FP1 Specimen

1) $f(x) = x^3 - 3x^2 + 5x - 4$

a) $f'(x) = 3x^2 - 6x + 5$

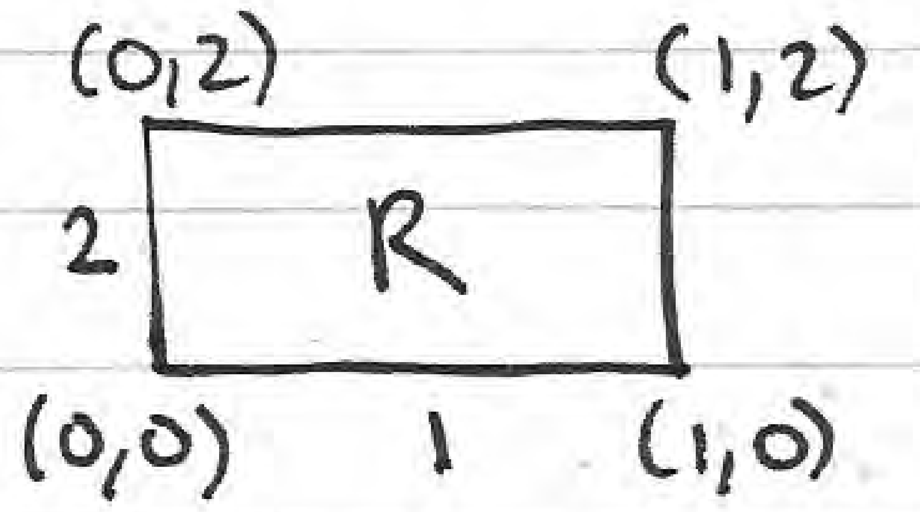
$x_0 = 1.4 \quad x_1 = 1.4 - \dfrac{f(1.4)}{f'(1.4)} = \underline{1.455} \ (3dp)$

2) $AR = \begin{pmatrix} a & 4 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 0 & a & a+8 & 8 \\ 0 & -1 & 1 & 2 \end{pmatrix}$

$(0,0)\,;(a,-1)\,;(a+8,1)\,;(8,2)$

b) $\det A = a + 4$

c)



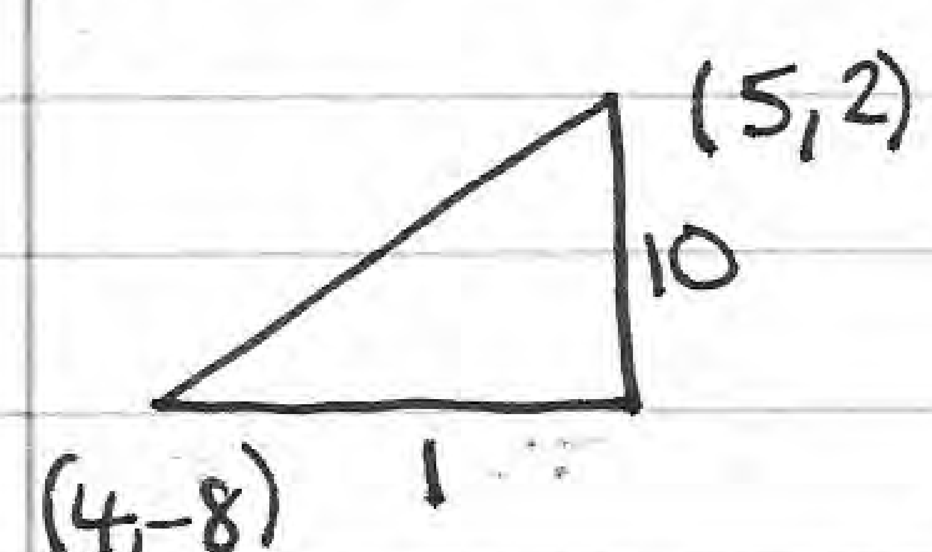
Area $R = 2$ Area Image $= 18$

$\Rightarrow \det A = 9 \Rightarrow 9 = a + 4 \Rightarrow \underline{a = 5}$

3) $R^2 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ rotation $90°$ clockwise about origin.

c) $R =$ rotation $45°$ clockwise about origin.

4) $f(x) = 2^x - 6x \quad f(4) = -8 \quad f(5) = 2$



$m = \frac{10}{1} \quad y - 2 = 10(x - 5)$

$y = 0 \Rightarrow -\frac{1}{5} = x - 5 \Rightarrow \underline{x = 4\frac{4}{5}}$

5) $\sum r^2 - \sum r - \sum 1 = \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1) - n$

$= \frac{1}{6}n\left[(n+1)(2n+1) - 3(n+1) - 6\right] = \frac{1}{6}n\left[2n^2 + 3n + 1 - 3n - 3 - 6\right]$

$= \frac{1}{6}n(2n^2-8) = \frac{1}{3}n(n^2-4) = \frac{1}{3}n(n+2)(n-2)$ #

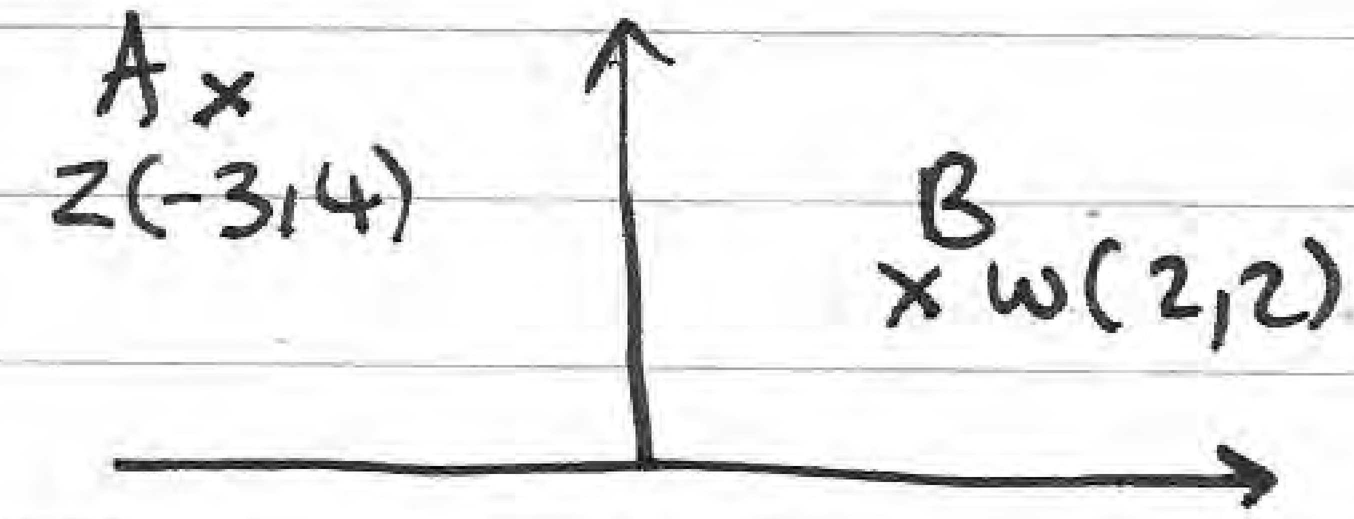
b) $\sum_{10}^{40} r^2-r-1 = \frac{1}{3}(40)(42)(38) - \frac{1}{3}(9)(11)(7)$
$= \underline{21049}$

6) $z=-3+4i \qquad |z| = \sqrt{3^2+4^2} = 5$

b) $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{4}{-3} \quad \theta = \tan^{-1}\left(\frac{4}{-3}\right) = -0.927$
$+\pi$
$\arg(z) = 2.21$

c) $w = \frac{(-14+2i)\times(-3-4i)}{(-3+4i)\times(-3-4i)} = \frac{42-8i^2+56i-6i}{9-16i^2} = \frac{50+50i}{25}$

$w = \underline{2+2i}$ d)



7) $y^2=4ax \quad (4t^2, 8t) \Rightarrow 64t^2 = 4a(4t^2)$
$\Rightarrow 64t^2 = a\times 16t^2 \Rightarrow \underline{a=4}$

b) $\frac{d}{dx}y^2 = \frac{d}{dx}16x \Rightarrow 2y\frac{dy}{dx} = 16 \Rightarrow \frac{dy}{dx} = \frac{8}{y} = \frac{8}{8t} \Rightarrow m_t = \frac{1}{t}$

$y-8t = \frac{1}{t}(x-4t^2) \Rightarrow yt - 8t^2 = x-4t^2 \Rightarrow yt = x+4t^2$ #

c) directrix $x+4=0 \quad (-4, 15)$

$15t = -4+4t^2 \Rightarrow 4t^2-15t-4=0 \quad (4t+1)(t-4)=0$
$t=-\frac{1}{4} \quad t=4$

$t=4 \quad (64, 32) \qquad t=-\frac{1}{4} \quad \left(\frac{1}{4}, -2\right)$

8) $f(x)=2x^3-5x^2+px-5$

a) $1-2i \Rightarrow$ other solution is $1+2i$ $\quad \alpha+\beta=2$

$\alpha\beta=1-4i^2=5$

b) $(x^2-2x+5)(2x-1)=0 \Rightarrow x=1-2i, 1+2i, \frac{1}{2}$

c) $10x+2x=12x\equiv px \Rightarrow p=12$

9) $\begin{pmatrix}2&1\\-1&0\end{pmatrix}^n=\begin{pmatrix}n+1&n\\-n&1-n\end{pmatrix}$

$n=1$ $\begin{pmatrix}2&1\\-1&0\end{pmatrix}^1=\begin{pmatrix}2&1\\-1&0\end{pmatrix}$ $\quad n=1$ $\begin{pmatrix}1+1&1\\-1&1-1\end{pmatrix}=\begin{pmatrix}2&1\\-1&0\end{pmatrix}$ ✓

$n=k+1 = \begin{pmatrix}2&1\\-1&0\end{pmatrix}^{k+1}=\begin{pmatrix}2&1\\-1&0\end{pmatrix}\begin{pmatrix}2&1\\-1&0\end{pmatrix}^k$

$=\begin{pmatrix}2&1\\-1&0\end{pmatrix}\begin{pmatrix}k+1&k\\-k&1-k\end{pmatrix}=\begin{pmatrix}2k+2-k&2k+1-k\\-k-1&-k\end{pmatrix}=\begin{pmatrix}k+2&k+1\\-(k+1)&-k\end{pmatrix}$

#

true for $n=1$, $n=k+1$ if true for $n=k$

$\therefore$ by induction true for all $n\in\mathbb{Z}^+$

b) $f(n)=4^n+6n-1$ $\quad n=1$ $f(1)=4+6-1=9=3\times3$ ✓

$n=k+1 \Rightarrow f(k+1)=4^{k+1}+6(k+1)-1=4\times4^k+6k+5$

$f(k+1)-f(k)=4\times4^k+6k+5-4^k-6k+1=3\times4^k+6$

$\Rightarrow f(k+1)=3[4^k+2]+f(k)$

true for $n=1$, true for $n=k+1$ if true for $n=k$ $\therefore$ by induction true for all $n\in\mathbb{Z}^+$