	A1
			(6)
(b)	$\sum_{r=2n+1}^{4n}\left(2r-1\right)^2=\mathrm{f}\left(4n\right)-\mathrm{f}\left(2n\right)\text{ or }\mathrm{f}\left(2n+1\right)$	Require some use of the result in part (a) for method.	M1
	$=\frac{1}{3}4n\left(4.\left(4n\right)^2-1\right)-\frac{1}{3}.2n\left(4.\left(2n\right)^2-1\right)$	Correct expression	A1
	$=\frac{2}{3}n\left[128n^2-2-16n^2+1\right]$		
	$=\frac{2}{3}n\left[112n^2-1\right]$	Accept $a=\frac{2}{3},b=112$	A1
			(3)
			Total 9

Question Number	Scheme	Notes	Marks
6.	$xy = c^2$ at $\left(ct, \frac{c}{t}\right)$.		
(a)	$y = \dfrac{c^2}{x} = c^2x^{-1} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = -c^2x^{-2} = -\dfrac{c^2}{x^2}$	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = k\,x^{-2}$	
	$xy = c^2 \Rightarrow x\dfrac{\mathrm{d}y}{\mathrm{d}x} + y = 0$	Correct use of product rule. The sum of two terms, one of which is correct **and** rhs = 0	M1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{dy}{dt}.\dfrac{dt}{dx} = -\dfrac{c}{t^2}.\dfrac{1}{c}$	their $\dfrac{dy}{dt} \times \left(\dfrac{1}{\text{their } \dfrac{dx}{dt}}\right)$	
	$\dfrac{dy}{dx} = -c^2x^{-2}$ or $x\dfrac{dy}{dx} + y = 0$ or $\dfrac{dy}{dx} = \dfrac{-c}{t^2}.\dfrac{1}{c}$ or equivalent expressions	Correct differentiation	A1
	$y - \dfrac{c}{t} = -\dfrac{1}{t^2}(x - ct) \qquad (\times t^2)$	$y - \dfrac{c}{t} =$ their $m_T(x - ct)$ or $y = mx + c$ with their m_T and $(ct, \dfrac{c}{t})$ in an attempt to find 'c'. **Their m_T must have come from calculus and should be a function of *t* or *c* or both *c* and *t*.**	dM1
	$t^2y + x = 2ct$ (Allow $x + t^2y = 2ct$)	Correct solution only.	A1*
			(4)
	(a) Candidates who derive $x + t^2y = 2ct$, by stating that $m_T = -\dfrac{1}{t^2}$, with no justification score <u>no</u> marks in (a).		
(b)	$y = 0 \Rightarrow x = \dfrac{ct^4 - c}{t^3} \Rightarrow A\left(\dfrac{ct^4 - c}{t^3}, 0\right)$	$\dfrac{ct^4 - c}{t^3}$ or equivalent form	B1
	$y = 0 \Rightarrow x = 2ct \Rightarrow B(2ct, 0)$.	$2ct$	B1
			(2)
(c)	$\text{AB} = \text{"}2ct\text{"} - \text{"}\dfrac{ct^4 - c}{t^3}\text{"}$ **or** $\text{PA} = ct^{-3}\sqrt{t^4 + 1}$ and $\text{PB} = ct^{-1}\sqrt{t^4 + 1}$	Attempt to subtract their *x*-coordinates either way around.	M1
	Area APB $= \dfrac{1}{2} \times \textit{their AB} \times \dfrac{c}{t}$	Valid complete method for the area of the triangle in terms of *t* or *c* and *t*.	M1
	$= \dfrac{1}{2}\left(2ct - \dfrac{ct^4 - c}{t^3}\right)\dfrac{c}{t} = \dfrac{c^2\left(t^4 + 1\right)}{2t^4}$		
	$= 8\left(1 + \dfrac{1}{t^4}\right)$ or $\dfrac{8\left(t^4 + 1\right)}{t^4}$ or $\dfrac{8t^4 + 8}{t^4}$ or $8 + \dfrac{8}{t^4}$	Use of $c = 4$ and completes to one of the given forms oe simplest form. Final answer should be positive for A mark.	A1
			(3)
			Total 9

Question Number	Scheme	Notes	Marks
7.(i)(a)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$		B1
(b)	$\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$		B1
(c)	$\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$	M1: Multiplies their (b) x their (a) in the correct order A1: Correct matrix Correct matrix seen M1A1	M1A1
			(4)
(ii)	Area triangle T = $\frac{1}{2}\times(11-3)\times k = 4k$	M1: Correct method for area for T A1: 4k	M1A1
	$\det\begin{pmatrix} 6 & -2 \\ 1 & 2 \end{pmatrix} = 6\times 2 - 1\times(-2)(=14)$	M1: Correct method for determinant A1: 14	M1A1
	Area triangle $T = \frac{364}{"14"}(=26) \Rightarrow 4k = 26$	Uses 364 and their determinant correctly to form an equation in k.	M1
	$k = \frac{26}{4}\left(=\frac{13}{2}\right)$	Accept $k = \pm\frac{13}{2}$ or $k = -\frac{13}{2}$	A1
			(6)
			Total 10

Question Number	Scheme	Notes	Marks
8.(a)	$m=\dfrac{4k-8k}{k^2-4k^2}\left(=\dfrac{4}{3k}\right)$	Valid attempt to find gradient in terms of k	M1
	$y-8k=\dfrac{4}{3k}\left(x-4k^2\right)$ or $y-4k=\dfrac{4}{3k}\left(x-k^2\right)$ or $y=\dfrac{4}{3k}x+\dfrac{8k}{3}$	M1: Correct straight line method with their gradient in terms of k. If using $y=mx+c$ then award M provided they attempt to find c A1: Correct equation. If using $y=mx+c$, awardwhen they obtain $c=\dfrac{8k}{3}$ oe	M1A1
	$3ky-24k^2=4x-16k^2\Rightarrow 3ky-4x=8k^2$ * or $3ky-12k^2=4x-4k^2\Rightarrow 3ky-4x=8k^2$ *	Correct completion to printed answer with at least one intermediate step.	A1*
			(4)
(b)	(Focus) (4, 0)	Seen or implied as a number	B1
	(Directrix) x = -4	Seen or implied as a number	B1
	Gradient of l_2 is $-\dfrac{3k}{4}$	Attempt negative reciprocal of grad l_1 as a function of k	M1
	$y-0=\dfrac{-3k}{4}(x-4)$	Use of their changed gradient and numerical Focus in either formula, as printed oe	M1, A1
	$x=-4\Rightarrow y=\dfrac{-3k}{4}(-4-4)$	Substitute numerical directrix into their line	M1
	$(y=)6k$	oe	A1
			(7)
			Total 11

Question Number	Scheme	Notes	Marks
9.	$f(n) = 8^n - 2^n$ is divisible by 6.		
	$f(1) = 8^1 - 2^1 = 6$,	Shows that $f(1) = 6$	B1
	Assume that for $n = k$, $f(k) = 8^k - 2^k$ is divisible by 6.		
	$f(k+1) - f(k) = 8^{k+1} - 2^{k+1} - \left(8^k - 2^k\right)$	Attempt $f(k+1) - f(k)$	M1
	$= 8^k(8-1) + 2^k(1-2) = 7\times 8^k - 2^k$		
	$= 6\times 8^k + 8^k - 2^k \left(= 6\times 8^k + f(k)\right)$	M1: Attempt $f(k+1) - f(k)$ as a multiple of 6 A1: rhs a correct multiple of 6	M1A1
	$f(k+1) = 6\times 8^k + 2f(k)$	Completes to $f(k+1)$ = a multiple of 6	A1
	If the result is **true for** $n = k$, then it is now **true for** $n = k+1$. As the result has been shown to be **true for** $n = 1$, then the result is **true for all** n $(\in \mathbb{Z}^+.)$		A1cso
		Do not award final A if n defined incorrctly e.g. 'n is an integer' award A0	
			(6)
			Total 6
Way 2	$f(1) = 8^1 - 2^1 = 6$,	Shows that $f(1) = 6$	B1
	Assume that for $n = k$, $f(k) = 8^k - 2^k$ is divisible by 6.		
	$f(k+1) = 8^{k+1} - 2^{k+1} = 8\left(8^k - 2^k + 2^k\right) - 2.2^k$	Attempts $f(k+1)$ in terms of 2^k and 8^k	M1
	$f(k+1) = 8^{k+1} - 2^{k+1} = 8\left(f(k) + 2^k\right) - 2.2^k$	M1:Attempts $f(k+1)$ in terms of $f(k)$ A1: rhs correct and a multiple of 6	M1A1
	$f(k+1) = 8f(k) + 6.2^k$	Completes to $f(k+1)$ = a multiple of 6	A1
	If the result is **true for** $n = k$, then it is now **true for** $n = k+1$. As the result has been shown to be **true for** $n = 1$, then the result is **true for all** n $(\in \mathbb{Z}^+.)$		A1cso
Way 3	$f(1) = 8^1 - 2^1 = 6$,	Shows that $f(1) = 6$	B1
	Assume that for $n = k$, $f(k) = 8^k - 2^k$ is divisible by 6.		
	$f(k+1) - 8f(k) = 8^{k+1} - 2^{k+1} - 8.8^k + 8.2^k$	Attempt $f(k+1) - 8f(k)$	M1
		Any multiple m replacing 8 award M1	
	$f(k+1) - 8f(k) = 8^{k+1} - 8^{k+1} + 8.2^k - 2.2^k = 6.2^k$	M1: Attempt $f(k+1) - f(k)$ as a multiple of 6 A1: rhs a correct multiple of 6	M1A1
	$f(k+1) = 8f(k) + 6.2^k$	Completes to $f(k+1)$ = a multiple of 6	A1
		General Form for multiple m $f(k+1) = 6.8^k + (2-m)(8^k - 2^k)$	
	If the result is **true for** $n = k$, then it is now **true for** $n = k+1$. As the result has been shown to be **true for** $n = 1$, then the result is **true for all** n $(\in \mathbb{Z}^+.)$		A1cso

Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Further Pure Mathematics FP1R (6667/01R)

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Summer 2014
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎣ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme		Marks
1.	$\mathrm{f}(z) = 2z^3 - 3z^2 + 8z + 5$		
	$1 - 2\mathrm{i}$ (is also a root)	seen	B1
	$(z-(1+2\mathrm{i}))(z-(1-2\mathrm{i})) = z^2 - 2z + 5$	Attempt to expand $(z-(1+2\mathrm{i}))(z-(1-2\mathrm{i}))$ or any valid method to establish the quadratic factor e.g. $z = 1 \pm 2\mathrm{i} \Rightarrow z - 1 = \pm 2\mathrm{i} \Rightarrow z^2 - 2z + 1 = -4$ $z = 1 \pm \sqrt{-4} = \dfrac{2 \pm \sqrt{-16}}{2} \Rightarrow b = -2, c = 5$ Sum of roots 2, product of roots 5 $\therefore z^2 - 2z + 5$	M1A1
	$\mathrm{f}(z) = \left(z^2 - 2z + 5\right)\left(2z + 1\right)$	Attempt at **linear** factor with their *cd* in $(z^2 + az + c)(2z + d) = \pm 5$ Or $(z^2 - 2z + 5)(2z + a) \Rightarrow 5a = 5$	M1
	$(z_3) = -\dfrac{1}{2}$		A1
			(5)
			Total 5

Question Number	Scheme		Marks
2.	$f(x) = 3\cos 2x + x - 2$		
(a)	f(2) = -1.9609...... f(3) = 3.8805......	Attempts to evaluate both $f(2)$ and $f(3)$ and evaluates at least one of them correctly to awrt (or trunc.) 2 sf.	M1
	Sign change (and $f(x)$ is continuous) therefore a **root** α is between $x = 2$ and $x = 3$	Both values correct to awrt (or trunc.) 2 sf, sign change (or a statement which implies this e.g. -1.96.. < 0 < 3.88..) and conclusion.	A1
			(2)
(b)	$\frac{\alpha - 2}{"1.9609..."} = \frac{3 - \alpha}{"3.8805..."}$	Correct linear interpolation method. It must be a <u>correct statement</u> using their f(2) and f(3). Can be implied by working below.	M1
	If any "negative lengths" are used, score M0		
	$(3.88...+1.96....)\alpha = 3\times 1.96 + 2\times 3.88$		
	$\alpha_2 = \frac{3\times 1.96.. + 2\times 3.88..}{1.96... + 3.88...}$	Follow through their values if seen explicitly.	A1ft
	$\alpha_2 = 2.336$	cao	A1
			(3)
(c)	f(0) = +(1) **or** f(-1) = -(4.248)	Award for correct sign, can be in a table.	B1
	f(-0.5) (= -0.879.....)	Attempt f(-0.5)	M1
	f(-0.25) (= 0.382....)	Attempt **f(-0.25)**	M1
	$\therefore\ -0.5 < \beta < -0.25$	oe with no numerical errors seen	A1
			(4)
			Total 9

Question Number	Scheme		Marks
3.(i)(a)	Rotation of 45 degrees anticlockwise, about the origin	B1: Rotation about (0, 0) B1: 45 degrees (anticlockwise) -45 or clockwise award B0	B1B1
			(2)
(b)	$\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$	Correct matrix	B1
			(1)
(ii)	$\frac{224}{16}(=14)$	Correct area scale factor. Allow ± 14	B1
	$\det \mathbf{M} = 3\times 3 - k \times -2 = 14$	Attempt determinant and set equal to their area scale factor	M1
		Accept $\det \mathbf{M} = 3\times 3 \pm 2k$ only	
	$k = 2.5$	oe	A1
			(3)
			Total 6

Question Number	Scheme		Marks
4.(a)	$z=\frac{p+2\mathrm{i}}{3+p\mathrm{i}}.\frac{3-p\mathrm{i}}{3-p\mathrm{i}}$	Multiplying top and bottom by Conjugate	M1
	$=\frac{3p-p^2\mathrm{i}+6\mathrm{i}+2p}{9+p^2}$	At least 3 correct terms in the numerator, evidence that $\mathrm{i}^2=-1$and denominator real.	M1
	$=\frac{5p}{p^2+9}, \qquad +\frac{6-p^2}{p^2+9}\mathrm{i}$	Real + imaginary with i factored out. Accept single denominator with numerator in correct form. Accept 'a= ' and 'b='.	A1, A1
			(4)
(b)	$\arg(z)=\arctan\left(\frac{\frac{6-p^2}{p^2+9}}{\frac{5p}{p^2+9}}\right)$	Correct method for the argument. Can be implied by correct equation for p	M1
	$\frac{6-p^2}{5p}=1$	Their $\arg(z)$ in terms of $p=1$	M1
	$p^2+5p-6=0$	Correct 3TQ	A1
	$(p+6)(p-1)=0\Rightarrow x=$	M1:Attempt to solve their quadratic in p	M1
	$p=1, p=-6$	A1:**both**	A1
			(5)
			Total 9
(a) **Way 2**	$a+b\mathrm{i}=\frac{p+2\mathrm{i}}{3+p\mathrm{i}}$	Equate to $a+b\mathrm{i}$ then rearrange and equate real and imaginary parts.	M1
	$3a-pb=p,\ ap+3b=2$	Two equations for a and b in terms of p and attempt to solve for a and b in terms of p	dM1
	$=\frac{5p}{p^2+9}, \qquad +\frac{6-p^2}{p^2+9}\mathrm{i}$	Real + imaginary with i factored out. Accept single denominator with numerator in correct form. Accept 'a= ' and 'b='.	A1, A1
			(5)

Question Number	Scheme		Marks
5.(a)	$r\left(r^2-3\right)=r^3-3r$	r^3-3r	B1
	$\sum_{r=1}^{n} r\left(r^2-3\right)=\sum_{r=1}^{n} r^3-3\sum_{r=1}^{n} r$		
	$=\frac{1}{4}n^2(n+1)^2-\frac{3}{2}n(n+1)$	M1: An attempt to use at least one of the standard formulae correctly. A1: Correct expression	M1A1
	$=\frac{1}{4}n(n+1)\left(n(n+1)-6\right)$	Attempt factor of $\frac{1}{4}n(n+1)$ before given answer	M1
	$=\frac{1}{4}n(n+1)\left(n^2+n-6\right)$		
	$=\frac{1}{4}n(n+1)(n+3)(n-2)$	cso	A1
			(5)
(b)	$\sum_{r=10}^{50} r\left(r^2-3\right)=\mathrm{f}(50)-\mathrm{f}(9\,or\,10)$	Require some use of the result in part (a) for method.	M1
	$=\frac{1}{4}(50)(51)(53)(48)-\frac{1}{4}(9)(10)(12)(7)$	Correct expression	A1
	= 1621800 - 1890		
	= 1619910	cao	A1
			(3)
			Total 8

Question Number	Scheme		Marks
6.(a)	$\mathbf{A}+\mathbf{B}=\begin{pmatrix}1 & 2\\ -1 & 1\end{pmatrix}$	M1:Correct attempt at matrix addition with 3 elements correct A1: Correct matrix	M1A1
	$\mathbf{2A}-\mathbf{B}=\begin{pmatrix}5 & 1\\ -2 & -1\end{pmatrix}$	M1: Correct attempt to double **A** and subtract **B** 3 elements correct A1: Correct matrix	M1A1
	$(\mathbf{A}+\mathbf{B})(\mathbf{2A}-\mathbf{B})=\begin{pmatrix}1 & 2\\ -1 & 1\end{pmatrix}\begin{pmatrix}5 & 1\\ -2 & -1\end{pmatrix}$		
	$\begin{pmatrix}1 & 2\\ -1 & 1\end{pmatrix}\begin{pmatrix}5 & 1\\ -2 & -1\end{pmatrix}=\begin{pmatrix}1 & -1\\ -7 & -2\end{pmatrix}$	M1: Correct method to multiply A1: cao	M1A1
			(6)
(a) Way 2	$(\mathbf{A}+\mathbf{B})(\mathbf{2A}-\mathbf{B})=2\mathbf{A}^2+2\mathbf{BA}-\mathbf{AB}-\mathbf{B}^2$	M1: Expands brackets with at least 3 correct terms A1: Correct expansion	M1A1
	$\mathbf{A}^2=\begin{pmatrix}3 & 2\\ -2 & -1\end{pmatrix},\mathbf{BA}=\begin{pmatrix}-3 & -1\\ -1 & 0\end{pmatrix},$ $\mathbf{AB}=\begin{pmatrix}-2 & 3\\ 1 & -1\end{pmatrix},\mathbf{B}^2=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$	M1: Attempts $\mathbf{A}^2$, $\mathbf{B}^2$ and **AB** or **BA** A1: Correct matrices	M1A1
	$2\mathbf{A}^2+2\mathbf{BA}-\mathbf{AB}-\mathbf{B}^2=\begin{pmatrix}1 & -1\\ -7 & -2\end{pmatrix}$	M1: Substitutes into their expansion A1: Correct matrix	M1A1
(b)	$\mathbf{MC}=\mathbf{A}\Rightarrow\mathbf{C}=\mathbf{M}^{-1}\mathbf{A}$	May be implied by later work	B1
	$\mathbf{M}^{-1}=\frac{1}{-2-7}\begin{pmatrix}-2 & 1\\ 7 & 1\end{pmatrix}$	An attempt at their $\frac{1}{\det \mathbf{M}}\begin{pmatrix}-2 & 1\\ 7 & 1\end{pmatrix}$	M1
	$\mathbf{C}=\frac{1}{-2-7}\begin{pmatrix}-2 & 1\\ 7 & 1\end{pmatrix}\begin{pmatrix}2 & 1\\ -1 & 0\end{pmatrix}$	Correct order required and an attempt to multiply	dM1
	$\mathbf{C}=-\frac{1}{9}\begin{pmatrix}-5 & -2\\ 13 & 7\end{pmatrix}$	oe	A1
			(4)
			Total 10
(b) Way 2	$\begin{pmatrix}1 & -1\\ -7 & -2\end{pmatrix}\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\begin{pmatrix}2 & 1\\ -1 & 0\end{pmatrix}$	Correct statement	B1
	$a-c=2,\ b-d=1$ $-7a-2c=-1,\ -7b-2d=0$	Multiplies correctly to obtain 4 equations	M1
	$a=\frac{5}{9},b=\frac{2}{9},c=-\frac{13}{9},d=-\frac{7}{9}$	M1: Solves to obtain values for a, b, c and d A1: Correct values	M1A1

Question Number	Scheme		Marks
7.(a)	$y=2a^{\frac{1}{2}}x^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=a^{\frac{1}{2}}x^{-\frac{1}{2}}$ or $y^2=4ax \Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x}=4a$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}p}.\frac{\mathrm{d}p}{\mathrm{d}x}=2a.\frac{1}{2ap}$	$\frac{\mathrm{d}y}{\mathrm{d}x}=kx^{-\frac{1}{2}}$ or $ky\frac{\mathrm{d}y}{\mathrm{d}x}=c$ their $\frac{\mathrm{d}y}{\mathrm{d}p}\times\left(\frac{1}{\text{their }\frac{\mathrm{d}x}{\mathrm{d}p}}\right)$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=a^{\frac{1}{2}}x^{-\frac{1}{2}}$ *or* $2y\frac{\mathrm{d}y}{\mathrm{d}x}=4a$ *or* $\frac{\mathrm{d}y}{\mathrm{d}x}=2a.\frac{1}{2ap}$	Correct differentiation	A1
	At *P*, gradient of normal = -*p*	Correct normal gradient with no errors seen.	A1
	$y-2ap=-p(x-ap^2)$	Applies $y-2ap=\text{their } m_N\left(x-ap^2\right)$ or $y=\left(\text{their } m_N\right)x+c$ using $x=ap^2$ *and* $y=2ap$ in an attempt to find c. **Their m_N must be different from their m_T and must be a function of *p*.**	M1
	$y+px=2ap+ap^3$ *	cso **given answer**	A1*
			(5)
(b)	$y-px=-2ap-ap^3$	oe	B1
			(1)
(c)	$y=0 \Rightarrow x=2a+ap^2$	M1: y = 0 in either normal or solves simultaneously to find x A1: y = 0 and correct x coordinate.	M1A1
			(2)
(d)	*S* is (a, 0)	Can be implied below	B1
	Area *SPQP'* = $\frac{1}{2}\times\left(\text{"}2a+ap^2\text{"}-a\right)\times 2ap\times 2$	Correct method for the area of the quadrilateral.	M1
	$=2a^2p\left(1+p^2\right)$	Any equivalent form	A1
			(3)
			Total 11

Question Number	Scheme		Marks
8.			
	$\frac{12}{7}c = -\frac{1}{t^2} \times -\frac{6}{7}c + \frac{2c}{t}$	Substitutes $\left(-\frac{6}{7}c,\ \frac{12}{7}c\right)$ into the equation of the tangent	M1
	$\frac{12}{7}c = -\frac{1}{t^2} \times -\frac{6}{7}c + \frac{2c}{t} \Rightarrow$ $6t^2 - 7t - 3 = 0$	Correct 3TQ in terms of t	A1
	$6t^2 - 7t - 3 = 0 \Rightarrow (3t+1)(2t-3) = 0 \Rightarrow t =$	Attempt to solve their 3TQ for t	M1
	$t = -\frac{1}{3},\ t = \frac{3}{2} \Rightarrow \left(-\frac{1}{3}c,\ -3c\right),\ \left(\frac{3}{2}c,\ \frac{2}{3}c\right)$	M1: Uses at least one of their values of t to find A or B. A1: Correct coordinates.	M1A1
			(5)
			Total 5

Question Number	Scheme		Marks
9.(a)	When n = 1, rhs = lhs = 2		B1
	Assume true for $n = k$ so $\sum_{r=1}^{k}(r+1)2^{r-1} = k2^k$		
	$\sum_{r=1}^{k+1}(r+1)2^{r-1} = k2^k + (k+1+1)2^{k+1-1}$	M1: Attempt to add (k + 1)$^{\text{th}}$ term A1: Correct expression	M1A1
	$= k2^k + (k+2)2^k$		
	$= 2\times k2^k + 2\times 2^k$		
	$= (k+1)2^{k+1}$	At least one correct intermediate step required.	A1
	If the result is **true for *n* = *k*** then it has been shown **true for *n* = *k* + 1.** As it is **true for *n* = 1** then it is **true for all *n*** (positive integers.)	cso, statements can be seen anywhere in the solution.	A1
		Do not award final A if *n* defined incorrectly e.g. '*n* is an integer' award A0	
			(5)
(b)	When $n = 1$ $u_1 = 4^2 - 2^4 = 0$	$4^2 - 2^4 = 0$ seen	B1
	When $n = 2$ $u_2 = 4^3 - 2^5 = 32$	$4^3 - 2^5 = 32$ seen	B1
	True for $n = 1$ and $n = 2$		
	Assume $u_k = 4^{k+1} - 2^{k+3}$ and $u_{k+1} = 4^{k+2} - 2^{k+4}$		
	$u_{k+2} = 6u_{k+1} - 8u_k$ $= 6(4^{k+2} - 2^{k+4}) - 8(4^{k+1} - 2^{k+3})$	M1: Attempts u_{k+2} in terms of u_{k+1} and u_k A1: Correct expression	M1A1
	$= 6.4^{k+2} - 6.2^{k+4} - 8.4^{k+1} + 8.2^{k+3}$		
	$= 6.4^{k+2} - 3.2^{k+5} - 2.4^{k+2} + 2.2^{k+5}$	Attempt u_{k+2} in terms of 4^{k+2} and 2^{k+5}	M1
	$= 4.4^{k+2} - 2^{k+5} = 4^{k+3} - 2^{k+5}$		
	So $u_{k+2} = 4^{(k+2)+1} - 2^{(k+2)+3}$	Correct expression	A1
	If the result **is true for *n* = *k*** and ***n* = *k* + 1** then it has been shown **true for *n* = *k* + 2**. As it is **true for *n* = 1 and *n* = 2** then it is **true for all *n*** (positive integers.)	cso, statements can be seen anywhere in the solution.	A1
		Do not award final A if *n* defined incorrectly e.g. '*n* is an integer' award A0	
			(7)
			Total 12